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Solving FQP Problem as a FLCP by Proposed Method

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ABSTRACT

This paper presents a new method to solve fuzzy quadratic programming problems(FQPP), where the cost co-efficient and the right hand side co-efficient are represented by triangular fuzzy numbers. The fuzzy number quadratic programming problem (FQPP) is transformed to fuzzy triangular number linear Complementarity problem (FLCP). Furthermore, new operations on triangular fuzzy number are used. Finally, A Numerical example is given to illustrate the efficiency of the proposed method.
Keywords : Function Principle,Fuzzy Quadratic Programming Problem, Fuzzy Linear Complementarity Problem, New operations on triangular fuzzy numbers, AMS Subject Classification: 65K05, 90C90, 90C70, 90C30.

I. INTRODUCTION

Many practical problems cannot be represented by linear programming model. Therefore, attempts were made to develop more general mathematical programming methods and many significant advances have been made in the area of nonlinear programming. The first major development was the fundamental paper by Kuhn-Tucker in 1951[3] which laid the foundations for a good deal of later work in nonlinear programming. The linear complementarity problem (LCP) is a well known problem in mathematical programming and it has been studied by many researchers. In 1968, Lemke [3] proposed a complementarity pivoting algorithm for solving linear complementarity problems. Since, the KKT conditions for quadratic programming problems can be written as a LCP, Lemke's algorithm can be used to solve quadratic programs.

Since then the study of complementarity problems has been expanded enormously. Also, iterative methods developed for solving LCPs hold great promise for handling very large scale linear programs which cannot be tackled with the well known simplex method because of their large size and the consequent numerical difficulties. In a recent review, Pankaj Gupta et al [4] gave a fuzzy approximation to an infeasible generalized linear complementarity problem.

This paper provides a new technique for solving fuzzy quadratic programming problem by converting it into a fuzzy linear complementarity problem. Also this paper provides a new method of carrying out the fuzzy complementary pivot algorithm without introducing artificial variables, under certain conditions.

This paper is organized as follows: Section 2 introduces triangular fuzzy numbers and a new operation on triangular fuzzy number for solving Fuzzy Linear complementarity problem. Fuzzy linear complementarity problem & conversion of FQPP into FLCP is described in section 3, Section 4 deals with Algorithmic Approach for LCP and section 5, the effectiveness of the proposed method is illustrated by means of an example. Finally, section 6 contains some concluding remarks.

II. PRELIMINARIES

2.1 Fuzzy set

A Fuzzy set Ã is defined by Ã = {x,μ_A(x)}; x€A, μ_A(x)€[0,1]. In the pair (x, μ_A(x)),the first element x belong to the classical set A,the second element μ_A(x),belong to the interval [0,1] called membership function.

2.2 Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A}=(a_1,a_2,a_3)$ and is defined by the membership function as

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, \text{ if } a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, \text{ if } a_2\le x\le a_3\\ 0, \text{ otherwise}\end{cases}$$

2.3 Negative Triangular Fuzzy Numbers

A negative triangular fuzzy numbers Ã is denoted as Ã = (a_1,a_2,a_3),where all a_i's<0 for all i=1,2,3

Note: when Ã= (-3,-2,-1) is a negative triangular fuzzy number, This can be written as i=1,2,3 Ã= -(1,2,3).

2.4 Positive Triangular Fuzzy Numbers

A positive triangular fuzzy numbers Ã is denoted as Ã = (a_1,a_2,a_3),where all a_i's>0 for all i=1,2,3

2.5 The Fuzzy Arithmetic Operations under Function Principle

Suppose $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then the fuzzy arithmetic operations under function principle are furnished below.

Addition:

$\tilde{A}+\tilde{B}$ = (a_1+b_1, a_2+b_2, a_3+b_3), where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

Subtraction:

$\tilde{A}-\tilde{B}$ = (a_1-b_3, a_2-b_2, a_3-b_1), where a_1, a_2, a_3, b_1, b_2 and b_3 are any real numbers.

Scalar Multiplication:

Let $\lambda \in R$, then $\lambda\tilde{A}$ = (λa_1, λa_2, λa_3), $\lambda \ge 0$ and

$\lambda\tilde{A}$ = (λa_3, λa_2, λa_1), $\lambda < 0$.

Multiplication:

$\tilde{A}.\tilde{B} = (c_1,c_2,c_3)$, where $T=\{a_1b_1,a_1b_3,a_3b_1,a_3b_3\}, T_1=a_2b_2, c_1=\min T, c_2=T_1, c_3=\max T$. If a_1, a_2, a_3, b_1, b_2, b_3 are all nonzero positive real numbers, then $\tilde{A}.\tilde{B}=(a_1b_1,a_2b_2,a_3b_3)$.

Division:

$\tilde{A}/\tilde{B}=(a_1/b_3, a_2/b_2, a_3/b_1)$, if all b_i's are non-zero positive real numbers.

2.6. Graded Mean Integration Method

Suppose $\tilde{A}$ = (a_1, a_2, a_3) is a given triangular fuzzy number. Then the defuzzification of the fuzzy number by graded mean integration method is

$$p(\tilde{A}) = \frac{(a_1+4a_2+a_3)}{6}.$$

2.7 New operation on triangular fuzzy number

Subtraction:

Suppose $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers.

Then Ã- $\tilde{B}$ = (a_1-b_1, a_2-b_2, a_3-b_3), the new subtraction operation exist only if the following condition is satisfied DP (Ã) ≥ DP ($\tilde{B}$). Where DP (Ã) = (a_3-a_1)/2 and DP ($\tilde{B}$) = (b_3-b_1)/2, where DP denotes Difference points of a triangular fuzzy number.

Division:

Suppose $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then $\frac{\tilde{A}}{\tilde{B}}$ is

$\frac{\tilde{A}}{\tilde{B}}$ = (a_1/b_1,a_2/b_2,a_3/b_3), the new division operation exists only if the following conditions are satisfied, into negative multiplication of positive numbers, where MP (Ã) = (a_3+a_1)/2, DP (Ã) = (a3-a1)/2, MP ($\tilde{B}$) = (b3+b1)/2 and DP ($\tilde{B}$) = (b3-b1)/2, where MP denotes midpoint and DP denotes Difference point of a triangular fuzzy number. (i.e.) $\left|\frac{DP(\tilde{A})}{MP(\tilde{A})}\right| \geq \left|\frac{DP(\tilde{B})}{MP(\tilde{B})}\right|$

III. FUZZY NUMBER LINEAR COMPLEMENTARITY PROBLEM (FLCP)

3.1. Fuzzy Linear Complementarity Problem (FLCP)

Assume that all parameters in (1) - (3) are fuzzy and are described by fuzzy numbers. Then, the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy numbers.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q}$$

$$\tilde{W}_j \geq 0, Z_j \geq 0, j = 1, 2, 3, \ldots\ldots\ldots n$$

$$\tilde{W}_j \tilde{Z}_j = 0, j = 1, 2, 3, \ldots\ldots\ldots\ldots n$$

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of fuzzy complementary variables.

Definition 3.2

A solution $(\tilde{W}, \tilde{Z})$ to the above system (1) - (3) is called a fuzzy complementary feasible solution, if $(\tilde{W}, \tilde{Z})$ is a fuzzy basic feasible solution to (1) and (2) with one of the pair $(\tilde{W}_j, \tilde{Z}_j)$ basic for each j =1, 2, 3…n.

3.3 Fuzzy Quadratic Programming Problem (QPP) as a fuzzy Linear Complimentarity Problem (LCP)

Consider the following Quadratic Programming Problem

Minimize $\tilde{f}(\tilde{x}) = \tilde{c}^t \tilde{x} + \frac{1}{2}\tilde{x}^t \tilde{H}\tilde{x}$

Subject to $\tilde{A}\tilde{x} \leq \tilde{b}$

and $\tilde{x} \geq 0$

where $\tilde{c}$ an n-vector of fuzzy numbers is, $\tilde{b}$ is an m-vector, Ã is an mxn fuzzy matrix and $\tilde{H}$ is an nxn fuzzy symmetric matrix. Let $\tilde{y}$ denotes the vector of slack variables and $\tilde{u}, \tilde{v}$ be the Lagrangian multiplier vectors of the constraints Ã $\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$ respectively, then the Kuhn-Tucker conditions can be written as

$$\tilde{A}\tilde{x} + \tilde{y} = \tilde{b}$$

$$-\tilde{H}\tilde{x} - \tilde{A}^t\tilde{u} + \tilde{v} = \tilde{c}$$

$\tilde{x}^t\tilde{v} = 0, \tilde{u}^t\tilde{y} = 0$ And $\tilde{x}$, $\tilde{y}$ $\tilde{u}, \tilde{v}$ ≥0 Now Letting

$$\tilde{M} = \begin{bmatrix} \tilde{0} & -\tilde{A} \\ \tilde{A}^t & \tilde{H} \end{bmatrix}, \tilde{q} = \begin{bmatrix} \tilde{b} \\ \tilde{c} \end{bmatrix}, \tilde{w} = \begin{bmatrix} \tilde{y} \\ \tilde{v} \end{bmatrix} \text{ and } \tilde{z} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}$$

the Kuhn-Tucker conditions can be expressed as the LCP.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q},$$

$$\tilde{W}^t\tilde{Z} = 0$$

$$(\tilde{W}, \tilde{Z}) \geq 0$$

IV. ALGORITHM FOR FUZZY LINEAR COMPLEMENTARITY PROBLEM

Lemke [5] suggested an algorithm for solving linear complementarity problems. Based on this idea, an algorithm for solving fuzzy linear complementarity problem is developed here.

Consider the FLCP$\left(\tilde{q}, \tilde{M}\right)$ of order n, suppose the fuzzy matrix $\tilde{M}$ satisfies the conditions: There exists a column vector of $\tilde{M}$ in which all the entries are strictly positive. Then a variant of the complementary pivot algorithm which uses no artificial variable at all can be applied on the FLCP $\left(\tilde{q}, \tilde{M}\right)$.The original tableau for this version of the algorithm is:

$\tilde{w}$	$\tilde{Z}$	
$\tilde{I}$	$-\tilde{M}$	$\tilde{q}$

We assume that $\tilde{q} \geq 0$.

Let s be such that $\tilde{M}_{.s} > 0$.So, the column vector associated with $\tilde{Z}_s$ is strictly negative in (4).Hence the variable $\tilde{Z}_s$ can be made to play the same role as that of the artificial variable $\tilde{Z}_0$

Step: 1

Determine t to satisfy $\left(\frac{\tilde{q}_t}{\tilde{m}_{ts}}\right)$ = minimum $\left\{\frac{\tilde{q}_i}{\tilde{m}_{is}} / i = 1\, to\, n\right\}$, and update the table by pivoting at row t and the table by pivoting at row t and $\tilde{Z}_s$ column. Thus, the right hand side constants vector becomes nonnegative after this pivot step.

Hence, $\left(\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_{t-1}, \tilde{Z}_s, \tilde{w}_{t+1}, ...\tilde{w}_n\right)$ is a feasible basic vector for (4), and if s = t, it is fuzzy complementary feasible basic vector and the solution corresponding to it is a FLCP $\left(\tilde{q}, \tilde{M}\right)$, terminate.

If s $\neq$ t, the feasible basic vector $\left(\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_{t-1}, \tilde{Z}_s, \tilde{w}_{t+1}, ...\tilde{w}_n\right)$ for (4) satisfies the following properties.

i. It contains exactly one basic variable from the complementary pair $(\tilde{W}_i, \tilde{Z}_i)$ for n-2 values of i (namely i $\neq$ s, t here).
ii. It contains both the variables from a fixed complementary pair (namely $(\tilde{W}_s, \tilde{Z}_s)$ here), as fuzzy basic variables.
iii. There exists exactly one fuzzy complementary pair in which both the variables are contained in this basic vector (namely $(\tilde{W}_t, \tilde{Z}_t)$ here).

For carrying out this version of the fuzzy complementary pivot algorithm, any feasible fuzzy basic vector for (4) satisfying (i), (ii), (iii) is known as an almost fuzzy complementary feasible basic vector.

Step: 2

In the canonical tableau of (4) w.r.t the initial almost fuzzy complementary feasible basic vector, the updated column vector of $\tilde{W}_t$ can be verified to be strictly negative. Hence if $\tilde{W}_t$ is selected as the entering variable into the initial basic vector, an almost complementary extreme half-line is generated. Hence the initial almost complementary BFS of (4) is at the end of an almost complementary ray.

Step: 3

Choose $\tilde{Z}_t$ as the entering variable into the initial almost complementary feasible basic vector $\left(\tilde{w}_1, \tilde{w}_2, ..., \tilde{w}_{t-1}, \tilde{Z}_s, \tilde{w}_{t+1}, ...\tilde{w}_n\right)$ .In all subsequent steps, the entering variable is uniquely determined by the complementary pivot rule. The algorithm can terminate in two possible ways:

i)At some stage one of the variables from the complementary pair $(\tilde{W}_s, \tilde{Z}_s)$ drops out of the basic vector or becomes equal to zero in the BFS of (7).The BFS of (4) at that stage is a solution of the FLCP$\left(\tilde{q}, \tilde{M}\right)$.

ii)At some stage of the algorithm, both the variables in the complementary pair $(\tilde{W}_s, \tilde{Z}_s)$ may be strictly positive in the BFS and the pivot column in that stage may turn out to be nonpositive, and in this case the algorithm stops with almost complementary ray.

V. ILLUSTRATIVE EXAMPLE

Consider the following fuzzy quadratic programming problem

Minimize Z = $-6\tilde{x}_1 - 3\tilde{x}_2 + 4\tilde{x}_1\tilde{x}_2 + 2\tilde{x}_1^{\,2} + 3\tilde{x}_2^{\,2}$

Subject to the constraints,

$\tilde{x}_1 + \tilde{x}_2 \le 1$; $2\tilde{x}_1 + 3\tilde{x}_2 \le 4$

$\tilde{x}_1, \tilde{x}_2 \ge 0$

Here $\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{1} \\ \tilde{2} & \tilde{3} \end{bmatrix}$, $\tilde{X} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$, $\tilde{b} = \begin{bmatrix} \tilde{1} \\ \tilde{4} \end{bmatrix}$, $\tilde{H} = \begin{bmatrix} \tilde{4} & \tilde{4} \\ \tilde{4} & \tilde{6} \end{bmatrix}$,

$\tilde{c} = \begin{bmatrix} -\tilde{6} \\ -\tilde{3} \end{bmatrix}$ Finally the fuzzy matrix $\tilde{M}$ is given by

$$\tilde{M} = \begin{bmatrix} \tilde{0} & -\tilde{A} \\ \tilde{A}^T & \tilde{H} \end{bmatrix} = \begin{bmatrix} \tilde{0} & \tilde{0} & -\tilde{1} & -\tilde{1} \\ \tilde{0} & \tilde{0} & -\tilde{2} & -\tilde{3} \\ \tilde{1} & \tilde{2} & \tilde{4} & \tilde{4} \\ \tilde{1} & \tilde{3} & \tilde{4} & \tilde{6} \end{bmatrix},$$

$$\tilde{q} = \begin{bmatrix} \tilde{b} \\ -\tilde{c} \end{bmatrix} = \begin{bmatrix} \tilde{1} \\ \tilde{4} \\ -\tilde{6} \\ -\tilde{3} \end{bmatrix}$$

Now, the fuzzy linear complementary problem is solved by the proposed algorithm and the results are tabulated below.

Table 5.1

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,1)	(0,0,0)	(0,0,0)	(0,0,0)	**(**0,0,0**)**	(0,0,0)	(1,2,3)	(1,1,1)	(3,6,9)
W_2	(0,0,0)	(1,1,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(1,1,1)	-(1,4,7)	(0,0,0)
W_3	(0,0,0)	(0,0,0)	(1,1,1)	(0,0,0)	-(1,2,3)	-(1,1,1)	-(1,2,3)	-(1,4,7)	-(1,3,5)
W_4	(0,0,0)	(0,0,0)	(0,0,0)	(1,1,1)	-(1,1,1)	(1,4,7)	-(1,4,7)	-(3,6,9)	(0,0,0)

We pivot at row 3 and the $\tilde{Z}_4$ column, for the next tableau we have $\tilde{y}_s = \tilde{z}_3$

Table 5.2

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,1,2)	(0,0,0)	(2,5,8)
W_2	(0,0,0)	(1,1,1)	-(1,1,1)	(0,0,0)	(1,2,3)	(1,1,1)	(2,3,40))	(0,0,0)	(1,3,5)
Z_4	(0,0,0)	(0,0,0)	$(-1,\frac{-1}{4},\frac{-1}{7})$	(0,0,0)	$(\frac{1}{7},\frac{1}{2},3)$	$(\frac{1}{7},\frac{1}{4},1)$	$(\frac{1}{7},\frac{1}{2},3)$	(1,1,1)	$(\frac{1}{7},\frac{3}{4},5)$
W_4	(0,0,0)	(0,0,0)	$-(\frac{3}{7},\frac{3}{2},9)$	(1,1,1)	$\left(\frac{-4}{7},2,26\right)$	$\left(\frac{10}{7},\frac{11}{2},16)\right)$	$\left(\frac{-4}{7},-1,20\right)$	(0,0,0)	$\left(\frac{3}{7},\frac{9}{2},45\right)$

Here $\tilde{y}_s = \tilde{z}_3$ enters the basis, by the minimum ratio test $\tilde{w}_2$ leaves the basis and for the next iteration $\tilde{y}_s = \tilde{z}_2$ we pivot at row 2 and the $\tilde{z}_3$ column

Table 5.3

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,1)	$-\left(1,\frac{1}{3},0\right)$	$\left(1,\frac{1}{3},0\right)$	(0,0,0)	$-\left(\frac{3}{2},\frac{2}{3},0\right)$	$-\left(1,\frac{1}{3},0\right)$	(0,0,0)	(0,0,0)	$\left(2,4,\frac{11}{2}\right)$
Z_3	(0,0,0)	$\left(\frac{1}{4},\frac{1}{3},\frac{1}{2}\right)$	$-\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$	(0,0,0)	$\left(\frac{1}{2},\frac{2}{3},\frac{3}{4}\right)$	$\left(\frac{1}{4},\frac{1}{3},\frac{1}{2}\right)$	(1,1,1)	(0,0,0)	$\left(\frac{1}{2},1,\frac{5}{4}\right)$
Z_4	(0,0,0)	$\left(\frac{3}{2},\frac{1}{6},\frac{1}{28}\right)$	$\left(\frac{-15}{28},\frac{-1}{12},\frac{1}{2}\right)$	(0,0,0)	$\left(\frac{1}{14},\frac{1}{6},\frac{3}{4}\right)$	$\left(\frac{-19}{4},\frac{1}{12},\frac{27}{28}\right)$	(0,0,0)	(1,1,1)	$\left(\frac{1}{14},\frac{1}{4},\frac{5}{4}\right)$
W_4	(0,0,0)	$\left(-10,\frac{1}{3},\frac{2}{7}\right)$	$\left(\frac{-65}{7},\frac{-11}{6},\frac{67}{7}\right)$	(1,1,1)	$\left(\frac{-109}{7},\frac{8}{3},\frac{185}{7}\right)$	$\left(\frac{-60}{7},\frac{35}{6},\frac{114}{7}\right)$	(0,0,0)	(0,0,0)	$\left(\frac{-172}{7},\frac{11}{2},\frac{320}{7}\right.$

Table 5.4

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,1)	$\left(\frac{-7}{6},\frac{-41}{105},0\right)$	$\left(\frac{-53}{12},\frac{24}{105},\frac{67}{42}\right)$	$\left(\frac{7}{12},\frac{6}{105},\frac{-1}{12}\right)$	$\left(\frac{-109}{12},\frac{-54}{105},\frac{122}{42}\right)$	(0,0,0)	(0,0,0)	(0,0,0)	$\left(\frac{-37}{3},\frac{453}{105},\frac{193}{6}\right)$
Z_3	(0,0,0)	$\left(\frac{10}{24},\frac{41}{105},\frac{31}{6}\right)$	$\left(\frac{-557}{84},\frac{-248}{105},\frac{127}{6}\right)$	$\left(\frac{-22}{21},\frac{-24}{105},\frac{53}{24}\right)$	$\left(\frac{-143}{84},\frac{162}{315},\frac{327}{4}\right)$	(0,0,0)	(1,1,1)	(0,0,0)	$\left(\frac{-77}{6},\frac{24}{35},\frac{101}{12}\right)$
Z_4	(0,0,0)	$\left(\frac{-937}{168},\frac{-69}{420},\frac{-63}{168}\right)$	$\left(\frac{-18}{215},\frac{8281}{1176},\frac{1211}{48}\right)$	$\left(\frac{-63}{112},\frac{-1}{70},\frac{133}{48}\right)$	$\left(\frac{-14473}{336},\frac{81}{630},\frac{3641}{168}\right)$	(0,0,0)	(0,0,0)	(1,1,1)	$\left(\frac{-5713}{84},\frac{96}{560},\frac{1535}{12}\right)$
Z_2	(0,0,0)	$\left(\frac{-7}{6},\frac{-6}{35},\frac{-1}{12}\right)$	$\left(\frac{-65}{12},\frac{-11}{35},\frac{67}{42}\right)$	$\left(\frac{1}{6},\frac{6}{35},\frac{7}{12}\right)$	$\left(\frac{-109}{12},\frac{8}{35},\frac{185}{42}\right)$	(1,1,1)	(0,0,0)	(0,0,0)	$\left(\frac{-43}{3},\frac{33}{35},\frac{80}{3}\right)$

Hence Min $\tilde{f}$ = (-893,-10.27, 2858.12)

VI. CONCLUSION

In this paper, a new approach for solving QPP by converting it into a linear complementarity problem with fuzzy parameters is suggested. Even though we are considering the Single objective case, this method can also be extended to multi- objective programming with fuzzy coefficients.

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