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# A New Dimension Algorithmic Approach to Solve Fuzzy Linear Complementarity Problem with Interval Numbers Approach

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ABSTRACT

New approach for solving fuzzy linear complimentarity problem (FLCP) based on Variable Dimension algorithm is proposed in this paper. Here the linear complimentarity problem is reduced to interval number linear complimentarity problem and modified arithmetic operations on interval numbers are used to solve the formulated model. At the end. Numerical examples, illustrating the proposed method are provided.
**Keywords:** Fuzzy Linear Complimentarity Problem, Interval Numbers, Modified Interval Arithmetic.

## I. INTRODUCTION

The linear Complementarity problem (LCP) is a well known mathematical programming and it has been studied by many Researchers. It has its wide applications in engineering, economics and many other scientific fields. In 1963, Lemke [7] proposed a complementarity pivoting algorithm for solving linear complementarity problems. Since, the Karush Kuhn Tucker conditions for quadratic programming problems can be written as a LCP, Lemke's algorithm can be used to solve quadratic programs. Since then the study of complementarity problems has expanded enormously. In 1967, Robert .L.Graves [20] proposed a principal pivoting method for solving LCPs.

In 1968, Cottle and Danzig [31] unified linear, quadratic programs and bimatrix games. In 1971, K.G.Murthy [31] developed an algorithm for solving parametric LCPs. Here in this study, fuzzy linear complementarity problems and fuzzy parametric linear complementarity problems have been formulated. Solution procedures are provided for all the models developed. Also, iterative methods developed for solving LCPs hold great promise for handling very large scale linear programs which cannot be tackled with the well known simplex method because of their large size and the consequent numerical difficulties. In a recent review, Pankaj Gupta et al [4] gave a fuzzy approximation to an infeasible generalized linear complementarity problem.

In 1984 "A Variable Dimension Algorithm for the linear complimentarity problem" developed by Ludo Vander Hayden. The variable dimension algorithm can also be applied to solve LCP's with co positive plus co-efficient matrices. This paper provides a variable dimension algorithm for the fuzzy linear complimentarity problem.

Given the n x n matrix m and the n-dimensional vector q, the linear complimentarity problem consists in finding nonnegative vector s and z which satisfy

$$W - Mz = q \qquad (I)$$

$$w,z \geq 0 \qquad (II)$$

$$w^t z = 0 \qquad (III)$$

Given the nonnegativity of the vectors s and z requires that $w_i z_i = 0$ for i = 1,2,…..n. Two such vectors are said to be complimentary. A Solution to this problem is immediately available upon inspection, when q is nonnegative, since we can set w = q and z = 0. When referring to our algorithm us simplicity assumes that q has atleast one negative co-ordinate.

This paper is organized as follows: Section 2 introduces triangular fuzzy numbers and a new operation on triangular fuzzy number for solving Fuzzy Linear complementarity problem. Fuzzy linear complementarity problem and conversion of FQPP into FLCP is described in section 3, Section 4 deals with an Algorithmic Approach for LCP and in section 5, and the effectiveness of the proposed method is illustrated by means of an example. Finally, section 6 contains some concluding remarks.

## II. PRELIMINERIES

### 2.1 INTERVAL NUMBERS

If $\tilde{A}$ is a triangular fuzzy number, We will let $\tilde{A}_\alpha$ = $[A_\alpha^-, A_\alpha^+]$ be the closed interval which is a $\alpha$ -cut for $\tilde{A}$ where $A_\alpha^-$ and $A_\alpha^+$ are its left and right end points respectively.

Let I and J be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.I = [a,b] , where a≤ b

J = [c,d] , where c≤d, When a= b and c=d, these interval numbers degenerate to a scalar real number.

### 2.2. ARITHMETIC OPERATIONS ON INTERVAL NUMBERS

The arithmetic operations on I and J are furnished below.

**Addition:**

I + J = [a,b] +[c,d] =[a + b, c+d ] , where a, b, c & d are any real numbers.

**Subtraction:**

I - J = [a,b] - [c,d] = [a-d, b-c ], where a, b, c & d are any real numbers.

**Scalar Multiplication:**

Let λ ∈ R, then λ[a,b] = [λa, λb], λ ≥ 0, λ[a,b] = [λb, λa ], λ < 0

**Multiplication:**

I.J=[a,b].[c,d]=[min(ac,ad,bc,bd),max(ac,ad,bc,bd)], where ac,ad,bc,bd are all arithmetic products.

**Division:**

I/ J =[a,b] / [c,d], provided $0 \notin [c,d]$,where1/d and 1/c are quotients.

### 2.2. Principal sub problem

Let $\tilde{J}$ {$\tilde{1}$, $\tilde{2}$, ……. $\tilde{n}$}. Denote $\widetilde{w_J}$ = ($\widetilde{w_J}$: $\tilde{j}$ € $\tilde{J}$), $\tilde{z}_J$ = ($\tilde{z}_J$: $\tilde{j}$ € $\tilde{J}$), $\tilde{q}_J$ = ($\tilde{q}_J$: $\tilde{j}$ € $\tilde{J}$),and the principal submatrix of m corresponding to $\tilde{J}$, $\widetilde{M_{JJ}}$ = ($\widetilde{m_{\iota J}}$: $\tilde{\iota}, \tilde{j}$ € $\tilde{J}$). The principal sub problem of the LCP (I) – (III) in the variables$\widetilde{w_J}$, $\tilde{z}_J$is the LCP ($\tilde{q}_J$, $\widetilde{M_{JJ}}$) of order $|\tilde{J}|$, the complimentary variables in it are {$\widetilde{w_J}$, $\tilde{z}_J$} for $\tilde{j}$ € $\tilde{J}$

### 2.3. Position 1 Solution

This is a solution ( $\widetilde{w_J}$ , $\tilde{z}_J$ ) for (I) satisfying the following properties:

(i)

here exists an index k such that

(ii)

$\widetilde{z_k}$ = 0 and $\widetilde{w_k}$ < 0.$\tilde{z}_J$ = 0 for $\tilde{j}$ > $\tilde{k}$

(iii)

f $\tilde{k}$ > 1, $\widetilde{w}^{(k-1)}$ = ($\widetilde{w}_1$, ……., $\widetilde{w}_{(k-1)}$) , $\tilde{z}^{(k-1)}$ = ($\tilde{z}_1$, ……., $\tilde{z}_{(k-1)}$ ) is a solution for the principal subproblem of (I) – (III) determined by the subset {$\tilde{1}$, $\tilde{2}$, ……. ($\widetilde{k-1}$)}, that is, $\widetilde{w}^{(k-1)}$ ≥ 0, $\tilde{z}^{(k-1)}$ ≥ 0 and( $\widetilde{w}^{(k-1)^T}$) $\tilde{z}^{(k-1)}$ = 0.

## III. A VARIABLE DIMENSION ALGORITHM

Before reviewing the algorithm we recall the usual nondegeneracy assumption for pivoting algorithm and introduce notation.

### 3.1. Assumption

Equation (I) is nondegenearte, every solution has at least n nonzero variables.

Given a positive integer $\tilde{k}$ ≤ $\tilde{n}$ and given vector $\tilde{x}$ € $\widetilde{R^n}$, $\widetilde{x^{(k)}}$ denotes the vector

[ $\widetilde{x_1}$ , $\widetilde{x_2}$ ,…….. $\widetilde{x_k}$ ] $\in \widetilde{R^k}$. Similarly the k[th] leading principal submatrix of $\widetilde{M}$ is denoted $\widetilde{M^{(k)}}$. Given LCP ($\widetilde{M}$, $\tilde{q}$) the k – problem is the subproblem LCp($\widetilde{M^{(k)}}$, $\widetilde{q^{(k)}}$).

Our variable dimension algorithm [Vander heyden, 1980] starts at the point $[\tilde{s}, \tilde{z}] = [\tilde{q}, \tilde{0}]$ and follows particular lines of solutions (I), called variable dimension lines.

### 3.2. Definition

A Variable Dimension line consists of a line of solutions $[\tilde{s}, \tilde{z}]$ for (I) –(III) verifying the following statements:

a. There exists an index k with $\widetilde{s_k} < 0$ and $\widetilde{z_k} <$ o:
b. $\widetilde{z_j} = 0$ for $\tilde{j} > \tilde{k}$;
c. if $\tilde{k} > \tilde{1}$ then $s^{\widetilde{(k-1)}}$ and $z^{\widetilde{(k-1)}}$ are nonnegative and complimentary.

### 3.3. Algorithm

The above line is a k-problem in that a point on the line would solve the k – problem if not for $\widetilde{s_k} < 0$. The intent of the algorithm in following the line is to reach an endpoint where $\widetilde{s_k}$ is zero. An endpoint is reached whenever a variable $\widetilde{w_h} = \widetilde{s_h}$ or $\widetilde{z_h}$, $\tilde{h} \le \tilde{k}$, becomes nonbasic, that is , equal to zero. The nondegeneracy assumption (3.I) insures that only one variable becomes zero at an endpoint. Distiquising three cases, we now show that an endpoint solves the LCP or uniquely leads to another variable dimension line, which the algorithm follows next. We characterize the new variable dimension line by identifying the variable which is zero at the endpoint and which becomes nonzero along the line.

**Initial step: Step 0:**

The initial basic vector is $\widetilde{w} = (\widetilde{w_1}, \widetilde{w_2}, \ldots\ldots\ldots \widetilde{w_k})$. The initial solution is the position 1 basic solution of (I) corresponding to it, define $\tilde{k}$ = minimum ($\tilde{\imath}$: $\widetilde{q_\imath} < 0$). Begin with the k – problem, by making a type 1 pivot step value of the nonbasic variable $\widetilde{y_k}$ from zero, as described below.

**Step.1: Type 1 pivot step, to Increase the value of a nonbasic variable from zero**

$\tilde{h} = \tilde{k}$ , $\widetilde{w_k} = \widetilde{s_k}$ (Dimension Increase) Let ($\widetilde{y_1}$ , $\widetilde{y_2}$ ,……… $\widetilde{y_k}$, $\widetilde{w_{(k+1)}}$, …… , $\widetilde{w_n}$) be the basic variable working for the $\tilde{k}$ - problem.

This is the desired situation where the endpoint $[\tilde{s}, \tilde{z}]$ solves the k – problem, and also solves the LCP if $\tilde{s} \geq 0$.Otherwise let $\tilde{g}$ = min ($\tilde{j}$/ $\widetilde{s_j} < 0$), $\tilde{j} > \tilde{k}$. The other variable dimension line incident to the endpoint is obtained by increasing $\widetilde{z_g}$($\widetilde{z_g} > 0$) and is a line of the $\tilde{g}$ problem is satisfied, the method is unable to proceed further and termination occurs with the conclusion that the method is unable to process this LCP. If condition $\widetilde{z_g}$($\widetilde{z_g} > 0$) is not satisfied define $\theta$ = Max {$\frac{\widetilde{q_\imath}}{\widetilde{a_\imath}}$: over $1 \le \tilde{\imath} \le \widetilde{(k-1)}$ such that $\widetilde{a_\imath} > 0$ and $\frac{\widetilde{q_k}}{\widetilde{a_k}}$, if $\widetilde{a_k} < 0$.}

**Step.2: Type 2 pivot step, to Decrease the value of a nonbasic variable from zero**

$\tilde{h} = \tilde{k}$, $\widetilde{w_k} = \widetilde{z_k}$ (Dimension Decrease). This pivot step will be made whenever we obtain a complementarity basic vector ($\widetilde{y_1}$ , $\widetilde{y_2}$ ,……… $\widetilde{y_k}$, $\widetilde{w_{(k+1)}}$, …… , $\widetilde{w_n}$) after doing some work on the complimentarity problem,with $\widetilde{w_k} = \widetilde{z_k}$.

The fact that the algorithm never returns to its starting point $[\tilde{s}, \tilde{z}]$ = $[\tilde{q}, 0]$ implies that $\tilde{k} > 1$ at this end point. The endpoint is a new solution for the (k-1) problem. Let $\tilde{g}$ = max ($\tilde{j}$ / $\widetilde{z_j} > 0$), $\tilde{g} < \tilde{k}$. The other variable dimension line incident to the endpoint is obtained by decreasing $\widetilde{s_g}$($\widetilde{s_g} < 0$) and is a line of the $\tilde{g}$ problem is satisfied, the method is unable to proceed further and termination occurs with the conclusion that the method is unable to process this LCP. If condition $\widetilde{s_g}$($\widetilde{s_g} < 0$) is not satisfied, define

$\theta$ = Min {$-(\frac{\widetilde{q_\imath}}{\widetilde{a_\imath}})$: over $1 \le \tilde{\imath} \le \tilde{g}$) such that $\widetilde{a_\imath} < 0$ and $\frac{\widetilde{q_k}}{\widetilde{a_k}}$, if $\widetilde{a_k} > 0$.}

**Step.3**

$\tilde{h} < \tilde{k}$ the endpoint is incident to another line of the k – problem obtained by increasing the variable Complementary to $\widetilde{w_h}$, namely $\widetilde{z_h}$ if $\widetilde{w_h} = \widetilde{s_h}$ and $\widetilde{s_h}$ if

$\widetilde{w_h} = \widetilde{z_h}$.Starting at $[\tilde{s},\tilde{z}] = [\tilde{q}, 0]$ the algorithm follows variable dimension lines and generates endpoints until it finds a solution for the LCP or generates an unbounded variable dimension line. Assuming nondegeneracy, the algorithm never visits an endpoint twice as the initial point is incident to precisely one variable dimension line, and any other endpoint is incident to at most two such lines. The finiteness of the number of variables dimension lines, and hence of endpoints, then proves that the algorithm must terminate with a solution if all variable dimension lines are bounded.

## IV. ILLUSTRATIVE EXAMPLE

Consider the LCP $(\tilde{q}, \tilde{m})$ where $\tilde{q} = \begin{bmatrix} -\tilde{1} \\ -\tilde{2} \\ -\widetilde{10} \end{bmatrix}$, $\widetilde{M} = \begin{bmatrix} \tilde{1} & \tilde{1} & \tilde{1} \\ \tilde{3} & \tilde{1} & \tilde{1} \\ \tilde{2} & \tilde{2} & \tilde{1} \end{bmatrix}$.

This problem converted into interval numbers, we get

$$\tilde{q} = \begin{bmatrix} [-\tilde{1},-\tilde{1}] \\ [-\tilde{2},-\tilde{2}] \\ [-\widetilde{10},-\widetilde{10}] \end{bmatrix},$$

$$\widetilde{M} = \begin{bmatrix} [\tilde{1},\tilde{1}] & [\tilde{1},\tilde{1}] & [\tilde{1},\tilde{1}] \\ [\tilde{3},\tilde{3}] & [\tilde{1},\tilde{1}] & [\tilde{1},\tilde{1}] \\ [\tilde{2},\tilde{2}] & [\tilde{2},\tilde{2}] & [\tilde{1},\tilde{1}] \end{bmatrix}.$$

Since $\tilde{k}$ = minimum $\{\tilde{\imath} : \widetilde{q_\imath} < 0\}$

$\tilde{\imath} = \tilde{1} \Rightarrow \widetilde{q_1} < 0, \tilde{\imath} = \tilde{2} \Rightarrow \widetilde{q_2} < 0\ \tilde{\imath} = \tilde{3} \Rightarrow \widetilde{q_3} < 0\ \therefore \tilde{k} = \tilde{1}$

The algorithm begins with $\tilde{k} = \tilde{1}$

Begin with the 1 – problem by making a type 1 pivot step to introduce the value of the non – basic variable $\widetilde{z_k}$ from 0.

### Initial Iteration:

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{w_1}$ | [1,1] | [0,0] | [0,0] | $[-1,-1]$ | $[-1,-1]$ | $[-1,-1]$ | $[-1,-1]$ |
| $\widetilde{w_2}$ | [0,0] | [1,1] | [0,0] | $[-3,-3]$ | $[-1,-1]$ | $[-1,-1]$ | $[-2,-2]$ |
| $\widetilde{w_3}$ | [0,0] | [0,0] | [1,1] | $[-2,-2]$ | $[-2,-2]$ | $[-2,-2]$ | $[-10,-10]$ |

$k = 1$, increase $\widetilde{z_1}$. In this type 1 pivot step, $\widetilde{w_1}$ drops from basic vector.

### Iteration: 1

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{z_1}$ | $[-1,-1]$ | [0,0] | [0,0] | [1,1] | [1,1] | [1,1] | [1,1] |
| $\widetilde{w_2}$ | $[-3,-3]$ | [1,1] | [0,0] | [0,0] | [2,2] | [2,2] | [1,1] |
| $\widetilde{w_3}$ | $[-2,-2]$ | [0,0] | [1,1] | [0,0] | [0,0] | [1,1] | $[-8,-8]$ |

$\widetilde{q_3} < 0, k = 3$, increase$\widetilde{z_3}$. In this type 1 pivot step, $\widetilde{w_2}$ drops from basic vector.

### Iteration: 2

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{z_1}$ | $[-0.5,-0.5]$ | $[-0.5,-0.5]$ | [0,0] | [1,1] | [0,0] | [0,0] | [0.5,0.5] |
| $\widetilde{z_3}$ | $[-1.5,-1.5]$ | [0.5,0.5] | [0,0] | [0,0] | [1,1] | [1,1] | [0.5,0.5] |
| $\widetilde{w_3}$ | $[-1.5,-1.5]$ | $[-1.5,-1.5]$ | [1,1] | [0,0] | $[-1,-1]$ | [0,0] | $[-8.5,-8.5]$ |

$\widetilde{q_3} < 0, k = 3$, increase$\widetilde{z_2}$(Complement of $\widetilde{w_2}$) In this type 1 pivot step, $\widetilde{z_3}$ drops from basic vector.

**Iteration: 3**

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{z_1}$ | [0.5,0.5] | $[-0.5,-0.5]$ | [0,0] | [1,1] | [0,0] | [0,0] | [0.5,0.5] |
| $\widetilde{z_2}$ | $[-1.5,-1.5]$ | [0.5,0.5] | [0,0] | [0,0] | [1,1] | [1,1] | [0.5,0.5] |
| $\widetilde{w_3}$ | $[-2,-2]$ | [0,0] | [1,1] | [0,0] | [0,0] | [1,1] | $[-8,-8]$ |

Again$\widetilde{q_3} < 0$, so need a type 2 pivot step, Decrease $\widetilde{w_2}$(non – basic variable), $\widetilde{z_1}$ drops.

**Iteration: 4**

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{w_2}$ | $[-1,-1]$ | [1,1] | [0,0] | $[-2,-2]$ | [0,0] | [0,0] | $[-1,-1]$ |
| $\widetilde{z_2}$ | $[-1,-1]$ | [0,0] | [0,0] | [1,1] | [1,1] | [1,1] | [1,1] |
| $\widetilde{w_3}$ | $[-2,-2]$ | [0,0] | [1,1] | [0,0] | [0,0] | [0,0] | $[-8,-8]$ |

$\widetilde{q_1} < 0$, $\widetilde{q_3} < 0$, so min(1,3) = 1 ⇒ k = 1, increase $\widetilde{w_1}$(complement of $\widetilde{z_1}$ ) $\widetilde{w_2}$drops.

**Iteration: 5**

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{w_1}$ | [1,1] | $[-1,-1]$ | [0,0] | [2,2] | [0,0] | [0,0] | [1,1] |
| $\widetilde{z_2}$ | [0,0] | $[-1,-1]$ | [0,0] | [3,3] | [1,1] | [1,1] | [1,1] |
| $\widetilde{w_3}$ | [0,0] | $[-2,-2]$ | [1,1] | [4,4] | [0,0] | [1,1] | $[-6,-6]$ |

$\widetilde{q_3} < 0$, k = 3, increase$\widetilde{z_3}$, $\widetilde{z_2}$ drops.

**Iteration: 6**

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{w_1}$ | [1,1] | $[-1,-1]$ | [0,0] | [2,2] | [0,0] | [0,0] | [1,1] |
| $\widetilde{z_3}$ | [0,0] | $[-1,-1]$ | [0,0] | [3,3] | [1,1] | [1,1] | [2,2] |
| $\widetilde{w_3}$ | [0,0] | $[-1,-1]$ | [1,1] | [1,1] | $[-1,-1]$ | [0,0] | $[-8,-8]$ |

$\tilde{q}_3 < 0$, k = 3, increase $\widetilde{w_2}$(complement of $\widetilde{z_2}$ ), $\widetilde{w_3}$(drops.

**Iteration: 7**

| Basic vector | $\widetilde{w_1}$ | $\widetilde{w_2}$ | $\widetilde{w_3}$ | $\widetilde{z_1}$ | $\widetilde{z_2}$ | $\widetilde{z_3}$ | $\tilde{q}$ |
|---|---|---|---|---|---|---|---|
| $\widetilde{w_1}$ | [1,1] | [0,0] | $[-1,-1]$ | [1,1] | [1,1] | [1,1] | [9,9] |
| $\widetilde{z_3}$ | [0,0] | [0,0] | $[-1,-1]$ | [2,2] | [2,2] | [1,1] | [10,10] |
| $\widetilde{w_2}$ | [0,0] | [1,1] | $[-1,-1]$ | $[-1,-1]$ | [1,1] | [0,0] | [8,8] |

Since all $\tilde{q_i} > 0$, It contains complementary feasible vector.

Thus $(\widetilde{w_1}, \widetilde{w_2}, \widetilde{w_3} : \widetilde{z_1}, \widetilde{z_2}, \widetilde{z_3})$ = { [9,9],[8,8],[0,0] : [0,0],[0,0],[10,10]}is a complementary feasible solution of this problem.

## V. CONCLUSION

In this paper, we have suggested a new algorithmic approach to solve fuzzy linear complementarity problem with interval numbers. As an illustration, a fuzzy linear complementarity problem with interval coefficients has been described and solved by Variable dimension algorithmic approach.

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