

Mathematical Modelling In Human Development with the Help of Innovative Technology

R. Dinesh¹, M. Muthuselvi²

¹M.Sc Mathematics, Department of Mathematics, Dr Sns Rajalakshmi College of Arts and Science, Coimbatore, Tamil Nadu, India

²Assistant Professor, Department of Mathematics, Dr Sns Rajalakshmi College of Arts And Science, Coimbatore, Tamil Nadu, India

ABSTRACT

This Paper deals with Mathematical Modelling in human development with the help of innovative technology. To build this Mathematical model three assumptions are taken. These assumptions helps to frame the system on nonlinear first order differential equations and the nonlinear differential equation is solved using variable separable method. Finally the solutions are resulted as graph using MATAB coding

Keywords : Innovation Technology, Human Development, Population, Ordinary Differential equation, MATLAB coding.

I. INTRODUCTION

Mathematical Modelling in terms of differential equation arises when the situation modelled involves some continuous variable(s) varying with respect to some other continuous variable(s) and it have some reasonable hypotheses about the rates of change of dependent variable(s) with respect to independent variable(s).

When it have one dependent variable x (say population size) depending on one independent variable (say time t), we get a mathematical model in terms of an ordinary differential equation of the first order, if the hypothesis is about the rate of change dx/dt . The model will be in terms of an ordinary differential equation on second order if the hypothesis involves the rate of change of dx/dt .

If there are a number of dependent continuous variables and only one independent variable, the hypothesis may give a mathematical model in terms of a system of first or higher order ordinary differential equations.

If there is one dependent continuous variable (say velocity of fluid u) and a number of independent continuous variables (say space coordinates x, y, z and time t), its helps to get a mathematical model in terms of a partial differential equation. If there are a number of dependent continuous variables and a number of independent continuous variables, it can get a mathematical model in terms of systems of partial differential equations.

In this Research paper investigate about Mathematical Modeling in human development with the help of Innovative technology.

II. PROBLEM

To build this model, let $x(t), y(t)$ are derived as human development and innovative technology at time t .

Let assume the following three conditions

- i. If the technology grows with innovation is a beneficial growth of human development

- ii. Without the innovation an arrival of new technology will decline at the human development.
- iii. If there is no innovative technology then the human development will decline at a rate proportional to the human development.

Using the above connection framing the system of non-linear first order Ordinary Differential Equation.

$$\frac{dx}{dt} = -ax + bxy \quad (1)$$

$$\frac{dy}{dt} = -py + qxy \quad (2)$$

Equation (1)&(2) are the system of non-linear First order ordinary differential equation .Which is derive from the above assumption .Here a, b, p, q>0

In this case, $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are vanishes,

∴ Equation (1) becomes

$$\begin{aligned} \frac{dx}{dt} &= 0 \\ -ax + bxy &= 0 \\ x(-a + by) &= 0 \\ -a + by &= 0 \\ y &= \frac{a}{b} \\ y = y_0 &= \frac{a}{b} \quad (3) \end{aligned}$$

Equation (2) becomes

$$\begin{aligned} \frac{dy}{dt} &= 0 \\ -py + qxy &= 0 \\ y(-p + qx) &= 0 \\ -p + qx &= 0 \\ x &= \frac{p}{q} \\ x = x_0 &= \frac{p}{q} \quad (4) \end{aligned}$$

If the initial Growth of human development and innovation technology are p/q and a/b respectively the population will not change with time.

These are equilibrium sizes of the population of human development and innovation technology of course x=0, y=0

Also gives another equilibrium position.

From equation (1)&(2)

$$\frac{2}{1} ==> \frac{dy}{dx} = \frac{-py + qxy}{-ax + bxy}$$

$$\frac{dy}{dx} = \frac{-py + qxy}{-ax + bxy} \quad (5)$$

$$\frac{dy}{dx} = \frac{y(-p + qx)}{x(-a + by)}$$

$$(-a + by) \frac{dy}{y} = (-p + qx) \frac{dx}{x}$$

$$\left(-\frac{a}{y} + \frac{by}{y}\right) dy = \left(-\frac{p}{x} + \frac{qx}{x}\right) dx$$

$$\left(-\frac{a}{y} + b\right) dy = \left(-\frac{p}{x} + q\right) dx$$

Integrating on both sides

$$\int \left(-\frac{a}{y} + b\right) dy = \int \left(-\frac{p}{x} + q\right) dx$$

$$-alog y + by = -plog x + qx + log d \quad (6)$$

$$sub t = 0, x(0) = x_0, y(0) = y_0$$

$$-alog y_0 + by_0 = -plog x_0 + qx_0 + log d$$

$$log d = -alog y_0 + by_0 + plog x_0 - qx_0$$

Substitute log d value in equation (6)

$$-alog y + by = -plog x + qx - alog y_0 + by_0 + plog x_0 - qx_0$$

$$alog y_0 - alog y - by_0 + by = -plog x + plog x_0 + qx - qx_0$$

$$alog \frac{y_0}{y} + plog \frac{x}{x_0} = b(y_0 - y) + q(x_0 - x)$$

(7)

Hence analytic solution of the model has obtained.

III. MATLAB CODING

To predict the result of equation (7), apply MATLAB coding to visualize the Mathematicalfunction.

```

1 - clear;
2 - clear all;
3 - X=-10:0.5:10;
4 - Y=-10:0.5:10;
5 - [x,y]=meshgrid(X,Y);
6 - a=12;
7 - b=16;
8 - p=15;
9 - q=12;
10 - x0=p/q;
11 - y0=a/b;
12 - Z=(a.*(log(y0./y)))+(p.*(log(x./x0)))-(b.*(y0-y))-(q.*(x0-x));
13 - contour(X,Y,Z);
    
```

Figure 1

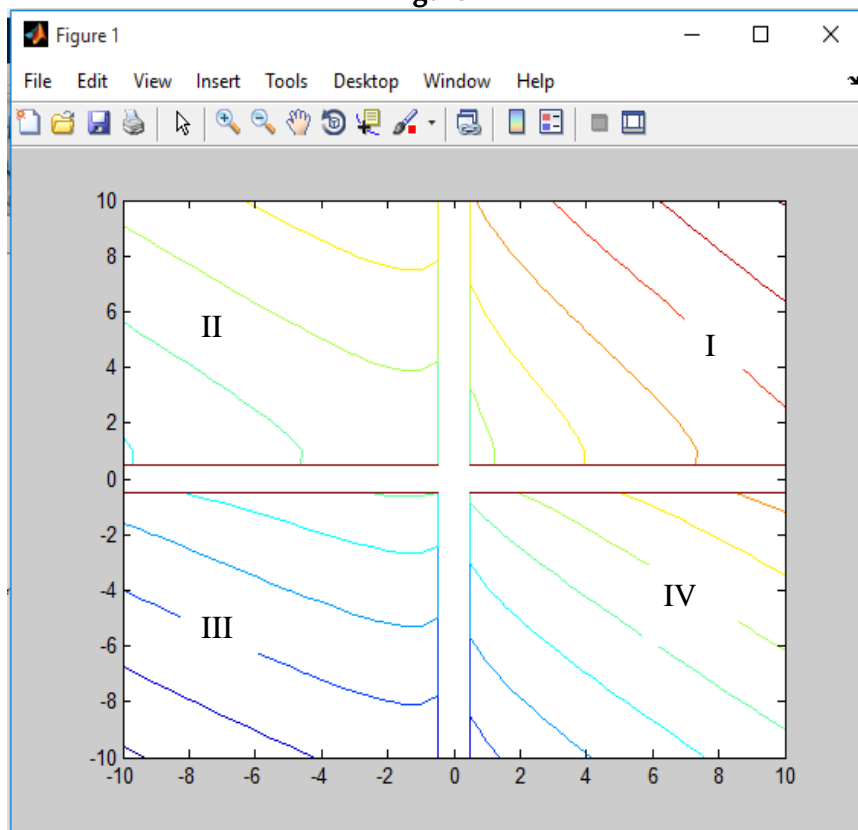


Figure 2

IV. RESULT

In this case $x_0 = p/q$, $y_0 = a/b$ while substituting different values of a, b, p & q , it obtain different types of graphs. But different values of a, b, p and q state the following results are common.

Thus through every point of the first quadrant of the x - y plane, there is a unique trajectory. No two trajectories can intersect, since intersection will imply to two different slopes at the same point.

If it start with $(0,0)$ or $(p/q, a/b)$, it get point trajectories. If it start with $x=x_0, y=0$, from (1) & (2), it find that x increase while remains zero. Similarly if it start with $x=0, y=y_0$, it find that x remains zero while

y decreases. Thus positive axis of x & y give two line trajectories.

Since no two trajectories intersect, no trajectories starting from a point situated within the first quadrant will intersect the x-axis & y-axis trajectories. Thus all trajectories corresponding to positive initial populations will lie strictly within the first quadrant. Thus the initial population are positive, the populations will be always positive. In this model if a, b, p and q are initially zero, it will be always remain zero.

The lines through (p/q, a/b) parallel to the axes of coordinates divide into four parts I, II, III & IV using equation (7) find that

$$\text{In I} \Rightarrow \frac{dx}{dt} > 0, \frac{dy}{dt} > 0, \frac{dy}{dx} > 0$$

$$\text{In II} \Rightarrow \frac{dx}{dt} < 0, \frac{dy}{dt} > 0, \frac{dy}{dx} < 0$$

$$\text{In III} \Rightarrow \frac{dx}{dt} < 0, \frac{dy}{dt} < 0, \frac{dy}{dx} > 0$$

$$\text{In IV} \Rightarrow \frac{dx}{dt} > 0, \frac{dy}{dt} < 0, \frac{dy}{dx} < 0$$

In I & IV, x(t) increases and in III & IV, y(t) decreases.

V. CONCLUSION

This model predicts that innovative technologies helps to give a better development in human growth.

VI. REFERENCES

- [1]. J N Kapur ,MATHEMATICAL MODELLING ,Published by New Age International (p) Ltd., First Edition:1998, New Delhi.
- [2]. M.R. Ball "Mathematics in the Social and Life Science", Ellis Horwood and John Wiley.
- [3]. J. S. Berry , D.N. Burghes , I.D. Huntly , D.J.G. James and A.O. Moscardini, "Teaching and Applying Mathematical Modelling", Ellis Horwood and John Wiley.
- [4]. F. Brauer and J.A. Nohal "Ordinary Differential Equations", N.A. Bejamirs, New York.
- [5]. D.N. Burghes "Mathematical Modelling in the Social Management and Life Science", Ellis Horwood and John Wiley.
- [6]. D.N. Burghes "Modelling with Differential Equations", Ellis Horwood and John Wiley.

- [7]. F. Chorlton "Ordinary Differential and Difference Equations" Von Nostrand, New York.
- [8]. R.A. Coddington and N. Levinson "Theory of Ordinary Differential Equations", Tata McGraw-Hill, New Delhi.
- [9]. M. Cross and A.O. Moscardini, "The Art of Mathematical Modelling", Ellis Horwood and John Wiley.
- [10]. C. Dyson , E. Ivery, "Principle of Mathematical Modelling" ,Academic Press, New York.
- [11]. EUD/UMAP "Undergraduate Mathematics and its Applications", Project Publications, EDC . Camb. Mass.
- [12]. L. Elsgotts "Differential Equations and Calculus of Variations", Mir Publishers, Moscow.
- [13]. G.N. Ewing, "Calculus of Variations with Applications", McGraw Hill, New York.
- [14]. M. Tenenbaum and H. Pollard "Ordinary Differential Equations", Harper and Row, New York.
- [15]. R. Weinstock "Calculus of Variations with Applications", McGraw-Hill, New York.