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A Study on Total Coloring and Finding the Total Chromatic Number of Cycle, Butterfly and Web Graph

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ABSTRACT

In graph theory total colouring and total chromatic number plays a major role. In this paper, we study the concepts of total colouring and total chromatic number. Hence we obtain the total chromatic number of Cycle graph, Butterfly graph and Web graph using an algorithm.
Keywords : Total Colouring, Total Chromatic Number, Cycle Graph, Butterfly Graph and Web Graph.

I. INTRODUCTION

Graph theory is the study of graphs, which are mathematical structures used to model pair wise relations between objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges, arcs, or lines. Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs. It has very profound and plentiful theoretical results and extensive practical applications.

The problem of colouring and colouring numbers are important contents in the field of the graph theory. Beginning with the origin of the four colour problem in 1852, the field of graph colourings has developed into one of the most popular areas of graph theory. Introducing graph theory with a coloring theme, Chromatic Graph Theory explores connections between major topics in graph theory and graph colourings as well as emerging topics. Graph colouring is nothing but a simple way of labelling graph components such as vertices, edges, and regions under some constraints. In a graph, no two adjacent vertices, adjacent edges, or adjacent regions are coloured with minimum number of colours. This number is called the chromatic number and the graph is called a properly coloured graph.

The theory and method of colouring in Discrete Mathematics occupies an important position. Graph colouring enjoys many practical applications as well as theoretical challenges. Besides the classical types of problems, different limitations can also be set on the graph or on the way a colour is assigned or even on the colour itself. In this paper we used the concepts of total colouring and chromatic number for different graphs and we formulated algorithm. The concept of total colouring of star, wheel, helm graph family is explored by Arundhadhi and Ilayarani[1]. Total colouring of simple graphs has been given in paper by L.Andersen[2].Total colourings of graphs has been proved by yap [3].

II. BASIC CONCEPTS

2.1 Total Colouring:

The assignment of colours to both the edges and vertices of a graph G, where the incident elements of

G are not coloured with the same colour is known as total colouring.

2.2 Total Chromatic Number:

The Total chromatic number ψ_T(G) of a graph G is the minimum number of colours needed to colour the edges and the vertices of G so that incident or adjacent elements have distinct colours.

2.3 Cycle graph:

A cycle is a path of edges and vertices wherein a vertex is reachable from itself. A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices connected in a closed chain.

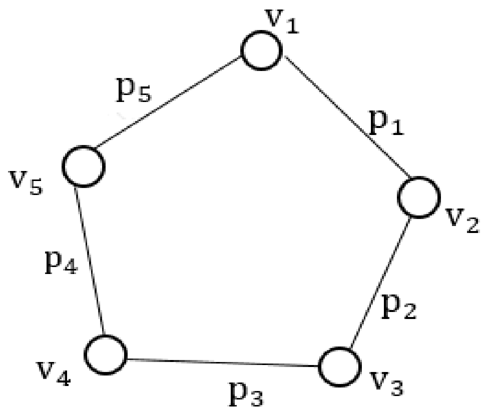

Figure 1. Cycle graph

2.3.1Algorithm for coloring cycle graph:

Input: C_m, m≥4

V← $\{v, v_1 \ldots\ldots v_m\}$

E← $\{p_i \leftarrow vv_{i+1}$ (i =1 to m-1)}

for i =1to m-1

{

a← $i+1$;

if a ≤ m-2;

v_i← a;

else

v_i← a-(m-3);

}

end for

end procedure

Output: vertex and edge coloured C_m

2.3.2 Theorem: The total chromatic number of cycle Graph C_m is m-1, m≥4

(i.e) $\psi_T(C_m)$ =m-1, m≥4

Proof:

Since $\Delta(C_m)$ = m, (m-1) colours are required for proper colouring and hence $\psi_T(C_m) \geq$ m -1 Now, the color class of 1 is{p_i (i =1to m)}. The color class of v_i =i+(m-4) if (1≤ i < 2), v_i =i+(m-4) if (1< i ≤ 2), v_i=i+(m-4) if (2< i ≤ 3),), v_i=i+(m-4) if (2< i ≤ 3), v_i=i-(m-2) if (3< i ≤ 4)}. The elements in each color classes are neither incident nor adjacent.

Therefore , the coloring given in the algorithm 2.3.1 is a total coloring of C_m.

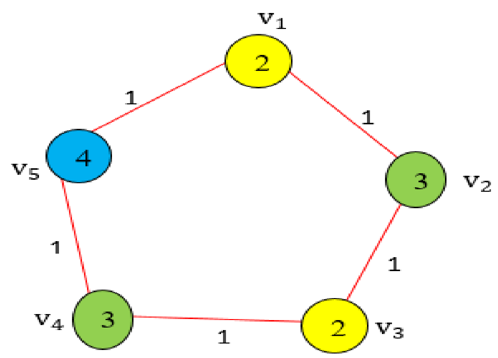

Figure-2: $\psi_T(C_5) = 4$

2.4 Butterfly Graph:

The butterfly graph is a planar undirected graph with 5 vertices and 6 edges. It is also called the bowtie graph and the hourglass graph.A butterfly graph can be constructed by joining two similar cycle graphs C_3 with a common vertex. Let B_m be a butterfly graphwhere m=5.

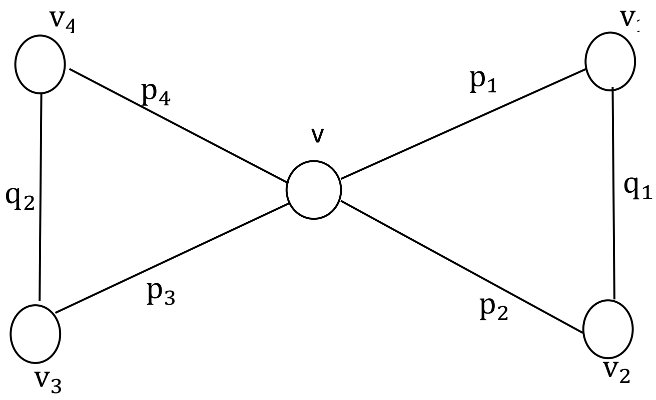

Figure 3. Butterfly Graph

2.4.1 Algorithm for coloring Butterfly graph:

Input: B_m, m≥4

V← $\{v, v_1 \ldots\ldots v_m\}$

E← $\{p_i \leftarrow vv_i$(i=1 to m)}

V← 1;

for i=1 to m

{

a← $i + 1$;

if a ≤ m-1;

p_i← a;

else

p_i← a-(m-2);

}

end for

for i=1 to m-1

{

b← $i + 2$;

if b ≤ m-1;

v_i ←b;

else

v_i ←b-(m-2);

}

end for

v_m ←2;

end for

end procedure

Output: vertex and edge coloured B_m.

2.4.2 Theorem: The total chromatic number of butterfly Graph B_m is m-1, m≥4

(i.e) $\psi_T(B_m)$ =m-1, m≥4

Proof:

Since $\Delta(B_m)$ = m, (m-1) colours are required for proper coloring and hence $\psi_T(w_m)$ ≥ m + 1 Now, the color class of 1 is{v and q_i (i =1to m)}. The color class of {vv_i=i+1 if (1≤ i ≤ 2), vv_i=i-1 if (3≤ i ≤ 4) and v_i =i+(m-2) if (1≤ i < 2), v_i =i+(m-4) if (1< i ≤ 2), v_i=i+(m-4) if (2< i ≤ 3),), v_i=i+(m-4) if (2< i ≤ 3), v_i=i-(m-2) if (3< i ≤ 4)}. The elementsin each color classes are neither incident nor adjacent.Therefore, the coloring given in the algorithm 2.4.1is a total coloring of B_m.

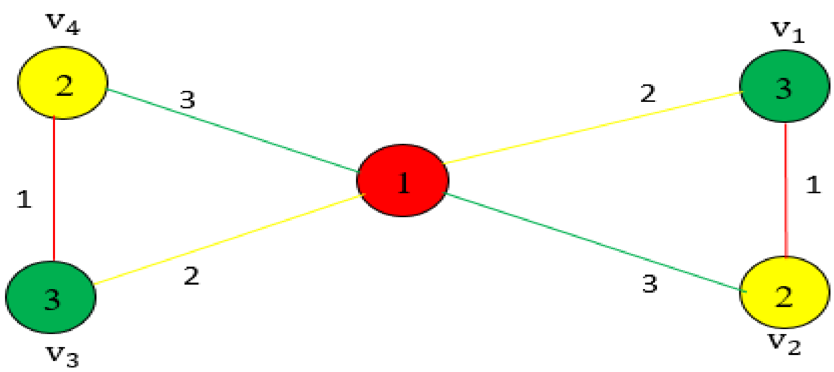

Figure-4: $\psi_T(B_4) = 3$

2.5 Wheel graph:

A single vertex is connected in a wheel graph to all vertices of a cycle,which has one internal vertex and m leaves and it is denoted byw_m,1-skeleton of an (m-1)-gonal pyramid is defined with m vertices in wheel graph.

2.6 Helm graph:

A helm graph denoted by H_m is a graph obtained by attaching a single edge and node to each node of the outer circuit of a wheel graph.

2.7 Web Graph:

A web graph denoted by W_m is a graph obtained by connecting each node by edges and by attaching single edges and nodes to each node of the outer circuit of a helm graph H_m.

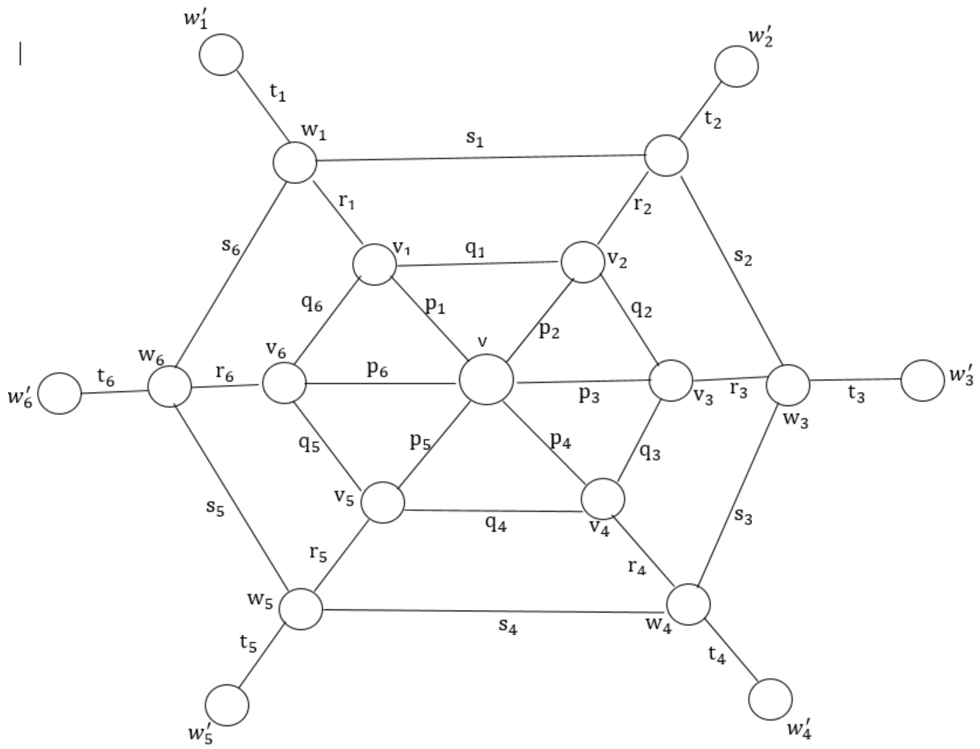

Figure 5. Web Graph

2.7.1Algorithm for coloring web graph:

Input: W_m, m≥4

V← $\{v, v_1 .... v_m, w_1 .... w_m, w_1' .... w_m'\}$

E← $\{p_i \leftarrow vv_i$(i =1 to m),

$q_i \leftarrow v_iv_{i+1}$(i =1 to m-1), $q_m \leftarrow v_mv_1$

$r_i \leftarrow v_iw_i$(i =1 to m)

$s_i \leftarrow v_iv_{i+1}$(i =1 to m) , $s_i \leftarrow v_mv_1$

$t_i \leftarrow w_iw_i^1$(i =1 to m-1)}

v← 1;

for i =1 to m

{

p_i← $i + 1$;

}

end for

for i =1 to m-1

{

end for a← $i + 5$;

if a ≤ m+1;

v_i← a;

else

v_i← a-m;

v_m ←2

for i=1 to m

{

b← $i + s4$;

if b ≤ m+1;

q_i ←b;

else

q_i←b-m;

}

end for

for i=1 to m

{

r_i ←1;

}

end for

```
for i=1 to m
{
c←i+1;
if c ≤ m+1;
wk←c;
else
wk←c-m
}
end for
for i=1 to m
{
d←i+4;
if d≤ m+1;
si←d;
else
si← d − m;
}
end for
for i=1 to m
{
g←i+2;
if g≤ m+1;
ti← g;
else
ti←g-m;
}
end for
for i=1 to m;
{
w'i ←1;
}
end for
end procedure
```

Output: vertex and edge coloured W_m.

2.7.2 Theorem: The total chromatic number of web Graph W_m is m+1, m≥4

(i.e) $\psi_T(W_m)$ =m+1, m≥4

Proof:

Since $\Delta(W_m)$ = m, (m+1) colors are required for proper coloring and hence $\psi_T(w_m) \geq$ m +1 Now, the color class of 1 is{v, r_i and w_i' (i =1to m)}.The color class of(2 ≤ i ≤ m + 1) is{p_{i-1},q_f,v_e,w_{i-1},s_j,t_h; f = i + 2 if 2≤ i ≤ 4 and f = i – 4if 5 ≤ i ≤ 7,e = i+1 if 2≤ i ≤ 5 and e = i-5 if 6 ≤ i ≤ 7 , j = i+2 if 2≤ i ≤ 4 and if 5 ≤ i≤ 7 , h = i+4 if i =2 and h= i – 2 if 3≤i≤7} . The elements in each color classes are neither incident nor adjacent.

Therefore , the coloring given in the algorithm 2.7.1 is a total coloring of W_m.

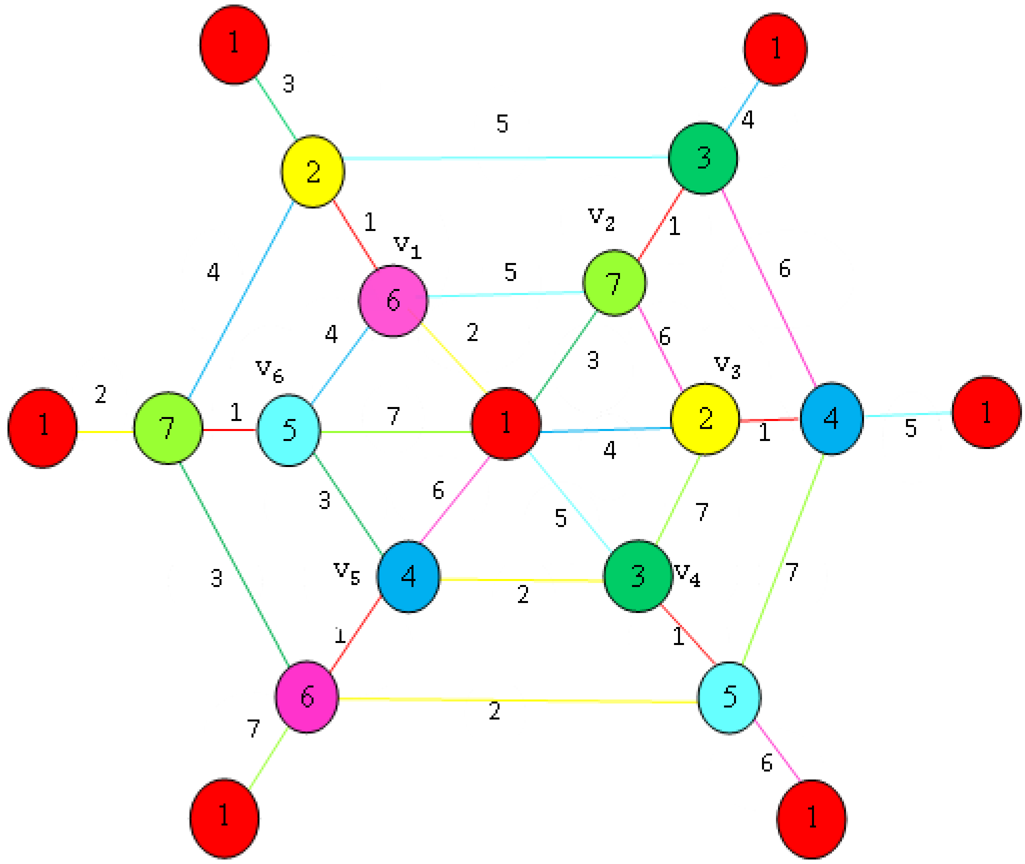

Figure 6. $\psi_T(W_6) = 7$

III. CONCLUSION

In this paper we have obtained the following results.

1. The total chromatic number of Cycle graph is $\psi_T(C_m) = m - 1$ m≥4
2. The total chromatic number of Butterfly graph is $\psi_T(B_m) = m - 1$, m≥4
3. The total chromatic number of Web graph is $\psi_T(W_m) = m + 1$, m≥4

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