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Algorithm on Vertex Coloring

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ABSTRACT

Graph theory is the study of graphs,which are mathematical structures used to model pairwise relations between objects. In our work we have used welsh powell algorithm on vertex coloring, in this paper we formulated the welsh-powell algorithm,it provide a greedy algorithm that runs on a static graph,this is an iterative greedy algorithm.

Keywords : Vertex Coloring, Edge Coloring, Face Coloring, Map Coloring

I. INTRODUCTION

Graph theory concerns the relationship among lines and points, a graph consists of some points and lines between them it is a branch of discrete mathematics and has found multiple applications in computer science,chemistry etc.

A graph is a pair of sets G = (V, E),therefore V is the set of vertices,E is a set of edges,graph theory studies the properties of various graphs. Graphs can be used to model many situations in the real world. Graph coloring is a special case of graph labelling. Graph coloring is the way of coloring the vertices of the graph with the minimum number of colors such that no two adjacent vertices share the same colors.

The paper written by Leonhard Euler on the seven bridges of Konigsberg and published in 1736 is regarded as the first paper in the history of graph theory.Swiss Mathematician Leonhard euler,who,as a consequence of the solution invented the branch of mathematics now known as graph theory.

Example:let G=(V,E) where
V={v_1,v_2,v_3,v_4,v_5},E={e_1,e_2,e_3,e_4,e_5,e_6,e_7}

and the ends of the edges is given by:

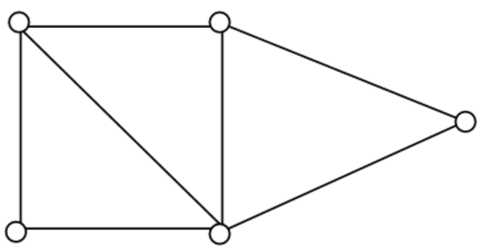
$e_1 \leftrightarrow (v_1, v_2)$,$e_2 \leftrightarrow (v_1, v_3)$,$e_3 \leftrightarrow (v_3,v_4)$,$e_4 \leftrightarrow (v_2,v_4)$,$e_5 \leftrightarrow (v_1, v_4)$, $e_6 \leftrightarrow (v_2, v_5)$,$e_7 \leftrightarrow (v_4,v_5)$

$v_1 e_1 v_2$

e_6

$e_2 e_4 v_5$

e_7



$v_3 e_3 v_4$

Fig 1. A graph G with 5 vertices and 7 edges

II. BASIC DEFINITION

2.1: Vertex Coloring:

Vertex coloring is an assignment of labels or colors to each vertex of a graph.Thus in an undirected graph there exists an edge between them, if no two distinct adjacent vertices have the same color is called "vertex coloring".vertex coloring is a loopless graph.

2.2: Edge Coloring:

An edge coloring of a graph is an assignment of colors to the edges of the graph,so that no two edges have the same color.It is one of the several different types of graph coloring is called "Edge Coloring".

2.3: Map Coloring:

A Map is defined to be a plane connected graph with no bridges.A Map is said to be a k-face colourable,if we color its region with atmost k colors in such a way, that no two adjacent regions (i.e.).,two region sharing a common boundary edges have the same color.

2.4: Face coloring:

Face coloring assign a color to each faces, so that no two faces that share a boundary have the same color.

2.5: Four Color Theorem:

The four color theorem states that, given any separation of a plane into contiguous regions, producing a figure called a map,no more than four colors are required to color the region of the map,so that no two adjacent region have the same color.

III. VERTEX COLORING PROBLEM

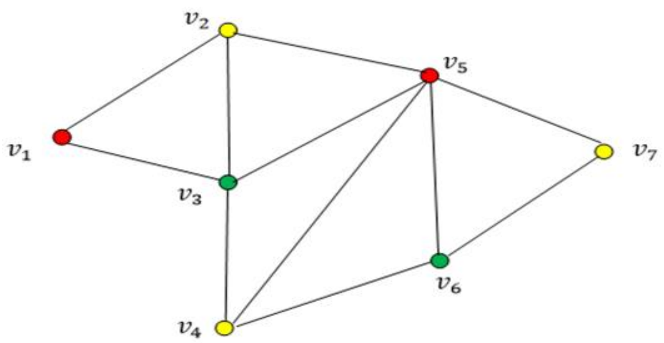


Fig 2 A graph with 7 vertices

Table I. Shows the adjacency matrix for the graph presented in fig.3 and table II shows the degree of vertices for this graph

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	1	1	0	0	0	0
v_2	1	0	0	0	1	0	0
v_3	1	0	0	1	1	0	0
v_4	0	0	1	0	1	1	0
v_5	0	1	1	1	0	1	1
v_6	0	0	0	0	1	0	1
v_7	0	0	0	0	1	1	0

Table II. The Degrees of the vertices

Vertices	v_1	v_2	v_3	v_4	v_5	v_6	v_7
Degrees	2	2	3	3	5	3	2

IV. VERTEX COLORING ALGORITHM IN GRAPHS

Assign color 1 to v_1,moving to vertex v_2 color it 1,if it is not adjacent to v_1;otherwise color it 2 proceeding to v_3,color it 1 if it is not adjacent to v_1;if it is adjacent to v_1;color it 2,if it is not adjacent to v_2;otherwise color it 3.

Case 1:
The vertices $v_1, v_2 \ldots\ldots v_7$,the colors available are 1,2……..7

C_1={1},C_2={1,2},C_3={1,2,3},C_4={1,2..4},C_5={1,2….5},C_6= {1,2,….6},C_7={1,2…..7}

Case 2:

i=1

1 is the first color in C_1,so we assign it to vertex v_1

Case 3:

Color the first vertex with the first color,v_2 and v_3 are adjacent to v_1,we getC_2={2},C_3={2,3}

i becomes 2 and we return to

2 is the first color in C_2,so we assign it to vertex v_2.

$v_5$are adjacent to v_2,

C_5={1,3..5}

i becomes 3 and we return to

Case 4:

3 is the first color in C_3,so we assign it to vertex v_3

v_4 and v_5 are adjacent to v_3

C_4={1,2,4},C_5 stays as {1,2,4,5}

i becomes 4 and we return to

Case 5:

2 is the first color in C_4,so we assign it to vertex v_4

v_5,v_6 is adjacent to v_4

C_5 stays as {1,3..5},C_6={1,3…6}

i becomes 5 and we return to

Case 6:

1 is the first color in C_5,so we assign it to vertex v_5

v_4,v_6,$v_7$is adjacent to v_5

C_4 = {2,3,4}, C_6={2,3..6},C_7 = {2,3….7}

i becomes 6 and we return to

Case 7:

3 is the first color in C_6,so we assign it to vertex v_6

v_7,v_4 is adjacent to v_6

C_7={1,2,4….7},C_4={1,2,4}

i becomes 7 and we return to

Case 8:

2 is the first color in C_7,so we assign to to vertex v_7

v_5 and $v_6$are adjacent to v_7

C_5 stays as {1,3..5},C_6={1,3…6}

i becomes 8 and we return to

Case 9:

v_1,v_5 are colored 1

v_2,v_4,v_7 are colored 2

v_3,v_6 are colored 3 which are mentioned below in table III, table IV and table V.

Table III

Vertex	v_1	v_5
Color(red)	1	1

Table IV

Vertex	v_2	v_4	v_7
Color(yellow)	2	2	2

Table V

Vertex	v_6	v_3
Color(green)	3	3

V. V. Welsh-Powell Algorithm:

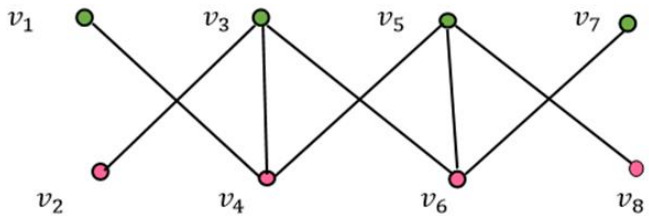


Fig 3. A graph with seven vertices

Table VI

The Degrees of the Vertices

Vertex	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
Valence	1	1	3	3	3	3	1	1

First find the degree of each vertex v_1,v_2…..v_7

List the vertices in order of descending valence (i.e) degree (v(i))>=degree(v(i+1))

Color the first vertex in the given list

Go down the sorted list and color every vertex not connected to the colored vertices above the samecolor,then cross out all the colored vertices in the list Repeat the process on the uncoloured vertices with a new color,always working in descending order of degree until all vertices are colored

Step 1:

List the vertices of G as x_1 , x_2 ….. x_8 ,so that d(x_1) ≥d(x_2)≥……d(x_8),list the colors available as 1,2…..8, Take x_1=v_3,x_2=v_4,x_3=v_1, x_4=v_5,x_5=v_7, x_6=v_2,x_7=v_6,x_8=v_8

C_1={1},C_2={1,2},C_3={1,2,3},C_4={1,2..4},C_5={1,2….5},C_6={1,2,….6},C_7={1,2…..7},C_8={1,2…..8}

Step 2:

x_1 is assigned color 1;(i.e)., v_3 has color 1

C_2={2},C_4={2,3,4},C_6={2,3..6},i becomes 2

Step 3:

x_2 is assigned color 2;(i.e)., v_4 has color 2

C_3={1,3},C_5={1,3….5},i becomes 3

Step 4:

x_3 is assigned color 1;(i.e)., v_1 has color 1

C_4 stays as {2,3,4},i becomes 4

Step 5:

x_4 is assigned color 1;(i.e)., v_5 has color 1

C_6 stays as {2,3..6},C_8={2.3,4…..8},i becomes 5

Step 6:

x_5 is assigned color 1;(i.e)., v_7 has color 1

C_6 stays as {2,3..6},i becomes 6

Step 7:

x_6 is assigned color 2;(i.e)., v_2 has color 2

C_3 stays as {1,3},i becomes 7

Step 8:

x_7 is assigned color 2;(i.e)., v_6 has color 2

C_3 stays as {1,3},C_5 stays as {1,3….5},C_7={1,3,4…..7},i becomes 8

Step 9:

x_8 is assigned color 2;(i.e)., v_8 has color 2

C_5 stays as {1,3….5}

Namely the top three vertices are colored 1 and the bottom three colored 2,so that we have a 2-coloring

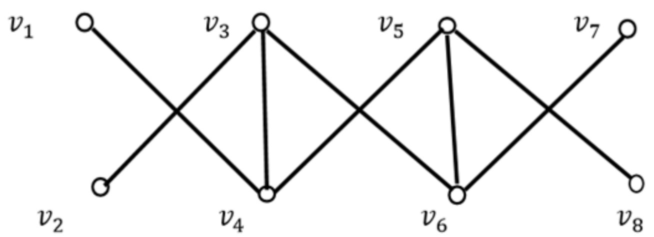


Fig 4 A Graph with seven vertices

All vertices are coloured in a descending order example:

v_3,v_4,v_5,v_6,v_1,v_2,v_7,v_8

v_3 color green

v_4 don't color green since it connect to v_3

v_5 color green

$v_6$don't color green since it connect to v_5

v_1 color green

v_2 don't color green since it connect to v_3

$v_7$color green

v_8 don't color green since it connect to v_5

Table VII

Vertex	v_3	v_5	v_1	v_7
Color(green)	1	1	1	1

We now ignore the vertices that have been already colored,we are left with v_4,v_6,v_2,v_8

Now color with a second color pink

$v_4$color pink

v_6 color pink

v_2 color pink

$v_8$color pink

Table VIII

Vertex	v_4	v_6	v_2	v_8
Color(pink)	2	2	2	2

VI. CONCLUSION

In this paper, we developed an algorithm for Welsh Powell and Vertex Coloringwith minimum different number of colors.

VII. REFERENCES

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