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A Comparative Analysis on Euclidean Measure In Intuitionistic Fuzzy Set And Interval-Valued Intuitionistic Fuzzy Set

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ABSTRACT

Intuitionistic Fuzzy set (IFS) was proposed in early 80™s. It is a well known theory. As a developer in Fuzzy Mathematics, interval" valued Intuitionistic Fuzzy sets (IVFS) were developed afterwards by Gargo and Atanssov. It has a wide range of applications in the field of Optimization and algebra. There are many distance measure in Fuzzy such as Hamming, Normalized Hamming, Euclidean, Normalized Euclidean, Geometric, Normalized Geometric etc¦ to calculate the distance between two fuzzy numbers. In this paper, the comparison between Euclidean distance measure in Intuitionistic Fuzzy set and interval " valued Intuitionistic Fuzzy sets is explored. The step-wise conservation of Intuitionistic Fuzzy set and interval " valued Intuitionistic Fuzzy sets is also proposed. A real life application for this comparison is explained briefly. This type of comparative analysis shows that the distance between Intuitionistic Fuzzy set and interval" valued Intuitionistic Fuzzy sets varies slightly due to boundaries of interval " valued Intuitionistic Fuzzy sets.

Keywords : Single Valued Neutrosophic Set (SVNS), Interval Neutrosophic Set (INS), Euclidean Distance.

I. INTRODUCTION

Euclidean distance is a one of the most commonly used distance measures that has been used to solve many theoretical and practical issues in fuzzy problem. This method is widely use in many fields such as communication [2], engineering [3], chemistry [4], biology [5] and many mathematical specifications such as optimization [6], discrete mathematics [7], statistics [8],operation research [9] and fuzzy mathematics [10] [11]. In fuzzy set theory, Euclidean distance is applied to calculate distances between fuzzy numbers or sets and as a method for decision-making in a situation where two fuzzy sets or fuzzy numbers appear at the same time. Usually, Euclidean distance is applied in a set of discrete fuzzy numbers or values in interval form.

On purpose of finding the distance, many related research have used Euclidean distance in their fuzzy decision problems. Euclidean distance is used for calculating the distance between two fuzzy numbers and using this distance method to calculate the distance of each alternative from the fuzzy positive ideal solution. Then a closeness coefficient of alternative is defined for determining the ranking order of all alternatives. Subsequently, the chosen alternative can be selected and the shortest distance from the ideal solution and the farthest distance from the negative ideal solution are obtained. The objective of this paper is to make a comparison between distance in intuitionistic fuzzy sets (IFS) and interval-valued intuitionistic fuzzy sets (IVIFS). Therefore, in this paper, the conversion of intuitionistic fuzzy sets into interval-valued intuitionistic fuzzy sets is proposed. The paper unfolds as follows.

Section 2 briefly introduces some preliminary definitions concerning fuzzy set theory, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. Section 3 explains the History of distance measures particularly Euclidean distance measures and some of its basic properties.

Section 4 proposes the conversion of IFS into IVIFS step by step. Section 5 described suggested method and brief discussion on Euclidean distance. Section 6 addresses a numerical example proposed for the conversion of IFS into IVIFS and comparison among them in Euclidean distance shown using distance. Conclusions appear in Section 7.

II. METHODS AND MATERIAL

2.1 Preliminaries

Fuzzy set theory, a well-known theory was proposed by Zadeh [12] and defines set membership as a possibility distribution. The general rule for this can expressed as:

$$f:[0,1]^n \to [0,1]$$

where n some number of possibilities. This basically states that we can take n possible events and use f to generate as single possible outcome. In fuzzy set theory, the degree of belonging of element to the set is represented by a membership value in the real interval [0, 1] and there exists degree of non-membership which is complementary in nature. From latter point of view, it is true and acceptable that grade of membership and non-membership are complementary [13].

2.1.1 Fuzzy sets

A fuzzy set A in X = {x} (where x stands for a generic element of X) is given by

$$A=\{\langle x,\mu_A(x)\rangle \mid x\in X\}$$

where the functions μ_A: X → [0,1] is the membership function of the fuzzy set A ;

$$\mu_A(x)\in[0,1]$$

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov[14]. The IFS make the descriptions of the objective more realistic, practical and accurate, making it very promising[12].

2.1.2 Intuitionistic Fuzzy Sets(IFS)

An intuitionistic fuzzy sets in X is given by

$$A=\{\langle x,\mu_A(x),\nu_A(x),\pi_A(x)\rangle \mid x\in X\}$$

Where

$$\mu_A : X \to [0,1]$$

$$\nu_A : X \to [0,1]$$

with the condition

$$0\le \mu_A+\nu_A\le 1$$

The numbers $\mu_A(x)$, $\nu_A(x)\in[0,1]$ denote a degree of membership and non-membership of x to A, respectively.

Obviously, each fuzzy set A’ corresponds to the following:

$$A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle \mid x\in X\}$$

After the work of Atanassov [14], again Gargo and Atanassov [15] introduced the interval-valued intuitionistic fuzzy sets (IVIFS). They have shown several properties on IVIFSs and shown some applications on IVIFSs.

2.1.3 Interval-Valued Intuitionistic Fuzzy Sets(IVIFS)

Let X ≠ ϕ be a given set. An IVIFS is an expression given by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$$

Where

$$\mu_A : X \to D[0,1]$$

$$\nu_A : X \to D[0,1]$$

With the condition

$$0\le \mu_{Au}(x)+\nu_{Au}(x)\le 1 \;\; x\in X$$

2.2 Distance Measures in fuzzy sets

2.2.1 History of distance measure

Fuzzy distance can be widely usage in attribute importance. Many fuzzy distance indices have been proposed since 1965. Some of the methods used crisp number to calculate the distance between two trapezoidal fuzzy numbers [1,2,3,4,5,6,7,8,9]. Human intuition says that the distance between two uncertain numbers should as a collection of points with different degrees of belongingness, then the distance between two fuzzy numbers is noting but the collection of pairwise distance between the elements of the respective fuzzy numbers [10,11,12]. Therefore, we pay to other methods for fuzzy distance, which used fuzzy distance to calculate the distance between two fuzzy numbers and introduce a fuzzy distance for normal fuzzy numbers.

One of the first fuzzy distance method was introduced by voxman [12] through using r-cut. In 2006, Chakraborty and Chakraborty [10] introduced a fuzzy value distance measured by using r-cut of two generalized fuzzy numbers. Then Guha and Chakraborty [13] showed that their previous method could be developed to a new similarity measure with the help of the fuzzy distance measure. It is obvious that, the distance between the fuzzy number A and zero is more suitable to be A. However, by Chakraborty method the distance between the fuzzy number A and zero is not.

2.2.2 Euclidean distance Measures:

A.Hajjari [16] presented a fuzzy Euclidean distance for fuzzy data. Hajjari first described positive fuzzy number, negative fuzzy number and fuzzy zero to represent some new definitions, and then the author discussed fuzzy absolute, equality and inequality of fuzzy numbers based on these concepts and some useful properties too. The aforementioned concepts used to produce the distance between two fuzzy numbers as a trapezoidal fuzzy number.

Abbasbandy and Hajighasemi [15] introduced a symmetric triangular fuzzy number (TFN) as a fuzzy distance based on r-cut concept. All the above-mentioned distance methods used the r-cut concept to calculate the fuzzy distance. There are some other distance methods, which have used other fuzzy concept. Many distance methods for fuzzy numbers have been discussed [16,17,18]. Most of these approaches applied r-cut concept and can consider general fuzzy numbers in one dimension space. Nowadays one of the most applicable types of fuzzy numbers is triangular fuzzy numbers.

The Euclidean distance measure

$$d_E(A,B)=\sqrt{\frac{1}{6}\sum_{i=1}^{n}\left\{\begin{array}{l}\left(u_{AL}(x_i)-u_{BL}(x_i)\right)^2+\left(\pi_{AL}(x_i)-\pi_{BL}(x_i)\right)^2+\\\left(v_{AL}(x_i)-v_{BL}(x_i)\right)^2+\left(u_{AU}(x_i)-u_{BU}(x_i)\right)^2+\\\left(\pi_{AU}(x_i)-\pi_{BU}(x_i)\right)^2+\left(v_{AU}(x_i)-v_{BU}(x_i)\right)^2\end{array}\right\}}$$

2.2.3 Properties of distance measure

A. Subsethood measure

It is well known that subsethood measures can be generated from distance measures. In fuzzy set theory, fuzzy subsethood is an important concept. Zadeh'ssubsethood definition is given by for fuzzy sets A and B

$$A\subseteq B\Leftrightarrow m_A(x)\le m_B(x),\forall x\in X$$

Since a element x in universal set X can belong to a fuzzy set A to varying degrees, it is more natural to consider an indicator of degree to which A is subset of B . In general, such an indicator is a mapping $I:FS(X)\times FS(X)\rightarrow[0,1]$, satisfying special properties, called an inclusion indicator or subsethood measure.

B. Fuzzy subsethood measure:

A mapping $\alpha : FS(X) \times FS(X) \rightarrow [0,1]$ is called a fuzzy subsethood measure, if α satisfies the following properties (for all $A, B, C \in FS(X)$):

(1) $\alpha(A,B) = 1$ if and only if $A \subseteq B$.

(2) Let $\left[\frac{1}{2}\right] \subseteq A$, where $\left[\frac{1}{2}\right]$ is the fuzzy set of X defined by $m_{\frac{1}{2}}(x) = \frac{1}{2}$ for each $x \in X$ .Then $\alpha(A, A^C) = 0$ if and only if $A = X$.

(3) If $A \subseteq B \subseteq C$, then $\alpha(C,A) \le \alpha(B,A)$; and if $A \subseteq B$, then $\alpha(C,A) \le \alpha(C,B)$.

III. RESULTS AND DISCUSSION

3.1 Conversion of intuitionistic fuzzy sets into interval-valued intuitionistic fuzzy sets

The proposed conversion suggest following steps:

Step 1: Consider the IFS is

$$A = \left\{ \langle x, \mu_A(x), \nu_A(x), \pi_A(x) \rangle \mid x \in X \right\}$$

Then the values of membership, non-membership and hesitation are identified.

Step 2: Using the value of membership, the lower bound and upper bound are calculate using the equation

$$\mu_{AL}(x) = \mu_A - \alpha_A \pi_A(x)$$

$$\mu_{Au}(x) = \mu_A - \alpha_A \pi_A(x)$$

Step 3: Using the value on non-membership, the lower bound and upper bound are calculate using the equation

$$\nu_{AL}(x) = \nu_A - \beta_A \pi_A(x)$$

$$\nu_{Au}(x) = \nu_A - \beta_A \pi_A(x)$$

Step 4: The values calculated are arrange in the IVIFS form

$$A = \left\{ \langle x, [\mu_{AL}(x), \mu_{AL}(x)], [\nu_{AL}(x), \nu_{AU}(x)], \pi_A(x) \rangle \mid x \in X \right\}$$

3.2 The Steps of Calculation Of Euclidean Distance

Note that it is possible to generalize this definition to all real numbers by using

$R^n \times R^n \rightarrow R$ [18].

In a direct way, the steps of calculation of Euclidean distance are involving two simple steps:

Step 1: Identify the values of membership and non membership on IFS.

Step 2: For $X = \{x_1, x_2, \ldots, x_n\}$ as the universal set and A and $B \in X$, the degree of membership and non membership are replaced into the Euclidean distance formula:

$$d_E(A,B) = \sqrt{\frac{1}{6} \sum_{i=1}^{n} \left\{ \begin{array}{l} (u_A(x_i) - u_B(x_i))^2 + \\ (\nu_A(x_i) - \nu_B(x_i))^2 \end{array} \right\}}$$

The similar steps are done for calculating Euclidean distance because its only differs on the formula.

Calculations on IVFSs are:

Step 1: Identify the values of membership and non membership on IFS.

Step 2: Let A and B be two IVIFS in X, $X = \{x_1, x_2, \ldots, x_n\}$ then substitute the interval of membership and non membership into the equation as follows:

$$d_E(A,B) = \sqrt{\frac{1}{6} \sum_{i=1}^{n} \left\{ \begin{array}{l} (u_{AL}(x_i) - u_{BL}(x_i))^2 + (\pi_{AL}(x_i) - \pi_{BL}(x_i))^2 + \\ (\nu_{AL}(x_i) - \nu_{BL}(x_i))^2 + (u_{AU}(x_i) - u_{BU}(x_i))^2 + \\ (\pi_{AU}(x_i) - \pi_{BU}(x_i))^2 + (\nu_{AU}(x_i) - \nu_{BU}(x_i))^2 \end{array} \right\}}$$

3.3 Numerical Examples

The calculations are computed using two steps of the Euclidean distance. The results of the distance based on IFS and IVIFS are presented as follows.

Let consider two IFSs A and B in X as follows:

$$A = \{<a, 0.5, 0.2, 0.3>, <b, 0.6, 0.2, 0.2>, <c, 0.9, 0.0, 0.1>\}$$

$$B = \{<a, 0.3, 0.6, 0.1>, <b, 0.4, 0.4, 0.2>, <c, 0.7, 0.2, 0.1>\}$$

Step 1: The values of membership and non-membership are identified:

$\mu_A(a) = 0.5 \qquad \mu_A(b) = 0.6$

$\mu_A(c) = 0.9$

$\nu_A(a) = 0.2 \qquad \nu_A(b) = 0.2$

$\nu_A(c) = 0.7$

Step 2: The value of the distance is calculated using the IFS Euclidean distance-based formula:

$$d_E(A,B) = \sqrt{\frac{1}{6}\sum_{i=1}^{n}\begin{Bmatrix}\left(u_A(x_i)-u_B(x_i)\right)^2 + \\ \left(v_A(x_i)-v_B(x_i)\right)^2\end{Bmatrix}}$$

$$d_E(A,B) = \sqrt{\frac{1}{6}\{(0.5-0.3)^2+(0.6-0.4)^2+(0.9-0.7)^2+(0.2-0.6)^2+(0.2-0.4)^2+(0.0-0.2)^2\}}$$

$$d_E(A,B) = 0.5$$

Using the same sets of IFSs,

$$A = \{<a, 0.5, 0.2, 0.3>, <b, 0.6, 0.2, 0.2>, <c, 0.9, 0.0, 0.1>\}$$

$$B = \{<a, 0.3, 0.6, 0.1>, <b, 0.4, 0.4, 0.2>, <c, 0.7, 0.2, 0.1>\}$$

The conversions into IVIFS are follows. First the IVIFS of set A is calculating:

Step 1: The values of membership, non-membership and hesitation are identified

$\mu_A(a) = 0.5 \qquad \mu_A(b) = 0.6 \qquad \mu_A(c) = 0.9$

$\nu_A(a) = 0.2 \qquad \nu_A(b) = 0.2 \qquad \nu_A(c) = 0.0$

$\pi_A(a) = 0.3 \qquad \pi_A(b) = 0.2 \qquad \pi_A(c) = 0.1$

Step 2: Using the value of membership, the lower bound and upper bound are calculate using the equation

$$\mu_{AL}(x) = |\mu_A(x) - \alpha_A \pi_A(x)|$$
$$\mu_{AU}(x) = \mu_A(x) + \alpha_A \pi_A(x)$$

$$\mu_{AL}(a) = |0.5 - 0.5(0.3)| = 0.35$$
$$\mu_{AU}(a) = 0.5 + 0.5(0.3) = 0.65$$

$$\mu_{AL}(b) = |0.6 - 0.5(0.2)| = 0.5$$
$$\mu_{AU}(b) = 0.6 + 0.5(0.2) = 0.7$$

$$\mu_{AL}(c) = |0.9 - 0.5(0.1)| = 0.85$$
$$\mu_{AU}(c) = 0.9 + 0.5(0.1) = 0.95$$

Step 3: Using the value on non-membership, the lower bound and upper bound are calculate using the equation

$$\nu_{AL}(x) = |\nu_A(x) - \beta_A \pi_A(x)|$$
$$\nu_{AU}(x) = \nu_A(x) + \beta_A \pi_A(x)$$

$$\nu_{AL}(a) = |0.2 - 0.5(0.3)| = 0.05$$
$$\nu_{AU}(a) = 0.2 + 0.5(0.3) = 0.35$$

$$\nu_{AL}(b) = |0.2 - 0.5(0.2)| = 0.1$$
$$\nu_{AU}(b) = 0.2 + 0.5(0.2) = 0.3$$

$$\nu_{AL}(c) = |0.0 - 0.5(0.1)| = 0.05$$
$$\nu_{AU}(c) = 0.0 + 0.5(0.1) = 0.05$$

Step 4: The values calculated are arrange in the IVIFS form

$$A = \{<a, [0.35, 0.65], [0.05, 0.35], 0.3>, <b, [0.5, 0.7], [0.1, 0.3$$

The steps of conversion, Step 1 until Step 4 are repeated to the IFS in set *B*. The conversion obtained the new sets of IVIFS:

$$A = \{<a, [0.35, 0.65], [0.05, 0.35], 0.3>, <b, [0.5, 0.7], [0.1, 0.3$$
$$B = \{<a, [0.25, 0.35], [0.55, 0.65], 0.1>, <b, [0.3, 0.5], [0.3, 0.5$$

The distance of IVIFSs is calculating using Euclidean distance:

Step 1: The values of membership and non-membership are identified.

$\mu_{AL}(a) = 0.35 \qquad \nu_{AL}(a) = 0.05$

$\mu_{AU}(a) = 0.65 \qquad \nu_{AU}(a) = 0.35$

$\mu_{AL}(b) = 0.5 \qquad \nu_{AL}(b) = 0.1$

$\mu_{AU}(b) = 0.7 \qquad \nu_{AU}(b) = 0.3$

$\mu_{AL}(c) = 0.85 \qquad \nu_{AL}(c) = 0.05$

$\mu_{AU}(c) = 0.95 \qquad \nu_{AU}(c) = 0.05$

$\mu_{BL}(a) = 0.25$ $\nu_{BL}(a) = 0.55$
$\mu_{BU}(a) = 0.35$ $\nu_{BU}(a) = 0.65$

$\mu_{BL}(b) = 0.3$ $\nu_{BL}(b) = 0.3$
$\mu_{BU}(b) = 0.5$ $\nu_{BU}(b) = 0.5$

$\mu_{BL}(c) = 0.65$ $\nu_{BL}(c) = 0.15$
$\mu_{BU}(c) = 0.75$ $\nu_{BU}(c) = 0.25$

Step 2: The value of Euclidean distance is calculated using the formula of Euclidean distance for IVIFS as follows

$$d_E(A,B) = \sqrt{\frac{1}{6}\sum_{i=1}^{n}\left\{\begin{array}{l}(u_{AL}(x_i)-u_{BL}(x_i))^2+(\pi_{AL}(x_i)-\pi_{BL}(x_i))^2+\\(v_{AL}(x_i)-v_{BL}(x_i))^2+(u_{AU}(x_i)-u_{BU}(x_i))^2+\\(\pi_{AU}(x_i)-\pi_{BU}(x_i))^2+(v_{AU}(x_i)-v_{BU}(x_i))^2\end{array}\right\}}$$

$$d_E(A,B) = \sqrt{\begin{array}{l}\frac{1}{6}\{(0.35-0.25)^2+(0.65-0.35)^2+(0.05-0.55)^2+\\(0.35-0.65)^2+(0.5-0.3)^2+(0.7-0.5)^2+\\(0.1-0.3)^2+(0.3-0.5)^2+(0.85-0.65)^2+\\(0.95-0.75)^2+(0.05-0.15)^2+(0.05-0.25)^2\}\end{array}}$$

$$d_E(A,B) = \sqrt{\frac{1}{6}\left[0.1^2+0.3^2+0.2^2+0.2^2+0.2^2+0.2^2+0.5^2+0.3^2+0.2^2+0.2^2+0.1^2+0.2^2\right]}$$

$$d_E(A,B) = 0.591$$

The distance between set A and B in IFS is 0.5 while the distance in IVIFS is 0.591. The values differ because in IVIFS, the existed values are in form of boundary but there is no boundary in IFS. So the usage in Euclidean distance also different between IFS and IVIFS because of theexisting boundaries in IVIFS.

IV. CONCLUSION

The Euclidean distance is one of the methods that use in finding the distance. In fuzzy, it is useable when there is two or more fuzzy numbers or fuzzy data. In this paper, the conversion of IFS into IVIFS was proposed and the distance for IFS and IVIFS was calculated using Euclidean distance. The values of distances are different.

At the end, the objective obtained as the calculation in IFS and IVIFS are shown. The finding is hoped to give a guide and clearer calculation between Euclidean distance in IFS and IVIFS as well as giving a rough idea on difference between them. Further research could be extended to refine the difference between these two distances.

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