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A Case study on an Economic problem by using Fuzzy linear Equations

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ABSTRACT

With uncertainty on the parameters, linear system of equations plays an important role in Economics and Finance. In Economics, linear systems of equations with uncertainty on parameters are widely used due to some imprecise data on the relation of linear system of equations. In this paper, an economic problem is solved by fuzzy version of linear system of equations.
Keywords : Triangular fuzzy number, Gaussian fuzzy number, linear system of fuzzy version, uncertainty

I. INTRODUCTION

After development of fuzzy set thoery researchers have successfully applied in economics. Buckley (1987) applied fuzzy mathematics in finance; in 1992 Buckley devised a technique to solve fuzzy equations in economics and finance, Calzi (1990), discussed a general setting for the fuzzy mathematics of finance, Chiu and Park (1994), studied Fuzzy Cash Flow Analysis, Dimitrovski and Matos (2000) applied Fuzzy set in engineering economic analysis, Kahraman (2001) made comparison study on Fuzzy versus probabilstic benefit/cost ratio analysis for public work projects.

Kahraman et.al. (2000) studied Justification of manufacturing technologies using fuzzy benefit/cost ratio analysis, Kahraman et.al. (2005) studied investment analyses under fuzziness Fuzzy Decision Analysis for Alternative Selection Using a Fuzzy Annual Worth Criterion, Omitaomu and Badiru (2007), discussed Fuzzy Present Value Analaysis Model For Evaluating Information System Projects.

System of lineat equations play a vital role in real life problems such as optimization, economics and in engineering. Due to presence of uncertainty/ imprecision in the variables of systems of linear equations variables may be considered as fuzzy numbers. Cong-Xing and Ming (1991) used the concept of embedding approach for fuzzy number space. By using this embedding concept Friedman et. al. (1991) proposed a general model for solving a fuzzy system of linear equation.

Wang et.al. (2001) discussed an interative method for solving a system of linear equation of the form $X = AX + U$ .Asady et.al. (2005) also studied a general fuzzy system and developed different methods using embedding approach. Solution of a system of linear equations with fuzzy numbers is also investigated by Horcik (2008). Vroman et.al. (2007) used the parametric form of fuzzy number to solve the fuzzy general linear systems. Recently, Li. et.al. (2010) presented a new algorithm to solve fuzzy system of linear equations. Sevastjanov and Dymova (2009) proposed a new method for both the interval and fuzzy systems.

Garg and Singh (2008) used numberical approach to solve fuzzy system of linear equations with Gaussian fuzzy membership function. Behera and Chakraverty (2012) very recently developed a new solution method which can handle both fuzzy real and complex system of linear equations. Also very recently Amirfakhrian (2012) proposed one solution method for solving system of fuzzy lineat equations using fuzzy distance approach.

Chakraverty and Behera (2012) developed centre and width based approach for solving fuzzy system of linear equations. Senthilkumara and Rajendran (2011) discussed and algorithmic approach for solving fuzzy linear systems. Authors [Das and Chakraverty (2012); Senthilkumara and Rajendran (2011)] also investigated fully fuzzy system of linear equations.

System of linear equations has a wide application in varying subjects including mathematics, physics, statistics, operation research, economics, finance and social sciences. Most of these applications are characterized by the lacking of the imprecision system of coefficients and improper information on actual parameters values.

Therefore, there is a need to review the mathematical models and numerical variable that are suitable to deal with these ambiguous values. To overcome these situations, researchers may introduce fuzzy numbers instead of crisp numbers.

II. METHODS AND MATERIAL

A. Basic Idea of Demand and Supply In Economics

The term 'demand' refers to the quantity demanded of a commodity per unit of time at a given price. It implies also a desire backed by ability and willingness to pay. The demand curve slopes downward to the right. The downward slope of demand reads the law of demand, i.e., the quantity of a commodity demanded per unit increases as its price falls, and vice versa.

Supply : Market supply means the quantity of a commodity which all its producers or sellers offer to sell at a given price, per unit of time.The law of supply can be stated as the supply of a product increases with the increase in its price and decreases with decrease in its price, other things remaining constant.The supply curve slopes upward. It indicates that supply increases with increase in its price.

B. Equilibrium of Demand and Supply

Equilibrium refers to a state of market in which the demanded of a commodity equals the quantity supplied of the commodity. The equality of demand and supply produces equilibrium.

C. Some basic Definitions Of Fuzzy Set Theory

Fuzzy number: A fuzzy number is a convex normalized fuzzy set of the real line *R* whose membership function is piecewise continuous.

Triangular fuzzy number: A triangular fuzzy number *A* can be defined as a triplet [*a*, *b*, *c*]. Its membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \end{cases}$$

D. Gaussian Fuzzy Number

A Gaussian fuzzy number can be expressed as and the membership function can be defined as

$$\mu_A(x) = \exp\left(\frac{-(x-\mu)^2}{2\sigma^2} \right)$$

Where μ represents the MFs centre and σ determines the MFs width.

E. Interval/Fuzzy Solutions Of System Of Linear Equations

Let the system of equations be

$$(\underline{a_{11}}, \overline{a_{11}})\ (\underline{x_1}, \overline{x_1}) + (\underline{a_{12}}, \overline{a_{12}})(\underline{x_2}, \overline{x_2}) + \ldots + (\underline{a_{1n}}, \overline{a_{1n}})\ (\underline{x_n}, \overline{x_n}) = (\underline{r_1}, \overline{r_1})$$

$$(\underline{a_{21}}, \overline{a_{21}})\ (\underline{x_1}, \overline{x_1}) + (\underline{a_{22}}, \overline{a_{22}})(\underline{x_2}, \overline{x_2}) + \ldots + (\underline{a_{2n}}, \overline{a_{2n}})\ (\underline{x_n}, \overline{x_n}) = (\underline{r_2}, \overline{r_2})$$

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$$(\underline{a_{n1}}, \overline{a_{n1}})\ (\underline{x_1}, \overline{x_1}) + (\underline{a_{n2}}, \overline{a_{n2}})(\underline{x_2}, \overline{x_2}) + \ldots + (\underline{a_{nn}}, \overline{a_{nn}})\ (\underline{x_n}, \overline{x_n}) = (\underline{r_n}, \overline{r_n})$$

The above equations may be written equivalently as :

$$\underline{a_{11}}\,\underline{x_1} + \underline{a_{12}}\ \underline{x_2}\ + \ldots + \underline{a_{1n}}\,\underline{x_n} = \underline{r_1}$$

$$\overline{a_{11}}, \overline{x_1} + \overline{a_{11}}\ \overline{x_2} + \ldots + \overline{a_{11}}\ \overline{x_n} = \overline{r_1}$$

$$\underline{a_{n1}}\,\underline{x_1} + \underline{a_{n2}}\ \underline{x_2}\ + \ldots + \underline{a_{nn}}\,\underline{x_n} = \underline{r_n}$$

$$\overline{a_{n1}}\,\overline{x_1} + \overline{a_{n2}}\ \overline{x_2} + \ldots + \overline{a_{nn}}\ \overline{x_n} = \overline{r_n}$$

F. Solving an Economic Problem Using The Above Method

Let us consider that demand and supply are liner functions of price:

$$q_d = a * p + b$$
$$q_s = c * p + d$$

At equilibrium point these quantities are equal. a and c are crisp real numbers; b and d are to be estimated. Here we take b and d are Gaussian fuzzy number and triangular fuzzy number respectively.

Let a = -3 and c=1; b=Gauss (15,1) and d=[5,7,9] α– cut of b and d are

$[15-\sqrt{-21na}, 15+\sqrt{-21na}]and[5+2a, 9-2a]$ respectively

Therefore, the fuzzy version of the system will be

$$([1,1](x_1, x_2)+[3,3](p_1, p_2)=[15-\sqrt{-21na}, 15+\sqrt{-21na}$$
$$([1,1](x_1, x_2)+[-1,-1](p_1, p_2)=[5+2a, 9-2a]$$

- **On solving we get**

$$x_1 = \frac{3a}{2} - \frac{\sqrt{2(-2ina}}{4} + \frac{5}{2}$$
$$p_1 = \frac{5}{2} - \frac{\sqrt{2(-2ina)}}{4}\frac{a}{2}$$
$$x_2 = \frac{\sqrt{2(-2ina}}{4} - \frac{3a}{2} + \frac{21}{2}$$
$$p_2 = \frac{a}{2} + \frac{\sqrt{2(-2ina)}}{4} + \frac{3}{2}$$

Therefore, a- cut of p is [p_1, p_2], from which fuzzy number p can be evaluated. That is, we can assume the price of a commodity at the point of equilibrium.

III. CONCLUSION

We have investigated the solution of a fuzzy system by converting fuzzy numbers into intervals. We have applied the methodology to an economic problem. Any economic problem relating to linear equations can be solved by fuzzy version of linear system if the data are imprecise.

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