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Ion Thermal Rogue Waves Behaviour in Pure Pair-Ion Plasma

H. N. Abd El-Razek, I. M. Rizk

Theoretical Physics Group, Department of Physics, Faculty of Science, Damietta University, New Damietta, Egypt

ABSTRACT

The properties of ion thermal rogue waves in unmagnified warm pair-ion plasma in the vicinity of the critical density of the negative ions are investigated. Using the reductive perturbation theory, an extended modified Korteweg-de Vries equation (EMKdV) is obtained. At small wave number and using the derivative expansion method the EMKdV equation is reduced to the corresponding nonlinear Schrödinger equation (NLSE). In the regions of the modulation instability, rogue waves are formed. It is found that the different system parameters have remarkable effects on both the possibility of forming the rogue waves as well as their amplitudes.

Keywords: Pair-Ion plasma, Rouge wave, Ion thermal plasma, Extended Modified Kortewegde Vries equation (EMKdV) and Nonlinear Schrodinger equation (NLSE)

I. INTRODUCTION

The ordinary plasma systems consist of ions and electrons, and many of the linear and nonlinear characteristics of the system depend on the big mass difference between ions and electrons. So scientists became possessed by trying both experimentally and theoretically changing this basic structure by adding new components or try to replace electrons by another negative ion species in order to change the basic system characteristics or to achieve certain goals. For example Schermann and Major [1] in 1978 succeeded in generating plasma system with negative ions instead of electrons and another species of positive ion during their project on single particle spectroscopy. In 2003 Oohara and Hatakeyama[2] developed a novel method for generating pure pair-ion plasma using the fullerene as an ion source, and consequently they achieved a great step in studying pair plasmas properties. Pair plasmas have their own thermodynamical properties, in pure pair-ion plasma. It is found that both kinds of the ions species have a single time scale to relax to thermodynamical equilibrium owing to their equal masses [3]. A pair-ion plasma is similar to an electron-positron plasma; however, the problem of annihilation is absent in a pair-ion plasma. The short life time of electron-positron plasma and low density production of positrons in laboratory experiments make it difficult to analyze various collective modes. One of the major obstacles in generating a stable electron-positron plasma is the weak source of positrons (10^6 positrons/s) obtained using the radioactive sources[4] and (10^8 -10^9 positrons/s) with accelerator based source[5]. Therefore the attention is focused on the stable generation of pair-ion plasmas in laboratory such as fullerene $C^{\pm}_{60}$ and hydrogen $H^{\pm}$ plasmas. Experimental observations[6] of a pair-ion fullerene $C^{\pm}_{60}$ plasma have invoked a great deal of interest in these topics. It has been reported that pure PI fullerene plasmas can support three kinds of electrostatic waves propagating parallel to an external static magnetic field. These waves are the ion plasma waves (IPW), the ion acoustic wave (IAW), and the third one has been named as the intermediate frequency wave (IFW). However, there are two observations on the above experiment[6]. First the IAW has frequency larger than the theoretically calculated one. The second is that the IFW has a mysterious feature that the group velocity is negative but the phase velocity is positive i.e. the mode looks like a backward wave. However, the IPW shows no special features in pair-ion plasma. Some theoretical investigations have shown that the acoustic speed becomes larger in a pair-ion plasma if it is not pure and contains a significant concentration of electrons[7]. Saleem[8] put a criterion to define a pure pair-ion plasma and suggested that the lighter elements (like H and He) are more suitable to produce pair-ion plasmas.

The nonlinear waves in plasma can be described either by the Korteweg--de Vries equation KdV family equations or the nonlinear Schrödinger equation (NLSE). The KdV equation describes the evolution of non-modulated wave, i.e. a bare pulse with no fast oscillations inside the packet, which is usually called the KdV soliton. The NLSE governs the dynamics of a modulated wave packet. In the NLSE, the nonlinearities are in balance with the wave group dispersion and the resulting stationary solutions of the NLSE have an envelope structure, called envelope soliton[9]. The NLSE, where the modulational instability (MI) phenomenon could be studied, is considered as one of the most important equations which governs the movement of the nonlinear structures in many branches of physics; condensed matter, nonlinear optics, plasma, and even biophysics[10]. When the wave packet is modulationaly unstable, it may dissociate into smaller wave packets or to single waves. One of the most interesting type of waves which could be formed in the case of the modulation instability is the rogue waves.

Rogue waves are nonlinear waves which are short-lived phenomena appearing suddenly out of nowhere, so they can be quite unexpected and mysterious[11]. The average height of the rogue waves can be two or even more times the height of the surrounding waves. Importantly, rogue waves have been observed in many fields; mid-ocean and coastal waters[12], optical systems[13], fiber optics[14], parametrically driven capillaey waves [15], Bose-Einstein condensates [16,17], super fluids[18], optical cavities[19], atmospheric physics and plasma physics[20].

To the authors knowledge the ion thermal rogue waves in a pair-ion plasma system without electrons has not been studied yet. So we tried here to cover this important subject and studying how may the formed wave may be affected by the presence of stationary dust particles. It is well known that in the presence of two opposite polarity species as fluids, there were some critical density at which the nonlinear coefficient of the KdV equation vanishes and this leads to terminate the nonlinear term of the KdV equation and the KdV equation becomes inadequate to describe the system. Moreover to study the evolution equation of the system in the vicinity of such critical density, new stretched independent variables are required. Introducing these new variables into the system of equations will lead to Extended Modified Korteweg-de Vries equation (EMKdV). Using the derivative expansion technique method, this new equation could be transformed into the corresponding NLSE which is valid at a small wave number, from which we can study the modulation instability (MI) and so the possibility of forming the rogue waves[21], which is the motive of this study. This paper is organized as follows: in Sec.2 the model equations are introduced and the derivation of the evolution equations EMKdV and converting it to the corresponding NLSE. In Sec.3 the numerical results are discussed and finally Sec.4 is devoted for the conclusion.

II. MODEL EQUATIONS AND DERIVATION OF NLS EQUATION

We consider a warm unmagnatized plasma system consisting of positive and negative ions as fluids. The system of normalized equations describing such system is given by

$$\frac{\partial n_p}{\partial t}+\frac{\partial}{\partial x}(n_p u_p)=0, \tag{1}$$

$$(\frac{\partial}{\partial t}+u_p\frac{\partial}{\partial x})u_p+\frac{1}{n_p}\frac{\partial n_p}{\partial x}+\frac{\partial \phi}{\partial x}=0, \tag{2}$$

$$\frac{\partial n_n}{\partial t}+\frac{\partial}{\partial x}(n_n u_n)=0, \tag{3}$$

$$(\frac{\partial}{\partial t}+u_n\frac{\partial}{\partial x})u_n+\frac{\sigma}{n_n}\frac{\partial n_n}{\partial x}-\frac{\partial \phi}{\partial x}=0, \tag{4}$$

and the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2}=n_n-n_p \tag{5}$$

In Eqs.(1)-(5), the subscripts p and n stand for positive ions and negative ions, respectively. All number densities are normalized with respect to the unperturbed number density of the positive ions $n_p^{(0)}$. The velocities u_p and u_n are normalized with respect to positive ion-thermal speed $C_{pi}=(T_p/m_p)^{1/2}$. The potential ϕ is normalized by the thermal potential T_p/e . The space and time are normalized by positive ion debye length

$\lambda_{Dp} = (T_e / 4\pi e^2 n_p^{(0)})^{1/2}$ and the inverse of the positive ion plasma period $\omega_{pp}^{-1} = (m_p / 4\pi e^2 n_p^{(0)})^{1/2}$, respectively. Here, e is the electron charge, T_p the positive ions temperature in energy unit, $\sigma = T_n / T_p$ is the negative to positive ions temperature ratio. The quasineutrality condition for this system is $n_n^{(0)} = n_p^{(0)}$. $n_n^{(0)}$ and $n_p^{(0)}$ are the unperturbed densities of negative ions and unperturbed positive ions density respectively.

To find the evolution equation of this system we follow the standard method of reductive perturbation technique[22] by introducing the following stretched coordinates

$$X = \varepsilon^{1/2}(x - \lambda t) \text{ and } \tau = \varepsilon^{3/2} t, \tag{6}$$

where λ is the phase velocity and ε is a small parameter. Furthermore, we expand all the physical quantities in Eqs.(1)-(5) as follows

$$n_p = 1 + \varepsilon n_p^{(1)} + \varepsilon^2 n_p^{(2)} + \varepsilon^3 n_p^{(3)}, \tag{7}$$

$$n_n = N_n + \varepsilon n_n^{(1)} + \varepsilon^2 n_n^{(2)} + \varepsilon^3 n_n^{(3)}, \tag{8}$$

$$u_p = \varepsilon u_p^{(1)} + \varepsilon^2 u_p^{(2)} + \varepsilon^3 u^{(3)}, \tag{9}$$

$$u_n = \varepsilon u_n^{(1)} + \varepsilon^2 u_n^{(2)} + \varepsilon^3 u_n^{(3)}, \tag{10}$$

and

$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)}. \tag{11}$$

Substituting the Eq.(6) and the Eqs.(7)-(11) into the set of the basic equations and separating the different orders of ε, we get from the first order $\lambda = (\frac{N_n+\sigma}{N_n+1})^{1/2}$. Proceeding to the next orders it is easy to obtain the KdV equation as

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial X} + \frac{1}{2} B \frac{\partial^3 \phi}{\partial X^3} = 0, \tag{12}$$

where

$$A = \frac{(3\lambda^2 - 1)(\lambda^2 - \sigma)^3 - N_n(3\lambda^2 - \sigma)(\lambda^2 - 1)^3}{2\lambda(\lambda^2 - 1)(\lambda^2 - \sigma)[N_n(\lambda^2 - 1)^2 + (\lambda^2 - \sigma)^2]}, \tag{13}$$

and

$$B = \frac{(\lambda^2 - \sigma)^2 (\lambda^2 - 1)^2}{\lambda(N_n(\lambda^2 - 1)^2 + (\lambda^2 - \sigma)^2)}. \tag{14}$$

There is a critical density value of the negative ions at which the nonlinear coefficient of this equation vanishes. In this case the stretched variables (6) are not valid to use any more, so we assume the new stretched variables[23,24] and

$$X = \varepsilon(x - \lambda t) \text{ and } \tau = \varepsilon^3 t, \tag{15}$$

Using this expansion with the previously assumed expansion in Eqs.(7)-(11) and following the method in Refs.23 and 24 into the set of the basic equations and separating the different orders of ε, we can easily obtain the modified KdV equation.

$$\frac{\partial \phi}{\partial \tau} + C\phi^2 \frac{\partial \phi}{\partial X} + \frac{1}{2} B \frac{\partial^3 \phi}{\partial X^3} = 0, \tag{16}$$

where

$$C = \frac{1}{4\lambda(\lambda^2 - 1)^3(\lambda^2 - \sigma)^3[N_n(\lambda^2 - 1)^2 + (\lambda^2 - \sigma)^2]} \times [(15\lambda^4 - 4\lambda^2 + 1)(\lambda^2 - \sigma)^5 + N_n(15\lambda^4 - 4\sigma\lambda^2 + \sigma^2)(\lambda^2 - 1)^5]. \tag{17}$$

In the vicinity of the critical density of negative ions, neither the KdV equation (12) nor the modified KdV equation (16) is adequate for describing the evolution of the system. So we have to look for a new evolution equation. Following the work done by Watanabe[25] and El-labany and El-Sheikh[23] we obtain

$$\frac{\partial \phi}{\partial \tau} + (A\phi + C\phi^2) \frac{\partial \phi}{\partial X} + \frac{1}{2} B \frac{\partial^3 \phi}{\partial X^3} = 0, \tag{18}$$

which describes the evolution equation in the vicinity of the critical density of the negative ions. The nonlinear term is a combination of the nonlinear term of the KdV equation (12) and the nonlinear term of the modified KdV equation (16).

Now in order to study the possibility of generation of rogue waves in this system, we should determine the regions of modulational instability in which the rogue waves could be formed. This could be achievable by converting Eq. (18) into the corresponding NLSE equation which is valid only for small wave numbers. To do so we apply the derivative expansion technique[26]. We assume a solution of Eq. (18) in the form of a weakly modulated sinusoidal wave by expanding ϕ as:

$$\phi(X,\tau)=\sum_{m=1}^{\infty}\varepsilon^m\sum_{l=-\infty}^{\infty}\phi_l^{(m)}(\zeta,\eta)\exp-il(kX-\omega\tau), \quad (19)$$

where k is the carrier wavenumber and ω is the frequency for the given wave. The stretched variables ζ and η are

$$\zeta=\varepsilon(X-v_g\tau)\text{ and }\eta=\varepsilon^2\tau \quad (20)$$

whith v_g is the group velocity, which will be determined later.

Assume that all perturbed quantities depend on the fast scales via the phase $(kX-\omega\tau)$ only, while the slow scales (ζ,η) enter the arguments of the l th harmonic amplitude $\phi_l^{(m)}$. Since $\phi(X,\tau)$ must be real, $\phi_l^{(m)}$ must satisfy the reality condition $\phi_{-l}^{(m)}=\phi_l^{(m)*}$, where the asterisk indicates the complex conjugate.The derivative operators appearing in the system of the basic equations become

$$\frac{\partial}{\partial X}\rightarrow\frac{\partial}{\partial X}+\varepsilon\frac{\partial}{\partial\zeta}\text{ and}$$
$$\frac{\partial}{\partial\tau}\rightarrow\frac{\partial}{\partial\tau}-\varepsilon v_g\frac{\partial}{\partial\zeta}+\varepsilon^2\frac{\partial}{\partial\eta} \quad (21)$$

Using Eqs. (19)-(21) into (18), we obtain

$$i\,(\omega l+Bk^3l^3)\phi_l^{(m)}-(v_g+3Bk^2l^2)\frac{\partial\phi_l^{(m-1)}}{\partial\zeta}$$
$$+\frac{\partial\phi_l^{(m-2)}}{\partial\eta}-3i\,Bkl\frac{\partial^2\phi_l^{(m-2)}}{\partial\zeta^2}$$
$$+C\sum_{m'=1}^{\infty}\sum_{l'=-\infty}^{\infty}(-il''k\,\phi_{l-l'-l''}^{(m-m'-m'')}\phi_{l'}^{(m')}\phi_{l''}^{(m'')}$$
$$+il''k\,\phi_{l-l'-l''}^{(m-m'-m'')}\phi_{l'}^{(m')}\frac{\partial\phi_{l''}^{(m'')}}{\partial\zeta})$$
$$+A\sum_{m'=1}^{\infty}\sum_{l'=-\infty}^{\infty}(-il'k\,\phi_{l-l'}^{(m-m')}\phi_{l'}^{(m')}+\phi_{l-l'}^{(m-m'-1)}\frac{\partial\phi_{l'}^{(m')}}{\partial\zeta})$$
$$+B\frac{\partial^3\phi_l^{(m-3)}}{\partial\zeta^3}=0 \quad (22)$$

The first-order approximation (m=1) with (l=1) yields the linear dispersion relation

$$\omega=-Bk^3 \quad (23)$$

For the first-order harmonic (l=1) of the second-order approximation (m=2),we find that

$$v_g=-3Bk^2 \quad (24)$$

which corresponds to the group velocity, $\partial\omega/\partial k$.
For the second harmonic ($l=2$) with m ■2 we have

$$\phi_2^{(2)}=(A/6Bk^2)\phi_1^{(1)^2}, \quad (25)$$

whereas the zero harmonic ($l=0$) for this order, gives

$$\phi_0^{(2)}=(-A/v_g)\left|\phi_1^{(1)}\right|^2 \quad (26)$$

Proceeding to the third-order approximation ($m=3$) and solving for the first harmonic equations ($l=1$),an explicit compatibility condition will be found, from which we can be easily show to be the NLSE

$$i\frac{\partial\Phi}{\partial\eta}+\frac{1}{2}P\frac{\partial^2\Phi}{\partial\zeta^2}+Q|\Phi|^2\Phi=0, \quad (27)$$

where the dispersion coefficient P is given by

$$P=6Bk, \quad (28)$$

and the nonlinear coefficient Q is given by

$$Q=-(\frac{A^2}{6Bk}-kC), \quad (29)$$

There are many solutions to the Eq(27). One of these solutions is the rational solution which is an appropriate form for describing the rogue wave solution for PQ > zero [27] which is a localized solution in both space and time and is given by

$$\Phi=\sqrt{\frac{P}{Q}}\left[\frac{4(1+2i\,PT)}{1+4\,P^{\,2}\eta^2+4\zeta^2}-1\right]\exp(i\,P\eta). \quad (30)$$

And is as shown in Fig. 1. This Fig. describes the variation of rogue waves in three dimensions for the case of positively charged dust grains.

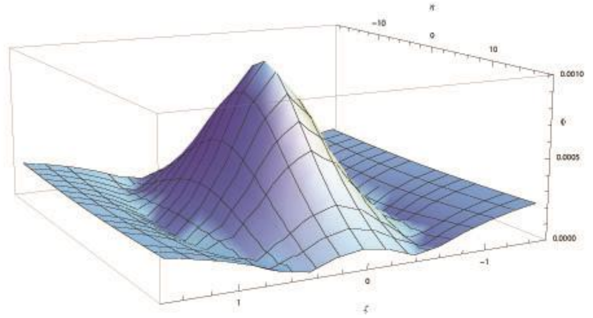

Figure 1: The Amplitude of the rogue waves Φ with ξ and η at σ =0.8 and k =0.05.

III. RESULTS AND DISCUSSION

The NLSE (27) supports many kinds of solutions and governs the modulation instability process in the system which happens due to the interchange of energy between the formed wave packet and the surrounding medium. If this balance is sustained then we will have a case of modulation stability (the value of the product of PQ is less than zero) and in this case the system may form what is called a dark soliton. Otherwise, the wave packet dissociates and may form grey solitons, bright solitons and many other types. One of them is the rogue waves. So before studying the behavior of these waves and how they are varying with different system parameters we have first to determine the modulation instability regions and determine how they vary with the system parameters. Fig. 2 represents the variation of the product PQ with the wave number k at different values of temperature ratio σ. It is noticed that decreasing the values of σ enhances the modulation stability of the system and so decreases of the probability of forming rogue waves.

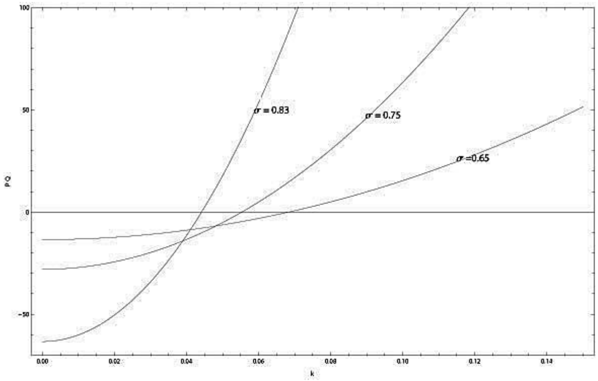

Figure 2: The variation of PQ with k at different values of σ where
σ =0.65, σ =0.75, σ =0.83.py

Fig. 3 shows the variation of the maximum amplitude of the rogue waves with σ . It is obvious that the maximum amplitude decreases with the increase of the temperature ratio.

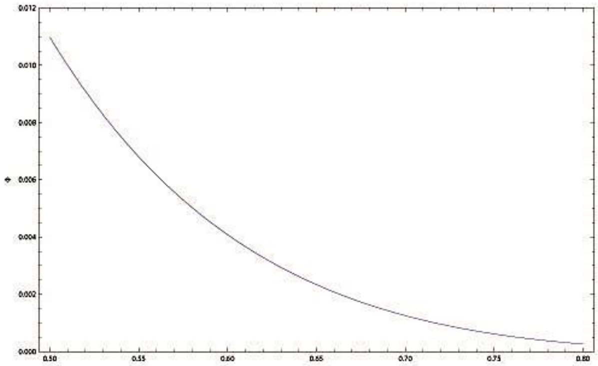
Figure 3: The variation of Φ with σ at k=0.132.

IV. CONCLUSION

The possibility of forming the ion thermal rogue wave is investigated through studying the modulation instability of the system at small wave numbers. It is found that the system pararmeters affect both the probability of forming the rogue waves as well as their amplitudes.

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