International Journal of Scientific Research in Science, Engineering and Technology
Print ISSN : 2395-1990
Online ISSN : 2394-4099
Available Online at : www.ijsrset.com
doi : https://doi.org/10.32628/IJSRSET241145

Solving Simple Hormonic Problems Using Adomian Decomposition Method

Irfan Ul Haq, Sandeep Kumar Tiwari, Pradeep Porwal, Naveed Ul Haq
School of Studies in Mathematics Vikram University, Ujjain, Madhya Pradesh, India

ARTICLEINFO

Article History:

Accepted: 25 June 2024
Published: 19 July 2024

Publication Issue :
Volume 11, Issue 4
July-August-2024

Page Number :
66-70

ABSTRACT

In this research paper, we propose classical numerical technique for solving some simple harmonic problems arising in some applications of science. Adomian decomposition method (ADM) are used. Some numerical examples have been solved to illustrate the accuracy and efficiency of this numerical method.

Keywords : Adomian Decomposition Method, Simple Harmonic Equations, Differential Equations.

I. INTRODUCTION

The Adomian decomposition method is applied for solving linear and non-linear differential and integral equations arising in mathematics, physics, biology and chemistry and up to large numbers of papers have been published. The main advantage of these methods is that it can be directly applied to all types of differential and integral equations .It reduces the size of computation work while still maintaining the accuracy of the numerical solution. In this paper we Adomian decomposition method (numerical method) to find the solution of simple harmonic problems.

11. Adomian Decomposition Method

Consider differential equation

$$Lu + Ru + Nu = (x) \tag{1}$$

Where N represents non-linear factor, L represents the highest order derivative which is supposed to be invertible and R represents a linear differential factor, whose order is less than L.From equation (1), we get

$$Lu = (x) - Ru - Nu \tag{2}$$

As L is invertible, therefore L^{-1} exists. Multiply Equation (3.2) with L^{-1}, we obtain

$$L^{-1}Lu = L^{-1}(x) - L^{-1}Ru - L^{-1}Nu \tag{3}$$

After simplification, from (3), we get

$$u = C + Dx + L^{-1}(x) - L^{-1}Ru - L^{-1}Nu \tag{4}$$

where C and D are constants of integration and can be obtained from the initial or boundary conditions. Adomian method approximate the solution of Equation (1) in the form of infinite series.

$$u(x) = \sum_{n=0}^{\infty} u_n\,(x) \tag{5}$$

and decomposing the non-linear operator N as

$$N(u) = \sum_{n=0}^{\infty} A_n \tag{6}$$

where A_n represents the Adomian polynomials as discussed in [2,3] and are givenby

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N(\textstyle\sum_{n=0}^{\infty}\lambda^i u_i\right]_{\lambda=0} \quad , \qquad n = 0,1,2,3, ...$$

Putting (3) and (6) into (4) we obtain

$$\sum_{n=0}^{\infty} u_n = C + Dx + L^{-1}G(x) - L^{-1}R\left(\sum_{n=0}^{\infty} u_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right)$$

The recursive relationship is found to be

$$u = G(x)$$

$$u_{n+1} = -L^{-1} \; - L^{-1}A_n$$

Using the above recursive relationship, we can make solution of u as

$$u = \lim_{n\to\infty} \phi_n\,(u) \tag{7}$$

Where

$$\phi_n(u) = \sum_{i=0}^{n} u_i \tag{8}$$

Numerical Problems

Example1. Consider the simple harmonic equation

$$\frac{d^2x}{dt^2} + \mu^2 x = 0 \tag{9}$$

The exact solution is $x(t) = cos\mu t$. Let $\mu = 1$ and initial condition $x(0) = 1, x'(0) = 0$. Apply Adomian decomposition method we obtain

$$L^{-1}(x) = -L^{-1}(x)$$

$$x(t) = x(0) + x'(0) - \int_0^t \int_0^t x(t)dtdt$$

$$x(t) = 1 - \int_0^t \int_0^t x(t)dtdt \tag{10}$$

$$x(t) = \sum_{n=0}^{\infty} x_n$$

From (10) we have

$$\sum_{n=0}^{\infty} x_n = 1 - \int_0^t \int_0^t \sum_{n=0}^{\infty} x_n(t)dtdt \tag{11}$$

$$x_0 + x_1 + x_2 \dots = 1 - \int_0^t \int_0^t (x_0 + x_1 + x_2)dtdt \tag{12}$$

From (12) we have

$$x_0 = 1$$

$$x_1 = -\int_0^t \int_0^t x_0 dtdt = -\frac{t^2}{2}$$

$$x_2 = -\int_0^t \int_0^t x_1 dtdt = \frac{t^4}{24}$$

$$x_3 = -\int_0^t \int_0^t x_2 dtdt = -\frac{t^6}{720}$$

The solution is

$$x = x_0 + x_1 + x_2 + x_3 + \cdots$$

$$x(t) = 1 - \frac{t^2}{2} + \frac{t^4}{24} - \frac{t^6}{720} + \cdots = cost$$

Example 2.

Consider the equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \mu^2 x = 0 \quad (13)$$

The exact solution is $x(t) = (1+t)e^t$. Let $\mu = 1,\ k = 1$ and the initial conditions

$x(0) = 1,\ x'(0) = 0$. Apply Adomian decomposition method we have

$$L^{-1}(x) = -L^{-1}(x) - L^{-1}(x')$$

$$x(t) = 1 + 2t - 2\int_0^t x\,dt - \int_0^t\int_0^t x\,(t)dtdt \quad (14)$$

Now

$$x(t) = \sum_{n=0}^{\infty} x_n$$

From (14) we have

$$\sum_{n=0}^{\infty} x_n = 1 + 2t - 2\int_0^t \sum_{n=0}^{\infty} x_n\,(t)dt - \int_0^t\int_0^t \sum_{n=0}^{\infty} x_n\,(t)dtdt \quad (15)$$

$$x_0 + x_1 + x_2 + \cdots = 1 + 2t - 2\int_0^t (x_0 + x_1 + x_2 + x_3 + \cdots)dt - \int_0^t\int_0^t x_0 + x_1 + x_2 + x_3 + \cdots$$

$$x_0 = 1 + 2t$$

$$x_1 = -2\int_0^t x_0\,dt - \int_0^t\int_0^t x_0\,dtdt = -2t - \frac{5}{2}t^2 - \frac{t^3}{3}$$

$$x_2 = -2\int_0^t x_1\,dt - \int_0^t\int_0^t x_1\,dtdt = 2t^2 + 2t^3 + \frac{3}{8}t^4 + \frac{1}{60}t^5$$

The solution is

$$x(t) = x_0 + x_1 + x_2 + \cdots = 1 - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{8} + \frac{t^5}{30} - \frac{t^6}{144} + \cdots$$

II. CONCLUSION

From the above numerical results it is conclude that Adomian decomposition method (ADM) and play a significant role for solving simple hormonic problems and nonlinear models arising in engineering. The numerical results obtained by both these methods are accurate and are approximately close to the exact solutions. These methods have much wider scope in future for solving two- and three-dimensional problems arising in different branches of engineering.

III. REFERENCES

[1] Qasim, A. F., Al- Rawi, E.S. (2018) "Adomian Decomposition Method with Modified Bernstein Polynomials for Solving Ordinary and Partial Differential Equations". Journal of Applied Mathematics, 2018, pp.1-9.

[2] Shiralashetti. S. C, Mundewadi. R. A, Naregal. S. S., Veeresh. B (2017) "Haar Wavelet Collocation Method for the Numerical Solution of Nonlinear Volterra- Fredholm-Hammerstein Integral Equations". Global Journal of Pure and AppliedMathematics, 13(2), pp. 463-474.

[3] Ramesh. T., Sivasangeetha. N. (2017) "Adomain Decomposition Method for Initial Value Problem". International Journal of Scientific Research in Science, Engineering and Technology (ijsrset.com) (3)6: Volume l3, pp.523-525.

[4] Kaliyappan.M., Hariharan. S.(2019) "Solving Nonlinear Differential Equations using Adomian Decomposition Method through Sagemath". International Journal of Innovative Technology and Exploring Engineering, 8(6), pp.510-515.

[5] Vahidi.A.R and Mokhtari.M (2008)"Adomian Decomposition Method for System of Linear Fredholm Integral Equations of the Second Kind" Applied Mathematical Sciences, Volume 2,pp. 57 - 62.

[6] Mustafa, I. (2005) "Numerical Solution of One Dimensional Nonlinear Burgers Equation and Convergence of Decomposition Method".Appl. Math. Comput., 170,pp.76-85.