

Effectiveness of Frequency-Domain Correlative Coding for MIMO-OFDM Systems Over Time Selective Rayleigh Fading Channels

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ABSTRACT

Multiple-input multiple-output (MIMO) antennas combined with orthogonal frequency division multiplexing (OFDM) has been investigated and proved to be very attractive for high-data-rate transmission communications. It has also been noted that, MIMO-OFDM systems are very vulnerable to time selective fading as channel time-variation destroys the orthogonality among subchannels, causing inter-carrier interference (ICI). Here, we apply frequency-domain correlative coding in MIMO-OFDM systems over frequency-selective, fastfading channels to reduce ICI. We derive the analytical expression of the carrier-to-interference ratio (CIR) to quantify the effect of time-selective fading and demonstrate the effectiveness of correlative coding in mitigating ICI in MIMO-OFDM systems which was reduced up to the lowest level of 8%.

Keywords: OFDM, Time-selective fading, inter-carrier interference (ICI), correlative coding

I. INTRODUCTION

OFDM, though effective in avoiding ISI due to multipath delay, is sensitive to time-selective fading, which destroys the orthogonality among different subcarriers in one OFDM symbol and thus causes ICI [Li, 1999]. If not compensated for, ICI will result in an error floor, which increases as Doppler shift and symbol duration increase. To combat ICI caused by time-selective fading or frequency offset in single-antenna OFDM systems, various methods [Armstrong, 1999], including frequency-domain correlative fading and partial response coding [Zhang, 2003], have been studied. The scheme [Zhao, 1998] can be viewed as a special type of frequency-domain partial response coding with correlation polynomial $F(D) = 1 - D$.

To improve spectral efficiency, MIMO antennas can be combined with OFDM to achieve spatial multiplexing, which forms MIMO-OFDM. Supports of high mobility (e.g., IEEE 802.16e) in MIMO-OFDM systems are critical for many

applications. Similar to single-antenna OFDM, MIMO-OFDM is also sensitive to channel time selectivity.

In this paper, we apply frequency-domain correlative coding originally proposed [Zhao, 1998] for single-antenna OFDM systems to MIMO-OFDM to improve system robustness to time-selective fading. While the analysis [Zhao, 1998] considered a simple case in which ICI is caused by a single parameter – the frequency offset normalized to the subcarrier separation, we consider a more comprehensive and realistic scenario which includes not only the spatial elements, but also the time-varying and frequency-selective aspects of the channel. We focus on deriving, via a rigorous analytical approach, a tractable, closed-form expression of CIR as a function of channel Doppler shift, number of subcarriers, OFDM symbol duration, and the power-delay profile of the multipath fading channel. With the CIR expression derived, we can quantify the impact of time-

selective fading and the improvement due to correlative coding in MIMO-OFDM.

II. METHODS AND MATERIAL

A. System Model

Consider a MIMO-OFDM system with N_t transmit antennas, N_r receive antennas, and N_s subcarriers which employs binary phase shift keying achieved through the frequency-domain polynomial $F(D) = 1 - D$ [Zhao 1998.], (BPSK) modulation. Input symbols $a_i \in \{1, -1\}$ are assumed to be i.i.d. with normalized energy. The correlative coding to encode a_i is which generates a new sequence $b_i = a_i - a_{i-1}$ with $E[b_i] = 0$ and otherwise.

$$E\{b_i b_j^*\} = \begin{cases} 2E\{a^2\} = 2, & i=j \\ E\{a^2\} = -1, & |i-j|=1 \\ 0 & \end{cases} \quad (1)$$

It is well known that the general form of MIMO-OFDM over slowly fading channels (i.e., the channel is time-invariant over several OFDM symbol periods) can be expressed as [Stuber, 2004]

$$\mathbf{y}_k = \mathbf{\Lambda}_k \mathbf{x}_k + \mathbf{n}_k \quad (2)$$

where \mathbf{x}_k and \mathbf{y}_k represent, respectively, the transmitted and received data for all antennas on subcarrier k , $\mathbf{\Lambda}_k$ is an $N_r \times N_t$ matrix whose (i, j) th element, $\{\mathbf{\Lambda}_k\}_{ij}$, denotes the channel frequency response between transmit antenna j and receive antenna i , and \mathbf{n}_k is an $N_r \times 1$ vector denoting the zero-mean AWGN with covariance $\sigma_n^2 \mathbf{I}_{N_r}$ for all antennas on subcarrier k .

B. Effects of Time-Selective Fading

In a time-selective fading environment, the $N_s N_r \times N_s N_t$ spatiotemporal channel matrix \mathbf{H} in one OFDM symbol period is expressed as

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_0(0) & \dots & \mathbf{H}_{L-1} & \dots & \mathbf{H}_1(0) \\ \vdots & & \ddots & & \vdots \\ \mathbf{0} & \dots & \mathbf{H}_{L-1}(N_s - 1) & \dots & \mathbf{H}_0(N_s - 1) \end{pmatrix} \quad (3)$$

where L is the number of resolvable paths and $\mathbf{0}$ is an $N_r \times N_t$ zero matrix. Each non-zero block of \mathbf{H} contains the $N_r \times N_t$ channel matrix $\mathbf{H}_l(n)$ for path l at time nT_s (T_s is the data symbol period).

Assuming a WSSUS channel, all elements of $\mathbf{H}_l(n)$ are modeled as independent complex Gaussian random variables with zero mean and equal variance. The channel is assumed to have an exponential power-delay profile $\theta(\tau_l) = e^{-\tau_l/T_{rms}}$, where τ_l is the delay of the l th path and T_{rms} is the rms delay spread. Since the channel is time-variant, the relationship between the channel coefficients for path l at times nT_s and $(n+m)T_s$ can be described as [Zheng, 2004]

$$\{\mathbf{H}_l(n+m)\}_{ij} = \alpha_m \{\mathbf{H}_l(n)\}_{ij} + \beta_{l,ij}(n+m) \quad (4)$$

Where

$$\alpha_m = \frac{E\{\{\mathbf{H}_l(n)\}_{ij}\{\mathbf{H}_l(n+m)\}_{ij}^*\}}{e^{-m/T_{rms}}} = J_0(2\pi m f_d T_s) \quad (5)$$

f_d is the maximum Doppler shift and $\beta_{l,ij}(n)$ are independent complex Gaussian random variables with zero mean and variance $e^{-(1-\alpha_m^2)}$.

It is observed that the channel matrix \mathbf{H} in (4.3) is no longer a block circulant matrix as the case of slowly fading channels. Consequently, $\mathbf{G} = (\mathbf{U} \otimes \mathbf{I}_{N_r}) \mathbf{H} (\mathbf{U} \otimes \mathbf{I}_{N_t})$ is no longer a block diagonal matrix, where \mathbf{U} is the unitary DFT matrix with

$$\{\mathbf{U}\}_{ij} = 1/\sqrt{N_s} e^{j\pi^2 \frac{-1}{N_s} ij}$$

$0 \leq i, j \leq N_s - 1$. This shows that time-selective fading causes ICI, which is $0 \leq i, j \leq N_s - 1$. This shows that time-selective fading causes ICI, which is represented by the off-diagonal blocks of \mathbf{G} . Let \mathbf{G}_{ij} denote the (i, j) th block of \mathbf{G} .

Eq. (4) can be rewritten as

$$y_k = \sum_{\substack{k'=0 \\ k' \neq k}}^{N_s-1} G_{kk'} x_k' + n_k, \quad k = 0, \dots, N_s - 1. \quad (6)$$

Let \mathbf{Y}_{ij} be an $N_s \times N_s$ matrix given by

$$Y_{ij} = \begin{pmatrix} \text{var}\{G_{00}\} & \dots & \text{var}\{G_{0,N_s-1}\} \\ \vdots & \ddots & \vdots \\ \text{var}\{G_{N_s-1,0}\} & \dots & \text{var}\{G_{N_s-1,N_s-1}\} \end{pmatrix}, \quad 1 \leq i \leq N_s, 1 \leq j \leq N_s. \quad (7)$$

As shown in APPENDIX A, \mathbf{Y}_{ij} has a circulant structure. Note that in this case CIR is the same for all subcarriers and is independent of the channel power-delay profile as well as the number of resolvable paths. Obviously, $C_{\text{corr}} \geq C, \forall k$.

Therefore, correlative coding effectively increase CIR. Note worthily, from (5), it is easy to see that although C_{corr} is different for different subcarriers, the difference diminishes as N_s increases. As indicated earlier, when frequency-domain correlative coding with $F(D) = 1 - D$ is used, the signals modulated on subcarriers are identical with alternate mark inversion code and $\{a_i\}$ can be recovered by using a ML sequence detector.

III. RESULTS AND DISCUSSION

In obtaining the numerical results, we consider a system with two transmit antennas and two receive antennas which employs BPSK modulation and adopt the ‘‘SUI-5’’ channel model [Falconer, 2002]. The time-selective Rayleigh fading channel is assumed to have three resolvable multipath components occurring at 0, 5, and 10 μ s. These paths are modeled as independent complex Gaussian random variables and the rms delay spread of the channel is 3.05 μ s. The maximum Doppler shift is calculated based on a carrier frequency of $f_c = 2$ GHz.

CIR levels versus T_s calculated using Eqs. (4) and (5) are plotted in Fig.1, where the vehicle speed applied is $v_s = 100$ Km/h. CIR curves of the MIMO-OFDM

system with different number of subcarriers in one OFDM symbol ($N_s = 8, 24, \text{ and } 128$) are compared

Table 1. PARAMETER VALUES USED IN THE SYSTEM SIMULATIONS

Cell Geometry	Hexagonal Array with side R = 1000 m
Carrier Frequency	$f_c = 2$ GHz
System Bandwidth	W = 5 MHz
Path Loss Exponent	$r = 3.7$
Vehicle speed	30km, 60km, 100km, 200km
Subcarriers	$N_s = 8, 24 \text{ and } 128$
Shadow Fading	Lognormal, with Standard Deviation $\sigma = 8$ dB
Multipath Fading	Rayleigh (K-factor = 0)
Antenna Pattern	Omnidirectional or Uniform over 120
Thermal Noise Density	$N_0 = -174$ dBm/Hz
Mobile Station's Noise Figure	NF = 8 dB
Transmit Power	PT = 5 W for $f_c = 2$ GHz PT = 31.25 W for $f_c = 5$ GHz
Median Cell-Boundary SNR	$\mu = 20$ dB
Transmit Antenna Array (BS) Length	BS = 3 m
Receive Antenna Array (MS) Length	MS = 0.1 m
AoD Statistics (at the Base Station)	Laplacian Power Angular Spectrum with Angular Spread $\theta_{BS} = 15^\circ = \pi/12$
AoA Statistics (at the Mobile Station)	Laplacian Power Angular Spectrum with Angular Spread $\theta_{MS} = 45^\circ = \pi/4$

In obtaining the numerical results, I consider a system with two transmit antennas and two receive antennas which employs BPSK modulation and adopt the ‘‘SUI-5’’ channel model. The time-selective Rayleigh fading channel is assumed to have three resolvable multipath components occurring at 0, 5, and 10 μ s. These paths are modeled as independent complex Gaussian random variables and the rms delay spread of the channel is 3.05 μ s. The maximum Doppler shift is calculated based on a carrier frequency of $f_c = 2$ GHz.

CIR levels versus T_s calculated using Eqs. (5) and (6) are plotted in Fig. 2 where the vehicle speed applied is $v_s = 100$ Km/h. CIR curves of the MIMO-OFDM system with different number of subcarriers in one OFDM symbol ($N_s = 8, 24, \text{ and } 128$) are compared. As shown in Fig. 1, frequency-domain correlative coding incorporated in this letter can effectively increase CIR and the improvement is proportional to the number of subcarriers. With $N_s = 128$, the improvement is observed to be as high as 3.0dB. The BER performances of MIMO-OFDM systems with and without frequency-domain correlative coding are compared in Fig.1., where $T_s = 5 \times 10^{-7}$ s and $v_s = 100$ Km/h are applied. The ML detection scheme is used when correlative coding is applied. The improvement in the BER performance is also found proportional to the number of subcarriers.

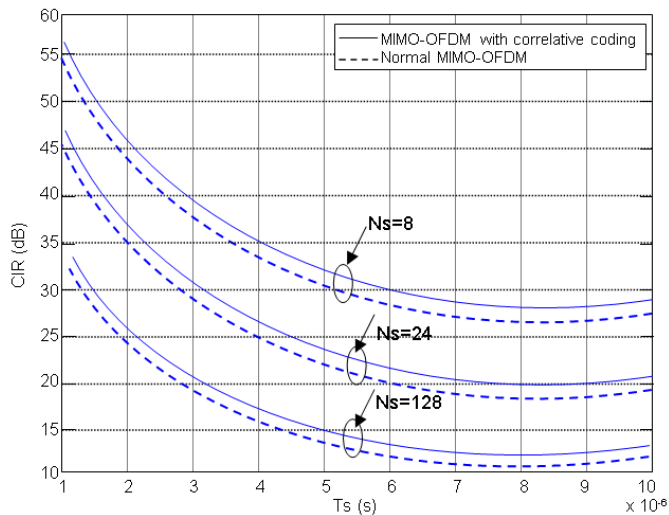


Figure 1. CIR curves of MIMO-OFDM systems with and without frequency-domain correlative coding.

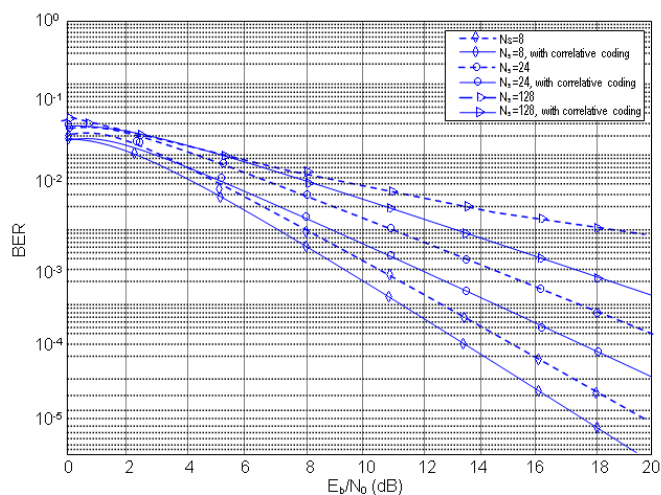


Figure 2. BER versus E_b/N_0 for MIMO-OFDM systems with and without frequency-domain correlative coding.

IV. CONCLUSION

We applied frequency-domain correlative coding to mitigate the effect of time-selective fading to the performance of MIMO-OFDM systems. We derived the analytical expression of CIR as a function of the maximum Doppler shift and power-delay profile of the channel, the number of subcarriers, and the OFDM symbol duration. The CIR expression can be used to quantify the amount of ICI caused by channel time variations. Numerical results indicate that a simple correlative coding scheme with correlation polynomial $F(D) = 1 - D$ can effectively increase CIR of a 128-

subcarrier MIMO-OFDM system by as much as 3.0dB, and the improvement further increases as the number of subcarriers becomes larger.

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