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Conformally Flat Spherically Symmetric Charged Perfect Fluid Distribution in Bimetric Theory of Gravitation

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ABSTRACT
In this paper the solution of Rosen's field equations of bimetric theory of gravitation for conformally flat spherically symmetric charged perfect fluid distribution have been obtained. The various physical properties of the model are also discussed. It is observed that the resulting model is expanding, non-rotating but shearing.
Keywords : Rosen's Field Equations, Perfect Charged Distribution
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I. INTRODUCTION

The bimetric theory of gravitation proposed by Rosen (1977), is the theory of gravitation based on two metrics. One g_{ij} is the fundamental metric tensor describes the gravitational potential and the second metric tensor γ_{ij} refers to the flat space-time and describes the inertial forces associated with the acceleration of the frame of reference. This theory has its own importance is that the Christoffel symbols in it behave like a tensor which are not in general relativity. Moreover the theory agrees with the general relativity up to the accuracy of the observations. However, the bimetric relativity does not account for black holes, which is a cretion of general relativity. In this bimetric theory of gravitation , at every point of the space-time, there associates two metrics

$$ds^2 = g_{ij}\ dx^i dx^j \qquad (1)$$

$$d\sigma^2 = \gamma_{ij}\ dx^i dx^j \qquad (2)$$

The field equations of this Rosen's bimetric theory of gravitation are

$$K_i^j = N_i^j - \frac{1}{2}\delta_i^j N = -8\pi k T_i^j \qquad (3)$$

where $N_i^j = \frac{1}{2}\gamma^{pr}\ (g^{sj} g_{si|p})_{|r}$, $\quad N = N_i^i$,

$$k = \sqrt{\frac{g}{\gamma}} \qquad (4)$$

The Einstein's field equations for conformally flat spherically symmetric charged perfect fluid distribution have been solved by S. N. Pandey and R. Tiwari (1981) [6] and they discussed its physical properties. In this paper the solution of Rosen's field equations of bimetric theory of gravitation for conformally flat spherically symmetric charged perfect distribution have been obtained. The various physical properties of the model are also discussed. It is observed that the resulting model is expanding, non-rotating but shearing.

Solution of Rosen's Field Equations

The Rosen's Field Equations of bimetric theory of gravitation are given by equations (3) and (4).In these field equations T_{ij} , F_{ij} , and E_{ij} are energy-momentum tensor, electromagnetic field tensor, and electric field tensor respectively.

$$T_{ij} = (\in + p)v_i v_j + p g_{ij} + E_{ij} \qquad (5)$$

$$E_{ij} = \frac{1}{4\pi}[F_{ai}F_{bj}g^{ab} - \frac{1}{4}g_{ij}F_{ab}F^{ab}] \qquad (6)$$

where $\in -$ the density and p-the pressure of the perfect charged distribution.

The electromagnetic field tensor F_{ij} satisfies

$$F^{ij}_{|j} = 4\pi\rho v^i \quad (7)$$

$$F_{[ij|k]} = 0 \quad (8)$$

The stroke '|' denote the covarient derivative, ρ is the current density and the four velocity vector v^i satisfies

$$g_{ij} v^i v^j = -1 \quad (9)$$

We consider the metric

$$ds^2 = e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 - dt^2),$$
$$\lambda = \lambda(r,t)\,, \quad (10)$$

and flat space-time corresponding to metric (10) is

$$d\sigma^2 = -dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + dt^2 \quad (11)$$

Assume that the space-time filled with the matter consist of perfect charged distribution with electromagnetic field F_{ij} .

The surviving components of the Christoffel symbols for the flat metric (11) are

$$\Gamma^2_{12} = \Gamma^3_{13} = 1/r, \quad \Gamma^1_{22} = -r, \quad \Gamma^1_{33} = -r\sin^2\theta,$$
$$\Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{23} = \cot\theta\,.$$

Assume that the only surviving component of electromagnetic field $F^{ij`}$ is F^{14} and $v^i = (v^1, o, o, v^4)$, then the field equations (3) for the line element (10) takes the form

$$-\frac{\lambda''}{2} - \frac{\lambda'}{r} + \frac{\ddot{\lambda}}{2} = 8\pi e^{\lambda}[(\in + p)v_1^2 + pe^{\lambda}] - (F_{14})^2$$

$$-\frac{\lambda''}{2} - \frac{\lambda'}{r} + \frac{\ddot{\lambda}}{2} = 8\pi p e^{2\lambda} + (F_{14})^2$$

$$\frac{\lambda''}{2} + \frac{\lambda'}{r} - \frac{\ddot{\lambda}}{2} = 8\pi e^{\lambda}[(\in + p)v_4^2 - pe^{\lambda}] + (F_{14})^2$$

Equation (9) gives

$$(v_4^2 - v_1^2) = e^{\lambda}$$

From the differential equation (12), (13), (14), we write

$$8\pi e^{\lambda}[(\in + p)v_1^2] - 2(F_{14})^2 = 0 \quad (16)$$

$$8\pi e^{\lambda}[(\in + p)v_4^2] + 2(F_{14})^2 = 0 \quad (17)$$

which yields

$$\Rightarrow \quad v_1^2 + v_4^2 = 0$$

and

$$(F_{14})^2 = -2\pi e^{2\lambda}(\in + p) \quad (19)$$

Further the equation (12), (13), (14), gives

$$\lambda'' + 2\frac{\lambda'}{r} - \ddot{\lambda} = 4\pi e^{2\lambda}(\in - 3p) \quad (20)$$

This is a non-linear partial differential equation, and therefore difficult to solve. Sothat one can assume $\in = 3p$. Then equation (20) reduces to

$$\lambda'' + 2\frac{\lambda'}{r} - \ddot{\lambda} = 0 \quad (21)$$

which has solution

$$\lambda = \frac{F_1(t+r)}{2r} + \frac{F_2(t-r)}{r} \quad (22)$$

where F_1 and F_2 are arbitrary functions of $(t + r)$ and $(t - r)$ respectively.

Thus $e^{\lambda} = e^{\left(\frac{F_1(t+r)}{2r} + \frac{F_2(t-r)}{r}\right)}$ and the line-element (10) for the perfect charged distribution would be

$$ds^2 = e^{\left(\frac{F_1}{2r} + \frac{F_2}{r}\right)}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\; d\phi^2 - dt^2)$$

(23)

This is the model for the perfect charged distribution in the bimetric theory of relativity.

3. Some Physical Properties

The condition $(\in = 3p)$ shows that the matter is radiating and under this circumstance, we obtained the pressure and density as

$$8\pi p = -(F_{14})^2\, e^{-2\left(\frac{F_1}{2r} + \frac{F_2}{r}\right)}$$

(12)

$$8\pi \in = -3(F_{14})^2\, e^{-2\left(\frac{F_1}{2r} + \frac{F_2}{r}\right)}$$

(13)

The velocity components are

$$v_1^2 = -v_4^2 = -\frac{1}{2} e^{\left(\frac{F_1}{2r} + \frac{F_2}{r}\right)},$$

(14)

(26)

(15)

and the non-vanishing component of electromagnetic field is

$$(F_{14})^2 = -8\pi p\, e^{2\left(\frac{F_1}{2r} + \frac{F_2}{r}\right)}$$

(27)

(18)

The current density for the model (23) would be

$$\rho = -\frac{\sqrt{2}}{r^2} \; e^{-\frac{3}{2}\left[\frac{F1}{2r}+\frac{F2}{r}\right]} \frac{\partial}{\partial r}\left[r^2 \left(-8\pi p \; e^{2\left(\frac{F1}{2r}+\frac{F2}{r}\right)} \right)^{\frac{1}{2}} \right] \tag{28}$$

The acceleration is given by $\dot{v}_i = v_{i|j} v^j$ and its non vanishing components are,

$$\dot{v}_1 = \dot{v}_4 = \frac{1}{4}(\lambda' + i\dot{\lambda}) \tag{29}$$

The expression for expansion ϕ, the rotation w_{ij} and shear forces σ_{ij} for the model (23) are given by

$$\phi = v^i_{|i}$$

$$-w_{ij} = \frac{1}{2}(v_{i|j} - v_{j|i}) + \frac{1}{2}(\dot{v}_i v_j - \dot{v}_j v_i)$$

$$\sigma_{ij} = \frac{1}{2}(v_{i|j} + v_{j|i}) + \frac{1}{2}(\dot{v}_i v_j + \dot{v}_j v_i) - \frac{\phi}{3}(g_{ij} + v_i v_j),$$

respectively (for details see Ellis 1971).

For the model (23), we calculated ϕ as

$$\phi = \frac{3}{2\sqrt{2}}(\dot{\lambda} - i\lambda') - \frac{i\sqrt{2}}{r}$$

shows that the model is expanding if

$(\dot{\lambda} - i\lambda') > \frac{4}{3}\frac{i}{}\frac{1}{r}$, it is contracting if

$(\dot{\lambda} - i\lambda') < \frac{4\,i}{3}\frac{1}{r}$ and it is neither expanding nor contracting for $(\dot{\lambda} - i\lambda') = \frac{4\,i}{3}\frac{1}{r}$.

The components $w_{ij} = 0$ suggested that the model is non-rotating .

The nonvanishing components of shear forces are

$$\sigma_{11} = \frac{1}{\sqrt{2}}\left(\frac{i}{3r} - \frac{\dot{\lambda}}{2}\right) e^{\lambda|2}$$

$$\sigma_{22} = -\frac{1}{\sqrt{2}} r \left(\frac{i}{3} + r\dot{\lambda}\right) e^{\lambda|2}$$

$$\sigma_{33} = -\frac{1}{\sqrt{2}}\left(\dot{\lambda} + \frac{i}{3r}\right) e^{\lambda|2}\, r^2 \sin^2\theta$$

$$\sigma_{44} = \frac{1}{\sqrt{2}}\left(\frac{\dot{\lambda}}{2} - \frac{i}{3r}\right) e^{\lambda|2}$$

$$\sigma_{14} = \frac{1}{\sqrt{2}}\left(\frac{(3i\dot{\lambda} + \lambda')}{4} + \frac{1}{3r}\right) e^{\lambda|2}.$$

Since the components of shearing forces exists, the resulting model (23) is shearing.

II. CONCLUSION

The solution of Rosen's field equations of bimetric theory of gravitation for conformally flat spherically symmetric charged distribution have been derived. It is observed that under the condition $(\in = 3p)$ the resulting model is expanding if $(\dot{\lambda} - i\lambda') > \frac{4\,i}{3}\frac{1}{r}$.It is non-rotating but shearing.

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IV. REFERENCES

[1]. R. Bali and Anjali : Bianchi Type I magnetized string cosmological model in general relativity, Astrophys Space Sci., 302 (2006), 201-205.

[2]. R. Bali and S. Dave : Bianchi Type- IX string cosmological models with bulk viscous fluid in general relativity, Astrophys. Space Sci., 288 (2003), 503-509

[3]. R. Bali and R. D. Upadhaya : LRS Bianchi Type I string dust magnetized cosmological models, Astrophys. Space Sci., 283 (2003), 97-108.

[4]. R. Bali and D. Singh : Bianchi Type –V bulk viscous fluid string dust cosmological model in general relativity , Astrophys.Space Sci., 300 (2005), 387-394.

[5]. R. Bali and U.K. Pareek : Bianchi Type I String dust cosmological model with magnetic field in general relativity, Astrophys.Space Sci., 312(2007), 305-310.

[6]. M. S. Borkar and S. S. Charjan), Bianchi Type I bulk viscous fluid string dust cosmological model

with magnetic field in bimetric theory of gravitation, An Int. J. AAM 5(1), (2010), 96-109.

[7]. M. S. Borkar and S. S. Charjan : Bianchi Type I Magnetized dust cosmological model in bimetric theory of gravitation, An Int. J. AAM 5 (2), , (2010) p.p. 1660-1671.

[8]. N. P. Gaikwad, M. S. Borkar, and S. S. Charjan ; Bianchi Type I massive string Magnetized baratropic perfect fluid cosmological model in bimetric theory of gravitation Chin. Phys. Lett. 28(8), (2011).

[9]. M. S. Borkar, S. S. Charjan, LRS Bianchi Type I Magnetized Cosmological Model with --Term in Bimetric Theory of Gravitation, American Journal of Pure Applied Mathematics, 2(1), Jan.2013 pp.7-12

[10]. M. S. Borkar, S. S. Charjan, Bianchi Type I Magnetized Cosmological Model with --Term in Bimetric Theory of Gravitation, An Int. J. AAM 8 (1), (June 2013 (116-127).

[11]. M. S. Borkar, S. S. Charjan, V.V. Lepse LRS Bianchi Type I Cosmological Models with Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN:2319-765X. 10(3) Ver. I (May-Jun. 2014), PP 62-69 www.iosrjournals.org www.iosrjournals.org

[12]. M.S. Borkar, S.S. Charjan and V.V. Lepse, Bianchi Type-I Hyperbolic Models with Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, An Int. J. AAM 11 (1), (June 2016), pp. 192 - 214

[13]. S.W. Hawking and G.F. R. Ellis: The large scale structure of space-time, Cambridge University Press, Cambridge (1973) , P.91

[14]. M. Isrelit : Spherically Symmetric fields in Rosen's bimetric theories of gravitation, General Relativity and Gravitation, 13(7), (1981), 681-688.

[15]. T. M. Karade : Spherically Symmetric Space Times in bimetric Relativity Indian J. Pure-appl. Math,11(9), (1980), 1202-1209.

[16]. S .D. Katore and R.S. Rane: Magnetized cosmological models in biometric theory of gravitation, Pramana J. Phys. 67(2), (2006), 237-227.

[17]. G.S. Khadekar and S.D. Tade : String cosmological models in five dimensional biometric theory of gravitation, Astrophys. Space Sci.DoI 10.1007\S10509-007-9410-2. (2007).

[18]. K. D. Krori, T. Choudhuri, and C. R. Mahant: String in some Bianchi Type cosmologies, General Relativity and Gravitation 26(3), (1994), 265-274.

[19]. P. S. Letelier : Clouds of strings in general relativity, Physical review D 20(6), (1979), 1294-1302.

[20]. P. S. Letelier: String cosmology, Phy. review D 28(10), (1983), 2414-2419.

[21]. S.N. Pandey and R.Tiwari, Indian J. pure appl. Maths, 12(2) : 261-264 (Feb.1981)

[22]. D.R.K. Reddy and N. Venkateshwara Rao : On some Bianchi Type Cosmological models in bimetric theory of gravitation, Astrophys. Space Sci 257, (1998), 1293-298

[23]. N. Rosen: A Theory of Gravitation, Annls of Phys, 84, (1974), 455-473

[24]. N. Rosen : A Topics in Theoretical and Experiential Gravitation Physics, edited by V. D .Sabtta and J. Weber. Plenum Press, London , (1977), pp 273-94

[25]. John Stachel : Thickening the string I,The String Perfect dust, Phy. review D21(8) , (1980).

[26]. P. K. Sahoo, B. Mishra and A. Ramu : Bianchi Types V and Vo cosmic string coupled with Maxwell fields in bimetric theory , Int. J. Theor Phys. (2010) DOI 10.1007/s 10773-010-0531-y

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