

# Massive String Cosmological Model for Perfect Fluid Distribution with Cosmological Term $\Lambda$ In Bimetric Theory of Gravitation

S. S. Charjan

Department of Mathematics, K. Z. S. Science College, Bramhani, Kalmeshwar, Dist. Nagpur, Maharashtra, India

**ABSTRACT :** Massive string magnetized cosmological model for perfect fluid distribution with cosmological term  $\Lambda$  is deduced by solving Rosen's field equations. To deduce a determinate solution, the condition  $A = B^n$  (where,  $n > 0$  real constants), between the metric potentials is used. It is seen that Massive string Bianchi Type I magnetized cosmological model (36) in bimetric theory of gravitation exists. It never goes to vacuum model. When  $u \rightarrow -\infty$ , then  $\theta \rightarrow -\infty$ ,  $\sigma \rightarrow -\infty$  and when  $u \rightarrow \infty$ , then  $\theta \rightarrow \infty$ ,  $\sigma \rightarrow \infty$ . This confirms that the model is expanding as well as shearing. The expansion and the shear in the model increases as cosmic time  $u$  increases. Further when  $u \rightarrow \infty$ , then  $\epsilon = \lambda \rightarrow \infty$ ,  $p \rightarrow \infty$ . When  $u \rightarrow -\infty$  then  $\epsilon = \lambda \rightarrow 0$  and  $p \rightarrow 0$ , which shows that our model goes over to vacuum model, when the cosmic time  $u$  is minus infinity. In the special case, in the absence of magnetic field i.e.,  $K = 0$ , it is seen that, our model (36) exists. Also the expansion and shear in the model increases, as the cosmic time  $u$  increases.

**Keywords:** Bimetric Gravitational Theory, Electromagnetic Field, Cosmology

**Mathematics Subject Classification 2010:** 83-XX, 83C50, 83F05

## I. INTRODUCTION

Several new theories of gravitation have been formulated which are considered to be alternatives to Einstein's theory of gravitation. The most important among them is Rosen's bimetric theory of gravitation [1 - 2]. The Rosen's bimetric theory is the theory of gravitation based on two metrics. One is the fundamental metric tensor  $g_{ij}$  describes the gravitational potential and the second metric  $\gamma_{ij}$  refers to the flat space-time and describes the inertial forces associated with the acceleration of the frame of reference. The metric tensor  $g_{ij}$  determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric  $\gamma_{ij}$  refers to the geometry of the empty universe (no matter but gravitation is there) and describe the inertial forces. The metric tensor  $\gamma_{ij}$  has no direct physical significance but appears in the field equations. Therefore it interacts with  $g_{ij}$  but not directly with matter. One can regard  $\gamma_{ij}$  as giving the geometry that would exist if there were no matter. In the absence of matter one would have  $g_{ij} = \gamma_{ij}$ . Moreover, the bimetric theory also satisfied the covariance and equivalence principles; the formation of general relativity. This theory agrees with the present observational facts pertaining to general relativity [3 - 4]. Thus at every point of space-time in Rosen's bimetric theory of gravitation, there are two metrics

$$ds^2 = g_{ij} dx^i dx^j \quad (1)$$

$$d\eta^2 = \gamma_{ij} dx^i dx^j \quad (2)$$

The field equations of Rosen's bimetric theory of gravitation are

$$N_i^j - \frac{1}{2} N \delta_i^j = -8\pi k T_i^j \quad (3)$$

where  $N_i^j = \frac{1}{2} \gamma^{pr} (g^{sj} g_{si|p})_{|r}$ ,  $N = N_i^i$ ,  $k = \sqrt{\frac{g}{\gamma}}$  with  $g = \det(g_{ij})$  and  $\gamma = \det(\gamma_{ij})$ . Here the vertical bar (|) stands for  $\gamma$ -covariant differentiation and  $T_i^j$  is the energy-momentum tensor of matter fields. Cosmic string arises during the phase transition after the big-bang explosion in the evolution of early stage of universe and formation of Galaxies. The general treatment of the string was initiated by Letelier and Stachel [5 - 7]. The cosmic strings have stress energy and couple to the gravitational field and give rise to density perturbations. The magnetic field is present in galactic and intergalactic system and therefore the magnetic field with energy density and string tension density of the cosmic string have been taken in the formation of our model in bimetric theory of gravitation.

In the context of general relativity cosmic strings do not occur in Bianchi Type models (see K. D. Krori et al.) [8]. In it, some Bianchi Type cosmological models – two in four and one in higher dimensions – are studied by Krori et al. They have shown that the cosmic strings do not occur in Bianchi Type V cosmology. Bali and Dave [9], Bali and Upadhaya [10], Bali and Singh [11], Bali and Pareek [12] have investigated Bianchi Type IX, I and V string cosmological models under different physical conditions in general relativity.

Recently, Borkar et al. have developed the models like Bianchi Type I string dust cosmological model with magnetic field in bimetric relativity [13], LRS Bianchi Type I string dust magnetized cosmological models in bimetric theory of relativity [14], The charged perfect fluid distribution in bimetric theory of relativity [15], Bianchi Type I bulk viscous fluid string dust cosmological model with magnetic field in bimetric relativity [16] and Bianchi Type I magnetized cosmological model in bimetric theory of gravitation [17]. Sahoo et al. [18] have investigated Bianchi Types V and VI<sub>0</sub> cosmic strings coupled with Maxwell fields in bimetric theory of gravitation. Gaikwad et al. [19] have developed Bianchi Type I massive string barotropic perfect fluid cosmological model in the bimetric theory of gravitation.

In this paper Massive string magnetized cosmological model for perfect fluid distribution with cosmological term  $\Lambda$  is deduced by solving Rosen's field equations. To deduce a determinate solution, the condition  $A = B^n$  (where,  $n > 0$  real constants), between the metric potentials is used.

It is seen that Massive string Bianchi Type I magnetized cosmological model (36) in bimetric theory of gravitation exists. It never goes to vacuum model. When  $u \rightarrow -\infty$ , then  $\theta \rightarrow -\infty$ ,  $\sigma \rightarrow -\infty$  and when  $u \rightarrow \infty$ , then  $\theta \rightarrow \infty$ ,  $\sigma \rightarrow \infty$ . This confirms that the model is expanding as well as shearing. The expansion and the shear in the model increases as cosmic time  $u$  increases. Further when  $u \rightarrow \infty$ , then  $\epsilon = \lambda \rightarrow \infty$ ,  $p \rightarrow \infty$ . When  $u \rightarrow -\infty$  then  $\epsilon = \lambda \rightarrow 0$  and  $p \rightarrow 0$ , which shows that our model goes over to vacuum model, when the cosmic time  $u$  is minus infinity. In the special case, in the absence of magnetic field i.e.,  $K = 0$ , it is seen that, our model (36) exists. Also the expansion and shear in the model increases, as the cosmic time  $u$  increases.

## II. SOLUTION OF ROSEN'S FIELD EQUATIONS

We consider an LRS Bianchi Type I metric is in the form

$$ds^2 = dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2) \quad (4)$$

where  $A$  and  $B$  are functions of  $t$  alone.

The flat metric corresponding to metric (4) is

$$d\Omega^2 = \Omega dt^2 + dx^2 + dy^2 + dz^2 \quad (5)$$

The energy momentum tensor for the string dust with magnetic field is taken as

$$T_i^j = (\rho + p) v_i v^j + p g_i^j - \lambda x_i x^j + E_j^i \quad (6)$$

$$\text{with } v^i v_i = -x^i x_i = -1 \quad (7)$$

$$\text{and } v^i x_i = 0 \quad (8)$$

In this model,  $\rho$  and  $\Omega$  denote the proper energy density and the string tension density,  $x^i$  is the unit space like vector specifying the direction of string and  $v^i$  is the unit time like vector .

The co-moving coordinate system is chosen as  $v^i = (0,0,0,1)$  ;  $x^i = (\frac{1}{A}, 0,0,0)$

If  $\rho_p$  is the particle density of configuration, then

$$\rho = \rho_p + \lambda$$

of the system of strings respectively. The electromagnetic field  $E_{ij}$  is given by Lichnerowicz (1967)

$$E_{ij} = \bar{\mu} \left[ |h|^2 \left( v_i v_j + \frac{1}{2} g_{ij} \right) - h_i h_j \right] \quad (9)$$

The four velocity vector  $v_i$  is given by

$$g_{ij} v^i v^j = \Omega \quad (10)$$

and  $\bar{\mu}$  is the magnetic permeability and  $h_i$  the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \varepsilon_{ijkl} F^{kl} v^j \quad (11)$$

where  $F_{kl}$  is the electromagnetic field tensor and  $\varepsilon_{ijkl}$  is the Levcivita tensor density.

Assume the comoving co-ordinates and so we have  $v^1 = v^2 = v^3 = 0, v^4 = 1$ . Further the magnetic field taken along x-axis so that

$$h_1 \neq 0, h_2 = h_3 = h_4 = 0$$

yield

$$F_{23} = \text{constant} = H \text{ (say)} \quad (10)$$

Due to the assumption of infinite electrical conductivity, we have

$$F_{14} = F_{24} = F_{34} = 0$$

The only non-vanishing component of  $F_{ij}$  is  $F_{23}$ .

So that

$$h_1 = \frac{AH}{\bar{\mu} B^2}$$

and

$$|h|^2 = \frac{H^2}{\bar{\mu}^2 B^4}$$

The components of electromagnetic field  $E_i^j$  are given by

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = -\frac{H^2}{2\bar{\mu} B^4}$$

$$T_1^1 = \left( p - \lambda - \frac{H^2}{2\bar{\mu} B^4} \right), T_2^2 = T_3^3 = \left( p + \frac{H^2}{2\bar{\mu} B^4} \right), T_4^4 = \left( -\rho - \frac{H^2}{2\bar{\mu} B^4} \right) \quad (12)$$

Rosen's Field Equations becomes

$$-\frac{A_{44}}{A} + \frac{2B_{44}}{B} + \frac{A_4^2}{A^2} - \frac{2B_4^2}{B^2} = 16\pi AB^2 \left( -p + \lambda + \frac{H^2}{2\mu B^4} \right) - 2\Lambda \quad (14)$$

$$\frac{A_{44}}{A} - \frac{A_4^2}{A^2} = 16\pi AB^2 \left( -p - \frac{H^2}{2\mu B^4} \right) - 2\Lambda \quad (15)$$

$$\frac{A_{44}}{A} + \frac{2B_{44}}{B} - \frac{A_4^2}{A^2} - \frac{2B_4^2}{B^2} = 16\pi AB^2 \left( \rho + \frac{H^2}{2\mu B^4} \right) - 2\Lambda \quad (16)$$

### III. SOLUTION OF FIELD EQUATIONS

The field equation (14) (15) and (16) are the arrangement of three equations with six unknowns  $A, B, \Lambda, \lambda, \rho$  and  $p$ . Thus initially the system is undetermined, so we need three more equations to attain the complete solution of the system. Therefore we supposed that expansion  $\theta$  is proportional to share  $\sigma$ .

$$A = B^n \quad \text{and} \quad \Lambda = \frac{\alpha}{AB^2} \quad (17)$$

$$\text{We assume the above conditions under two cases : (i) } \rho + \lambda = 0 \quad \text{and} \quad \text{(ii) } \rho - \lambda = 0 \quad (18)$$

**Case I :**  $\rho + \lambda = 0$

On adding equations (14) and (16) we get

$$\frac{4B_{44}}{B} - \frac{4B_4^2}{B^2} = 16\pi AB^2 \left( -p + \lambda + \rho + \frac{H^2}{\mu B^4} \right) - 4\Lambda \quad (19)$$

$$\frac{B_{44}}{B} - \frac{B_4^2}{B^2} = \frac{48\pi KB^{n-2}}{4-n} - \frac{2\alpha}{(4-n)B^{n+2}} \quad (20)$$

On putting  $B_4 = f(B)$  and  $B_{44} = f'f'$  in equation (20) we obtain

$$\frac{df^2}{dB} - \frac{2f^2}{B} = \frac{96\pi KB^{n-1}}{4-n} - \frac{4\alpha}{(4-n)B^{n+1}} \quad (21)$$

On integration, equation (21) leads to

$$f^2 = \frac{96\pi KB^n}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)B^n} + MB^2 \quad (22)$$

Here M is the constant of integration.

From equation (22), we get

$$\int \frac{dB}{\sqrt{\frac{96\pi KB^n}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)B^n} + MB^2}} = t + V \quad (23)$$

Here V is the constant of integration. Value of B can be obtained from equation (23).

Putting  $B=T$ ,  $x=X$ ,  $y=Y$  and  $z=Z$  metric (1) becomes

$$ds^2 = - \frac{dT^2}{\left[ \frac{96\pi K T^n}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)T^n} + MT^2 \right]} + T^{2n} dX^2 + T^2 (dY^2 + dZ^2) \quad (24)$$

**Case II :**  $\rho - \lambda = 0$

On subtracting equations (14) from (16) we get,

$$2 \left[ \frac{A_{44}}{A} - \frac{A_4^2}{A^2} \right] = 16\pi AB^2 P \quad (25)$$

(25)

Using (17), (18), and (15) in equation (25), we get

$$B_{44} - \frac{B_4^2}{B} = - \frac{16\pi K}{3n} B^{n-1} - \frac{2\alpha}{3n B^{(n+1)}} \quad (26)$$

On putting  $B_4 = f(B)$  and  $B_{44} = f'f'$  in equation (26) we obtain

$$\frac{df^2}{dB} - \frac{2f^2}{B} = \frac{-32\pi K B^{n-1}}{3n} - \frac{4\alpha}{3n B^{n+1}} \quad (27)$$

(27)

On integrating equation (27) we get,

$$f^2 = \frac{-32\pi K B^n}{3n(n-2)} + \frac{4\alpha}{3n(n+2)B^n} + NB^2 \quad (28)$$

Here N is the constant of integration.

$$\int \frac{dB}{\sqrt{\frac{-32\pi K B^n}{3n(n-2)} + \frac{4\alpha}{3n(n+2)B^n} + NB^2}} = t + U \quad (29)$$

Where U is the constant of integration. Value of B can be obtained from equation (29).

Hence, by appropriate transformation of coordinates

$B=T$ ,  $x=X$ ,  $y=Y$  and  $z=Z$  metric (1) becomes

$$ds^2 = - \frac{dT^2}{\left[ \frac{-32\pi K T^n}{3n(n-2)} + \frac{4\alpha}{3n(n+2)T^n} + N T^2 \right]} + T^{2n} dX^2 + T^2 (dY^2 + dZ^2)$$

Now choosing the cosmic time  $u = \pm \log T$ . For convenience, we can select  $u = -\log T$ , then the model (24) goes over to

$$ds^2 = - \frac{du^2}{\left[ M - \frac{32 \pi l}{(4-n)(n-2)} K(e^{-u})^{n-2} + \frac{4\alpha}{n} \right]} + (e^{-u})^2 [(e^{-u})^{2n-2} dX^2 + dY^2 + dZ^2] \quad (30)$$

#### IV. PHYSICAL AND GEOMETRICAL CHARACTERISTICS

For the model (24), energy density( $\rho$ ), string tension density( $\lambda$ ), particle energy density( $\rho_p$ ), pressure(P), expansion( $\theta$ ), share ( $\sigma$ ) are given by

$$\rho = -\lambda = \frac{K(4n+2)}{(4-n)} O^{-4} - \frac{\alpha(n-1)}{4\pi(4-n)} O^{-2(n+2)} \quad (31)$$

$$\rho_p = \frac{2K(4n+2)}{(4-n)} (e^{-u})^{-4} - \frac{\alpha(n-1)}{2\pi(4-n)} (e^{-u})^{-2(n+2)} \quad (32)$$

$$P = \frac{-(2n+4)K}{(4-n)} (e^{-u})^{-4} + \frac{\alpha(n-2)}{4\pi(4-n)} (e^{-u})^{-2(n+2)} \quad (33)$$

$$\theta = (n+2) \left[ \frac{96\pi K O^{n-2}}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)} \right]^{-2(n+2)} + M \Bigg]^{\frac{1}{2}} \quad (34)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \left[ \frac{96\pi K O^{n-2}}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)} \right]^{-2(n+2)} + M \Bigg]^{\frac{1}{2}} \quad (35)$$

For the model (30), energy density( $\rho$ ), string tension density ( $\lambda$ ), particle energy density ( $\rho_p$ ), pressure (P), expansion ( $\theta$ ), share ( $\sigma$ ) are given by

$$\rho = \lambda = \frac{\alpha(n-1)}{12\pi n} (e^{-u})^{2(n+2)-} - \frac{2K(2n+1)}{3n (e^{-u})^4} \quad (36)$$

$$\rho_p = 0 \quad (37)$$

$$P = -\frac{\alpha}{12\pi T^{2(n+2)}} (e^{-u})^{2(n+2)-} - \frac{2K}{3 (e^{-u})^4} \quad (38)$$

$$\theta = (n+2) \left[ \frac{4\alpha}{3n(n+2)O^{2(n+2)}} - \frac{32\pi K T^{n-2}}{3n(n-2)} + N \right]^{\frac{1}{2}} \quad (39)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \left[ \frac{4\alpha}{3n(n+2)(e^{-u})^{2(n+2)}} - \frac{32\pi K (e^{-u})^{(n-2)}}{3n(n-2)} + N \right]^{\frac{1}{2}} \quad (40)$$

#### V. SOLUTION IN THE ABSENCE OF MAGNETIC FIELD

**Case I:**  $\rho + \lambda = 0$

In the absence of magnetic field equation (24) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{4\alpha}{(4-n)(n+2)T^n} + MT^2\right]} + T^{2n}dX^2 + T^2(dY^2 + dZ^2)$$

Now choosing the cosmic time  $u = \pm \log T$ . For convenience, we can select  $u = -\log T$ , then above model goes over to

$$ds^2 = -\frac{du^2}{\left[M\epsilon^{\epsilon} + \frac{4\alpha}{(4-n)(n+2)}\right]} + (e^{-u})^{2n-2}dX^2 + dY^2 + dZ^2 \quad (41)$$

For the model (41), energy density( $\rho$ ), string tension density ( $\lambda$ ), particle energy density ( $\rho_p$ ), pressure (P), expansion ( $\theta$ ), share ( $\sigma$ ) are given by

$$\rho = -\lambda = -\frac{\alpha(n-1)}{4\pi(4-n)(n+2)} \quad (42)$$

$$\rho_p = -\frac{\alpha(n-1)}{2\pi(4-n)(e^{-u})^{2(n+2)}} \quad (43)$$

$$P = \frac{\alpha(n-2)}{4\pi(4-n)(e^{-u})^{2(n+2)}} \quad (44)$$

$$\theta = (n+2) \left[ \frac{4\alpha}{(4-n)(n+2)(n+2)} + M \right]^{\frac{1}{2}} \quad (45)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \left[ \frac{4\alpha}{(4-n)(n+2)(n+2)} + M \right]^{\frac{1}{2}} \quad (46)$$

**Case II :**  $\rho - \lambda = 0$

In the absence of magnetic field equation (30) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{4\alpha}{3n(n+2)T^n} + NT^2\right]} + T^{2n}dX^2 + T^2(dY^2 + dZ^2)$$

Now choosing the cosmic time  $u = \pm \log T$ . For convenience, we can select  $u = -\log T$ , then the above model goes over to

$$ds^2 = -\frac{du^2}{\left[N\epsilon^{\epsilon} + \frac{4\alpha}{3n(n+2)}\right]} + (e^{-u})^{2n}dX^2 + (dY^2 + dZ^2) \quad (47)$$

For the model (47), energy density( $\rho$ ), string tension density ( $\lambda$ ), particle energy density ( $\rho_p$ ), pressure (P), expansion ( $\theta$ ), share ( $\sigma$ ) are given by

$$\rho = \lambda = \frac{\alpha(n-1)}{12\pi n(n+2)} \quad (48)$$

$$\rho_p = 0 \quad (49)$$

$$P = -\frac{\alpha}{12\pi(e^{-u})^{2(n+2)}} \quad (50)$$

$$\theta = (n+2) \left[ \frac{4\alpha}{3n(n+2)(n+2)} + N \right]^{\frac{1}{2}} \quad (51)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \left[ \frac{4\alpha}{3n(n+2)(e^{-u})^{2(n+2)}} + N \right]^{\frac{1}{2}} \quad (52)$$

## VI. CONCLUSION

There is no big-bang and big crunch singularities in our model (30). From the equations (32), (33) and (34), it is seen that the such a Bianchi Type I magnetized cosmological model (36) in bimetric theory of gravitation exists. It never goes to vacuum model. When  $u \rightarrow -\infty$ , then  $\theta \rightarrow -\infty$ ,  $\sigma \rightarrow -\infty$  and when  $u \rightarrow \infty$ , then  $\theta \rightarrow \infty$ ,  $\sigma \rightarrow \infty$ . This confirms that the model is expanding as well as shearing. The expansion and the shear in the model increases as cosmic time  $u$  increases. Further when  $u \rightarrow \infty$ , then  $\epsilon = \lambda \rightarrow \infty$ ,  $p \rightarrow \infty$ . When  $u \rightarrow -\infty$  then  $\epsilon = \lambda \rightarrow 0$  and  $p \rightarrow 0$ , which shows that our model goes over to vacuum model, when the cosmic time  $u$  is minus infinity. In the special case, in the absence of magnetic field i.e.,  $K = 0$ , it is seen that, our model (36) exists. Also the expansion and shear in the model increases, as the cosmic time  $u$  increases.

## VII. ACKNOWLEDGEMENT

The author is grateful to Dr. M.S. Borkar, EX-PROFESSOR, Department of Mathematics, R.T.M. Nagpur University, Nagpur for his helpful guidance.

## VIII. REFERENCES

- [1]. R. Bali and Anjali : Bianchi Type I magnetized string cosmological model in general relativity, *Astrophys Space Sci.*,302 (2006), 201-205.
- [2]. R. Bali and S. Dave : Bianchi Type- IX string cosmological models with bulk viscous fluid in general relativity, *Astrophys. Space Sci.*, 288 (2003), 503-509
- [3]. R. Bali and R. D. Upadhaya : LRS Bianchi Type I string dust magnetized cosmological models, *Astrophys. Space Sci.*, 283 (2003), 97-108.
- [4]. R. Bali and D. Singh : Bianchi Type -V bulk viscous fluid string dust cosmological model in general relativity , *Astrophys.Space Sci.*, 300 (2005), 387-394.
- [5]. R. Bali and U.K. Pareek : Bianchi Type I String dust cosmological model with magnetic field in general relativity, *Astrophys.Space Sci.*, 312(2007), 305-310.
- [6]. M. S. Borkar and S. S. Charjan), Bianchi Type I bulk viscous fluid string dust cosmological model with magnetic field in bimetric theory of gravitation, *An Int. J. AAM*5(1), (2010), 96-109.
- [7]. M. S. Borkar and S. S. Charjan : Bianchi Type I Magnetized dust cosmological model in bimetric theory of gravitation, *An Int. J. AAM* 5 (2), , (2010) p.p. 1660-1671.
- [8]. N. P. Gaikwad, M. S. Borkar, and S. S. Charjan ; Bianchi Type I massive string Magnetized barotropic perfect fluid cosmological model in bimetric theory of gravitation *Chin. Phys. Lett.*28(8), (2011).
- [9]. M. S. Borkar, S. S. Charjan, LRS Bianchi Type I Magnetized Cosmological Model with --Term in Bimetric Theory of Gravitation, *American Journal of Pure Applied Mathematics*, 2(1), Jan.2013 pp.7-12
- [10]. M. S. Borkar, S. S. Charjan, Bianchi Type I Magnetized Cosmological Model with --Term in Bimetric Theory of Gravitation, *An Int. J. AAM* 8 (1), (June 2013 (116-127).



- [11]. M. S. Borkar, S. S. Charjan, V.V. Lapse LRS Bianchi Type I Cosmological Models with Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN:2319-765X. 10(3) Ver. I (May-Jun. 2014), PP 62-69 [www.iosrjournals.org](http://www.iosrjournals.org)
- [12]. Borkar, S.S. Charjan and V.V. Lapse, Bianchi Type-I Hyperbolic Models with Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, An Int. J. AAM 11 (1),(June 2016), pp. 192 - 214
- [13]. S.W. Hawking and G.F. R. Ellis: The large scale structure of space-time, Cambridge University Press, Cambridge (1973) , P.91
- [14]. M. Isrelit : Spherically Symmetric fields in Rosen's bimetric theories of gravitation, General Relativity and Gravitation, 13(7), (1981), 681-688.
- [15]. T. M. Karade : Spherically Symmetric Space Times in bimetric Relativity Indian J. Pure-appl. Math,11(9), (1980), 1202-1209.
- [16]. S .D. Katore and R.S. Rane: Magnetized cosmological models in biometric theory of gravitation, Pramana J. Phys.67(2), (2006), 237-227.
- [17]. G.S. Khadekar and S.D. Tade : String cosmological models in five dimensional biometric theory of gravitation, Astrophys. Space Sci.Doi 10.1007/S10509-007-9410-2. (2007).
- [18]. K. D. Krori, T. Choudhuri, and C. R. Mahant: String in some Bianchi Type cosmologies, General Relativity and Gravitation26(3), (1994), 265-274.
- [19]. P. S. Letelier : Clouds of strings in general relativity, Physical reviewD 20(6), (1979), 1294-1302.
- [20]. P. S. Letelier: String cosmology, Phy. reviewD 28(10), (1983), 2414-2419.
- [21]. S.N. Pandey and R.Tiwari, Indian J. pure appl. Maths, 12(2) : 261-264 (Feb.1981)
- [22]. D.R.K. Reddy and N. Venkateshwara Rao : On some Bianchi Type Cosmological models in bimetric theory of gravitation, Astrophys. Space Sci257, (1998), 1293-298
- [23]. N. Rosen: A Theory of Gravitation, Annls of Phys, 84, (1974), 455-473
- [24]. N. Rosen : A Topics in Theoretical and Experiential Gravitation Physics, edited by V. D .Sabtta and J. Weber. Plenum Press, London , (1977), pp 273-94
- [25]. John Stachel : Thickening the string I,The String Perfect dust, Phy. reviewD21(8) , (1980).
- [26]. P. K. Sahoo, B. Mishra and A. Ramu : Bianchi Types V and Vo cosmic string coupled with Maxwell fields in bimetric theory , Int. J. Theor Phys. (2010) DOI 10.1007/s 10773-010-0531-y