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Non-diagonal Cylindrically Symmetric Kaluza-Klein Cosmologies

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ABSTRACT

The geometry of three parameter family of non-diagonal cylindrically symmetric Kaluza-Klein cosmological models is described by the line element (4.1). The material distribution of model is a stiff-fluid ($pressure\ p = density\ \rho$). The pressure and density are given by $8\pi p = 8\pi\rho = [\,\alpha - \beta(3\beta+1)t^{-2\alpha}\,cosh^{-1}(mr)$, assumed $\beta > 0\ and\ \alpha > \beta(3\beta+1)$ and it is interesting that the parameter m is the measure of non- diagonality of the metric as well as of the inhomogeneity of matter. The family of our models have a big-bang singularity at t=0. All the physical and kinematical parameters remain well behaved for all t>0. Our models also admit dimensional reduction.

Keywords : Anomaly Detection, Cloud Computing, DDoS Attack, Feature Extraction, Feature Optimization

I. INTRODUCTION

The last two decades have witnessed an increase of interest in higher dimensional cosmology. According to this theory one assumes that the universe had a higher dimension than the usual four at a very early stage of its evolution. In Kaluza-Klein theory and superstring theory the dimension of the underlying space is taken to be greater than four. As the universe expands the extra dimensions contract, leaving behind the observable 4-dimensional space-time (Appelquist, Chodos and Freund [1987]). Chodos and Detweiler [19801 constructed some 5-dimensional vacuum solutions with Kasnerian time evolution in which the extra dimension shrank, while the usual 3-space expanded with time *t.* Recently 5-dimensional inhomogeneous cosmological models have been studied by many investigators.(Banerjee et al [1994], Chatterjee and Bhui [1990], Chatterjee et al [1993, 1994a, 1994b], Chodos and Detweiler [1980], Patel and Dadhich [1994a, 1994b]).

Inhomogeneity in cosmological models is relevant for many reasons (Krasinski [1997]), but principally to have general generic initial conditions and to facilitate the formation of large scale structures in the universe. In higher dimensions, several Kaluza-Klein (KK) extensions of Friedman -Robertson- Walker model have been discussed (Salidev [1984], Ishihara [1984], Chatterjee and Bhui [1990]). But all these extensions are big-bang singular. Benerjee et al [1995] have obtained the 5-dimensional analogues of the non-singular 4-dimansional inhomogeneous models discussed by Ruiz and Senvilla [1992] and Dadhich et al [1995]. Mars and Senovilla [1997] have obtained a non-diagonal G_2 separable perfect fluid model in 4-dimensions. The geometry of this inhomogeneous cosmological model is described by the line element

$$ds^2 = \cosh(2cx)\, t^{2\alpha} e^{c^2t^2}[dt^2 - dx^2] - t^2\cosh(2cx)\, dy^2 - \frac{(dz+ct^2dy)^2}{\cosh(2cx)} \tag{1.1}$$

where c and α are constants.

The energy density and pressure of the perfect fluid are given by

$$8\pi p = 8\pi\rho = \frac{\alpha e^{c^2t^2}}{\cosh(2cx)t^{2(\alpha+1)}} \tag{1.2}$$

In this paper, I wish to obtain perfect fluid cosmological models in 5-dimensional KK space-time which are generalizations of the solution (1.1). It turns out that the stiff-fluid character of the perfect fluid is maintained in 5-dimensions as well.

II. THE METRIC AND THE FIELD EQUATION

Consider the 5-dimensional cylindrically symmetric non-diagonal space time given by the line element

$$ds^2 = A^2[dt^2 - dr^2] - B^2(dz + Hd\phi)^2 - c^2d\phi^2 - E^2d\psi^2 \tag{2.1}$$

where A, B, C, E and H are functions of r and t. One can easily see that the 4-dimensional metric (1.1) is a particular case of the metric (2.1), Introducing the pentad

$$\theta^1 = Adr,\ \theta^2 = B(dz + Hd\phi),\ \theta^3 = cd\phi,\ \theta^4 = Ed\psi,\ \theta^5 = Adt \tag{2.2}$$

and hence we can express the metric (2.1) in the simple from

$$ds^2 = (\theta^5)^2 - (\theta^1)^2 - (\theta^2)^2 - (\theta^3)^2 - (\theta^4)^2 = g_{(ab)}\theta^a\theta^b \tag{2.3}$$

Here and in what follows the bracketed indices denote pentad components. Using the exterior calculus of differential forms and Cartan's equations of structure, one can easily find the pentad components $R_{(ab)}$ of the Ricci tensor for the metric (2.1) and the pentad (2.2). The non-zero $R_{(ab)}$ are listed below for ready reference

$$2A^2B^{-1}CR_{(23)} = \left[H'' + \frac{3H'B'}{B} - \frac{H'C'}{C} + \frac{H'E'}{E}\right] - \left[\ddot{H} + \frac{\dot{H}\dot{B}}{B} - \frac{\dot{H}\dot{C}}{C} + \frac{\dot{H}\dot{E}}{E}\right] \tag{2.4}$$

$$A^2R_{(15)} = \frac{\dot{B}'}{B} - \frac{\dot{C}'}{C} + \frac{\dot{E}'}{E} - \frac{\dot{A}}{A}\left[\frac{B'}{B} + \frac{C'}{C} + \frac{E'}{E}\right] - \frac{A'}{A}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{E}}{E}\right] + \frac{B^2\dot{H}H'}{2C^2} \tag{2.5}$$

$$A^2R_{(11)} = \frac{A''}{A} + \frac{B''}{B} + \frac{C''}{C} + \frac{E''}{E} - \frac{A'}{A}\left[\frac{A'}{A} + \frac{B'}{B} + \frac{C'}{C} + \frac{E'}{E}\right] - \frac{\dot{A}}{A}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{E}}{E} - \frac{\dot{A}}{A}\right] - \frac{\ddot{A}}{A} + \frac{B^2H'^2}{2C^2} \tag{2.6}$$

$$A^2R_{(22)} = \frac{B''}{B} + \frac{B'}{B}\left[\frac{C''}{C} + \frac{E''}{E}\right] - \frac{B^2H'^2}{2C^2} + \frac{B^2\dot{H}^2}{2C^2} - \frac{\ddot{B}}{B} - \frac{\dot{B}}{B}\left[\frac{\dot{C}}{C} + \frac{\dot{E}}{E}\right] \tag{2.7}$$

$$A^2R_{(33)} = \frac{C''}{C} + \frac{C'}{C}\left[\frac{B'}{B} + \frac{E'}{E}\right] - \frac{B^2H'^2}{2C^2} + \frac{B^2\dot{H}^2}{2C^2} - \frac{\ddot{C}}{C} - \frac{\dot{C}}{C}\left[\frac{\dot{B}}{B} + \frac{\dot{E}}{E}\right] \tag{2.8}$$

$$A^2R_{(44)} = \frac{E''}{E} + \frac{E'}{E}\left[\frac{B'}{B} + \frac{C'}{C}\right] - \frac{\ddot{E}}{E} - \frac{\dot{E}}{E}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] \tag{2.9}$$

$$A^2R_{(55)} = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{E}}{E} - \frac{A'}{A}\left[\frac{B'}{B} + \frac{C'}{C} + \frac{E'}{E} - \frac{A'}{A}\right] - \frac{\dot{A}}{A}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{E}}{E} + \frac{\dot{A}}{A}\right] - \frac{A''}{A} + \frac{B^2\dot{H}^2}{2C^2} \tag{2.10}$$

Here and in what follows an overhead dash and a dot indicate differentiation with respect to r and t respectively. Here ψ is taken in the form of a Kaluza-Klein parameter such that $\theta < \psi < 2\pi R_5$, where R_5 is the radius of KK circle.

Assume that the space-time filled with a perfect fluid distribution given by energy momentum tensor

$$T_{ik} = (p+\rho)v_iv_k - pg_{ik}\ ;\ v^iv_i = 1 \tag{2.11}$$

where ρ, p and v^i denote matter density, fluid pressure and the unit timelike flow vector respectively The Einstein field equations are

$$R_{ik} - \frac{1}{2}Rg_{ik} = -8\pi T_{ik} \tag{2.12}$$

where T_{ik} is given by (2.11). We have chosen the units such that G = 1, c = l . The filed equations (2.12) can be expressed in the pentad form as

$$R_{(ab)} = -8\pi\left[(p+\rho)v_{(a)}v_{(b)} - \frac{1}{3}(\rho - p)g_{(ab)}\right] \tag{2.13}$$

where $v_{(a)}$ are pentad components of v_i .

We are using comoving co-ordinates. Consequently we have $v_{(a)} = (0,0,0,0,1)$ (2.14)

In view of (2.14) the filed equations (2.13) give rise to the following relations.

$$R_{(23)} = 0,\ R_{(15)} = 0,\ R_{(11)} = R_{(22)} = R_{(33)} = R_{(44)} \tag{2.15}$$

$$8\pi\rho = -\frac{1}{2}\left[R_{(55)} + 4R_{(22)}\right],\ 8\pi p = -\frac{1}{2}\left[2R_{(22)} - R_{(55)}\right] \tag{2.16}$$

where $R_{(ab)}$ are given by (2.4 - 2.10)

III. THE STIFF-FLUID MODELS

Following Mars and Senovilla [1997] we assume the separability of metric potentials as

$$A = cosh^a(mr)T(t),\ B = cosh^b(mr)t^\beta\ C = cosh^c(mr)t^\gamma,\ E = cosh^e(mr)t^\delta \tag{3.1}$$

where $a, b, c, e, \beta, \gamma,$ *and* m are real constants. The choice of hyperbolic cosine function ensure that there will not by any spatial singularity in the models.
By using (3.1) in the equations (2.4) to (2.15) we get,

$$\dot{H} = Kt^{\gamma-\delta-3\beta} \tag{3.2}$$

where K is the constant of integration. On simplification the equation (2.16) gives us

$$\frac{\dot{T}}{T}(b+c+e) = \frac{1}{t}[b\beta + c\gamma + e\delta - \alpha(\beta+\gamma+\delta)] \tag{3.3}$$

The relation $R_{(22)} = R_{(33)}$ of (2.15) implies $2\beta + \delta = 0,\ b - c = -1,\ m^2 = K^2$

$$(\beta-\gamma)(\beta+\gamma+\delta-1) = 0,\ b+c+e = 0 \tag{3.4}$$

Let us now consider the relation $R_{(22)} = R_{(44)}$, this leads to

$$(\beta-\delta)(\beta+\gamma+\delta-1) = 0,\ b + \frac{1}{2} = e \tag{3.5}$$

From (3.4) and (3.5) one can calculate that

$$a = \frac{1}{2},\ b = -\frac{1}{2},\ c = \frac{1}{2},\ e = 0,\ \gamma = \beta + 1,\ \delta = -2\beta \tag{3.6}$$

With the help of (3.6) it is easy to see that the relation (3.3) is satisfied without any restriction on the function *T*(t). It now remains to solve the equation $R_{(22)} = R_{(11)}$ and hence the equation determines the function *T*(t) as

$$T(t) = t^a e^{\frac{m^2t^2}{8}} \tag{3.7}$$

where a is a constant of integration. And also $e = 0$ implies, $R_{(44)} = 0$ and consequently, we have $p = \rho$. So in the above set-up, only the inhomogeneous stiff- fluid models are possible. The pressure and density can be obtained from (2.16). They are given by

$$8\pi\rho = 8\pi p = [\alpha - \beta(1+3\beta)]t^{-2\alpha} cosh^{-1}(mr) \tag{3.8}$$

For $p > 0$ we must have $\alpha > \beta(1+3\beta)$.

Again by using (3.6) in (10.3.2) the metric potential H can be obtained as $H = \frac{1}{2}t^2$.

If $\alpha > \beta(1+3\beta)$, ρ is positive and consequently the following strong energy condition is satisfied. $R_{ik}v^i v^k = \frac{-16\pi}{3}(\rho + 2p)$

IV. DISCUSSION

The explicit form of the line element of our solution is

$$ds^2 = t^{2\alpha} e^{\frac{m^2t^2}{4}} \cosh(mr)\,[dt^2 - dr^2] - t^{2(1+\beta)} \cosh(mr)\, d\emptyset^2$$
$$-t^{2\beta} \cosh^{-1}(mr)\,(dz + \frac{m}{2}t^2 d\emptyset)^2 - t^{4\beta} \mathrm{d}\varphi^2 \tag{4.1}$$

Here m, α and β are three arbitrary parameters. Therefore have a three parameter family of stiff-fluid models in KK space-time. If choose $\beta > 0$, the coefficient of $\mathrm{d}\varphi^2$ in (4.1) tends to zero as t tends to infinity. Thus the dimensional reduction is always possible for our models. If $\beta = 0$ and φ =constant, the metric (4.1) reduces to the metric (1.1) discussed by Mars and Senovilla [1997] in connection with 4-dimensional G_2 separable stiff-fluid models.

If $\alpha = \beta(1+3\beta)$ the density ρ vanishes. Thus the matter free limit of our family of solution in the 5-dimensional non flat inhomogeneous empty space-time described by the line element

$$ds^2 = t^{2\beta(1+3\beta)} e^{\frac{m^2t^2}{4}} \cosh(mr)\,[dt^2 - dr^2] - t^{2(1+\beta)} \cosh(mr)\, d\emptyset^2$$
$$-t^{2\beta} \cosh^{-1}(mr)\,(dz + \frac{m}{2}t^2 d\emptyset)^2 - t^{4\beta} \mathrm{d}\varphi^2 \tag{4.2}$$

If $m = 0$ then the metric (4.1) becomes

$$ds^2 = t^{2\alpha}[dt^2 - dr^2] - t^{2(1+\beta)} d\emptyset^2 - t^{2\beta} dz^2 - t^{4\beta} \mathrm{d}\varphi^2 \tag{4.3}$$

The inhomogeneous Bianchi-I metric (4.3) describes a stiff-fluid model whose density ρ is given by

$$8\pi\rho = 8\pi p = [\alpha - \beta(1+3\beta)]t^{-2\alpha} \tag{4.4}$$

It is interesting to note that the parameter m occurring in our models is the measure of non-diagonality of the metric as well as of the inhomogeneity of matter.

When $\beta = 0$, we get the metric (4.1) becomes

$$ds^2 = t^{2\alpha} e^{\frac{m^2t^2}{4}} \cosh(mr)\,[dt^2 - dr^2] - t^2 \cosh(mr)\, d\emptyset^2 - \cosh^{-1}(mr)\,(dz + \frac{m}{2}t^2 d\emptyset)^2 - \mathrm{d}\varphi^2 \tag{4.5}$$

The metric (4.5) is an obvious 5-dimensional generalization of the stiff-fluid solution given by Mars and Senovilla [1997) with density given by

$$8\pi\rho = 8\pi p = \alpha t^{-2\alpha} cosh^{-1}(mr) \tag{4.6}$$

The expansionθ, the acceleration f, and the shear σ for the velocity field v' in five dimensions are defined by

$$\theta = v^i v_{k,i}, \quad f_i = v_{i,k} v^i, \quad \sigma^2 = \sigma_{ab}\sigma^{ab} \tag{4.7}$$

where

$$\sigma_{ab} = \frac{1}{2}[v_{i,k} + v_{k,i}] - \frac{1}{2}[v_i f_k + v_k f_i] - \frac{1}{4}\theta[g_{i,k} - v_i v_k] \tag{4.8}$$

Here the semicolon indicates covariant differentiation. For our family of solutions θ and f, are given by

$$\theta = \frac{1}{At}\left[\alpha + 1 + \frac{m^2t^2}{4}\right], \quad f = \left(-\frac{1}{2}m\tanh(mr), 0,0,0,0\right) \tag{4.9}$$

The expression for σ^2 is quite lengthy and complicated, therefore it is not given here.

If we choose $\alpha > \beta(1 + 3\beta)$ and positive, then $t = 0$ is an initial big-bang singularity at which density ρ and kinematical parameters diverge. We have verified that the fluid flow is irrotational.

When t tends to infinity, density ρ and the kinematical parameters tend to zero. When r tends to infinity, then also these quantities tend to zero. Here density ρ is a decreasing function of r and t. Thus the physical and kinematical parameters remain well behaved for all $t > 0$.

The models with separability assumption other than (3.1) are at present under investigation.

V. CONCLUDING REMARKS

Here we have considered a non-diagonal cylindrically symmetric metric. We have proved that only the stiff-fluid ($p = \rho$) models are possible in our set up. A three parameter family of stiff fluid cosmological models is obtained which admit the dimensional reduction. The models are inhomogeneous and have non zero shear, expansion and acceleration. The models have an initial big-bang singularity at $t=0$.

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