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Some Exact Solutions of String Cosmology in Bimetric Theory of Gravitation

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ABSTRACT

Following the techniques used by Letelier and Stachel some exact solutions of string Cosmology Bianchi I cosmological solutions of massive strings in the presence of magnetic field are obtained in bimetric theory of gravitation and their physical features are discussed.

Keywords. Bimetric theory; perfect fluid; cosmic string, magnetic field, Bianchi type-I

Mathematics Subject Classification 2020: 83D-XX, 83F-XX, 83F05

I. INTRODUCTION

Cosmic strings play an important role during an early stage of the evaluation of the universe (Kibble 1976) [27] and give rise to density perturbation which lead to the formation of galaxies (Zel'dovich 1980) [48]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general relativistic treatment of strings was initiated by Letelier (1979, 1983) [31-32]. Stachel (1980) [45] has considered a massless (geometric string) to develop a realistic treatment of strings. It is interesting to note that magnetic field is present in galactic and intergalactic spaces and plays an important role at cosmological scale. It is interesting to note that magnetic field plays a significant role at cosmological model. Melvin [35] suggested in the cosmological solution for dust and electromagnetic field that during the evolution of the universe, the matter was in highly ionized state and smoothly coupled with magnetic field and consequently form a neutral matter as a result of universe expansion. Hence in string dust universe the presence of magnetic field is not unrealistic. Banerjee et.al. [8] (1990) have investigated an axially symmetric Binachi Type I string dust cosmological model with magnetic field using the supplementary condition $\alpha(t) = a\beta(t)$ where α, β are metric potentials and a is the constant. Tikekar and Patel [46] (1992,1994) have investigated some string cosmological models in presence and absence of magnetic field. All these models are non-static and flow vector v^i is irrotational. Patel and Maharaj [36] (1996) have investigated stationary rotating world model with magnetic field. Singh and G.P. Singh [44] (1999) investigated string cosmological models in the presence of magnetic field in the context of the space-time with G_3 symmetry. Kilinc and Yavuz [29] (2000) have investigated the section of Einstein's field equation for inhomogeneous cylindrically symmetric space-time with string source in presence and absence of magnetic field. Balli and Dave (2001, 2003)[3]and [6] ; Bali and Upadhaya 2003 [5] ; Bali and Singh 2005) [7] have investigated Binachi type I,V, and IX string cosmological models under different physical conditions in general relativity.

Rosen (1973)[40] proposed bimetric theory of gravitation by assuming two metric tensors viz. a Riemannian metric tensor g_{ij} and the background metric tensor γ_{ij}. The metric tensor g_{ij} determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric γ_{ij} refers to the geometry of the empty (free from matter and radiation) universe and describe the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. Therefore it interacts with g_{ij} but not directly with matter. One can regard γ_{ij} as giving the geometry that would exists if there were no matter. In the absence of matter one would have $g_{ij} = \gamma_{ij}$. Moreover, the bimetric theory also satisfied the covariance and equivalence principles; the formation of general relativity. The theory agrees with the present observational facts pertaining to general relativity. Thus at every point of space-time, there are two line elements;

$$ds^2 = g_{ij} dx^i dx^j \tag{1}$$

$$d\eta^2 = \gamma_{ij} dx^i dx^j \tag{2}$$

The field equations of Rosen's bimetric theory of gravitation are

$$N_i^j - \frac{1}{2} N \delta_i^j + \wedge g_i^j = -8\pi k T_i^j \tag{3}$$

where $N_i^j = \frac{1}{2} \gamma^{pr} \left(g^{sj} g_{si|p} \right)_{|r}$, $N = N_i^i$, $k = \sqrt{\frac{g}{f}}$

together with $g = \det(g_{ij})$ and

$\gamma = \det(\gamma_{ij})$

Here the vertical bar (|) stands for γ-covarient differentation and T_i^j is the energy-momentum tensor of matter fields.

Rosen [40-43], Yilmaz [47], Karade and Dhoble [25], Karade [26], Israelit [23], Adhav et al [1] are some of the eminent authors who have studied several aspects of bimetric theory of gravitation. In particular, Reddy and Venkateswarlu [37], Reddy and Venkateswara Rao [38], Reddy [39] have established the non-existence of spatially homogeneous and isotropic cosmological models of Binachi types and Kantowski- Sachs in bimetric theory of gravitation when the field of gravitation is goverened by either perfect fluid or string dust. Further Borkar et al. Borkar et.al. [9-20], Gaikawad et.al.[21] have been investigated many magnetized cosmological models in bimetic theory of gravitation by using the techniques of Letelier and Stachel [31,32,45]

Recently, there has been a lot of interest in cosmological model on the basis of Rosen's bimetric theory of gravitation. The purpose of Rosen's bimetric theory is to get rid of the singularities that occur in general relativity that was appearing in the big-bang in cosmological models.

Bali R. and Umeshkumar Pareek [2] has investigated Binachi type I string dust cosmological model in general relativity for perfect fluid distribution in presence of magnetic field. The magnetic field is due to an electric current produced along x-axis. Following Bali and Jain [4], they have assumed that the eigen value (σ_1^1) of shear tensor (σ_i^j) is proportional to the expansion (θ) which is physically plausible condition. They have also assumed that the universe is filled with string dust and perfect fluid. The string dust condition leads to $\varepsilon = \lambda$ (Zel'dovich [48]) where ε is the rest energy density and λ the string tension density. They also discussed the behavior of the model in presence of magnetic field.

In this paper, we have investigated some exact solutions of Binachi Type I string Cosmological models in Rosen's bimetric theory of gravitation in presence of magnetic field by using the techniques used by Letelier and Stachel. The physical and geometrical significance of the model are also discussed.

The field equations of Rosen's bimetric theory of gravitation are

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where $N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj} g_{si|p}\right)_{|r}$, $N = N_i^i$, $k = \sqrt{\frac{g}{f}}$

together with $g = \det(g_{ij})$ and

$\gamma = \det(\gamma_{ij})$

Here the vertical bar (|) stands for γ-covarient differentation and T_i^j is the energy-momentum tensor of matter fields.

II. Metric and Field Equations:

We consider Binachi Type I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{4}$$

where A,B and C are functions of t-alone.

The flat metric corresponding to metric (4) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{5}$$

The energy momentum tensor for the string dust perfect fluid distribution with magnetic field is taken as

$$T_i^j = (\in + p)v_i v^j + pg_i^j - \lambda x_i x^j + E_i^j \tag{6}$$

with

$$v^i v_i = -x_i x^i = -1 \tag{7}$$

$$v^i x_i = 0 \tag{8}$$

In this model $\in$ and λ denote the rest energy density and the tension density of the system of strings respectively, p is the isotropic pressure, v^i is the flow vector and x_i the direction of strings.

The electromagnetic field E_i^j is given by Lichnerowicz [33]

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right] \tag{9}$$

The four velocity vector v_i is given by

$$g_{ij} v^i v_j = -1 \tag{10}$$

and $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} v^j \tag{11}$$

where F_{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the Levicivita tensor density.

Assume the commoving co-ordinates and so that we have $v^1 = v^2 = v^3 = 0, v^4 = 1$

Further we assume that current is flowing along x-axis, so magnetic field is in yz plane so that

$$h_1 \neq 0,\ h_2 = h_3 = h_4 = 0$$

The first set of Maxwell's equation

$$F_{[ij,k]} = 0 \tag{12}$$

Yield

$$F_{23} = constant = H\ (say)$$

Due to the assumption of infinite electrical conductivity we have $F_{14} = F_{24} = F_{34} = 0$. The only non-vanishing component of F_{ij} is F_{23}.

Hence

$$h_1 = \frac{AH}{\bar{\mu} BC} \tag{13}$$

Since

$$|h|^2 = h_l . h^l = h_1 h^1 = g^{11}(h_1)^2$$

$$|h|^2 = \frac{H^2}{\bar{\mu}^2 B^2 C^2} \tag{14}$$

From equation (9) we obtain

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{H^2}{2\bar{\mu} B^2 C^2} \tag{15}$$

Equation (6) leads to

$$T_1^1 = \left(P - \lambda - \frac{H^2}{2\bar{\mu}B^2C^2}\right), T_2^2 = T_3^3 = \left(P + \frac{H^2}{2\bar{\mu}B^2C^2}\right), T_4^4 = \left(-\in - \frac{H^2}{2\bar{\mu}B^2C^2}\right)$$

(16)

The Rosen's field equations (3) for the metric (4) and (5) with the help of (16) gives

$$\frac{-A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4^{\,2}}{A^2} - \frac{B_4^{\,2}}{B^2} - \frac{C_4^{\,2}}{C^2} = 16\pi ABC\left(-P + \lambda + \frac{H^2}{2\bar{\mu}B^2C^2}\right) - 2\wedge$$

(17)

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^{\,2}}{A^2} + \frac{B_4^{\,2}}{B^2} - \frac{C_4^{\,2}}{C^2} = 16\pi ABC\left(-P - \frac{H^2}{2\bar{\mu}B^2C^2}\right) - 2\wedge$$

(18)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4^{\,2}}{A^2} - \frac{B_4^{\,2}}{B^2} + \frac{C_4^{\,2}}{C^2} = 16\pi ABC\left(-P - \frac{H^2}{2\bar{\mu}B^2C}\right) - 2\wedge$$

(19)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^{\,2}}{A^2} - \frac{B_4^{\,2}}{B^2} - \frac{C_4^{\,2}}{C^2} = 16\pi ABC\left(\in + \frac{H^2}{2\bar{\mu}B^2C^2}\right) - 2\wedge$$

(20)

where $A_4 = \frac{dA}{dt}, B_4 = \frac{dB}{dt}, C_4 = \frac{dC}{dt} \quad etc.$

III. Solution of the Field Equations:

From equations (18) and (19), we obtain

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{C_4^{\,2}}{C^2} - \frac{B_4^{\,2}}{B^2} \tag{21}$$

Equations (17) and (18) leads to

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{A_4^{\,2}}{A^2} - \frac{B_4^{\,2}}{B^2} = 8\pi ABC\left(\lambda + \frac{2K}{B^2C}\right)$$

(22)

where

$$K = \frac{H^2}{2\bar{\mu}}$$

Equations (20) and (22) after using string dust condition $(\in = \lambda)$ (zel'dovicn[48]) , lead to

$$\frac{B_{44}}{B}-3\frac{A_{44}}{A}-\frac{C_{44}}{C}+3\frac{A_4^{\ 2}}{A^2}-\frac{B_4^{\ 2}}{B^2}+\frac{C_4^{\ 2}}{C^2}=16\pi A\left(\frac{K}{BC}\right)+2\wedge \tag{23}$$

Equations (17) to (20) are four equations, in six unknowns $A, B, C, \lambda, \in$ and P and therefore to deduce a determinate solution, we assume the condition that the component σ_1^1 of shear tensor σ_i^j is proportional to the expansion (θ) which leads to

$$A = N[BC]^{-n} = N[\mu]^{-n} \tag{24}$$

where $n > 0$ and N is proportionality constant.

Let

$$BC=\mu,\ \frac{B}{C}=v\ \ then\ B^2=\mu v, C^2=\frac{\mu}{v}$$

Now from equation (21) we write

$$\frac{(CB_4-BC_4)_4}{(CB_4-BC_4)}=\frac{B_4}{B}+\frac{C_4}{C} \tag{25}$$

On integrating above equation

$$C^2\left(\frac{B}{C}\right)_{,4}=lBC \tag{26}$$

where l is the constant of integration.

Now using assumption $BC=\mu\ and\ \frac{B}{C}=v$ equation (26) becomes

$$\frac{v_4}{v}=l \tag{27}$$

Now equation (23) after using (24) and assumptions $BC=\mu\ and\ \frac{B}{C}=v$ leads to

$$3n\frac{B_{44}}{B}-3\frac{A_{44}}{A}-\frac{C_{44}}{C}+3\frac{A_4^{\ 2}}{A^2}-\frac{B_4^{\ 2}}{B^2}+\frac{C_4^{\ 2}}{C^2}=16\pi A\left(\frac{K}{BC}\right)+2\wedge$$

Using (27) in (28), we get

$$2\mu_{44}-2\frac{\mu_4^2}{\mu}=\frac{4}{3n}\left[\frac{8\pi NK}{\mu^n}+\wedge\mu\right] \tag{29}$$

Now choosing $\mu_4=f(\mu)$, then after straight forward calculation, the above equation (29), reduces to

$$\frac{d}{d\mu}\left[f^2\right]-2\frac{f^2}{\mu}=\frac{4}{3n}\left[\frac{8\pi NK}{\mu^n}+\wedge\mu\right]$$

Which has a solution

$$f^2=L\mu^2-\frac{32\pi NK}{3n(n+1)\mu^{n-1}}+\frac{4\wedge\mu^2\log\mu}{3n}$$

where L is the constant of integration.

From equation (27) we write

$$\log v=\int\frac{l\ d\mu}{\sqrt{L\mu^2-\frac{32\pi NK}{3n(n+1)\mu^{n-1}}+\frac{4\wedge\mu^2\log\mu}{3n}}}+\log b$$

Now using the solution in equation (31) from $\frac{d\mu}{df}=f$, we write $-dt^2=-\frac{d\mu^2}{f^2}$ and then using the condition $A=N[\mu]^{-n}, BC=\mu\ and\ \frac{B}{C}=v$ the metric (4) having the form

$$ds^2=-\frac{d\mu^2}{\left[L\mu^2-\frac{32\pi NK}{3n(n+1)\mu^{n-1}}+\frac{4\wedge\mu^2\log\mu}{3n}\right]}+N^2\mu^{-2n}dx^2+\mu v dy^2+\frac{\mu}{v}dz^2 \tag{33}$$

After suitable transformation of co-ordinates i.e. putting $\mu = T, \quad Nx = X,\ y = Y,\ z = Z$
i.e. $\mu = T, \quad N\,dx = dX,\ dy = dY,\ dz = dZ$ above metric will be

$$ds^2 = -\frac{dT^2}{\left[LT^2 - \frac{32\pi NK}{3n(n+1)T^{n-1}} + \frac{4\wedge T^2 \log T}{3n}\right]} + T^{-2n}dx^2 + T\,v^2 dy^2 + \frac{T}{v}dz^2$$
(34)

This is the Binachi Type I string dust cosmological model in biometric theory of relativity.

IV. Some Physical and Geometrical features

The energy density $(\in)$, the string tension density (λ), the isotropic pressure (p) for the Binachi Type I string dust cosmological model (34) are given by

$$\in = \frac{(1-4n)K}{3nT^2} + \frac{(2n+1)\wedge}{24\pi\, N\, n + (1-n)} = \lambda$$

$$P = \frac{2(1-n-N)K}{3nT^2} + \frac{[(2n+1) - N(1+3n)]\wedge}{12\pi\, N\, nT^{(1-n)}} = \lambda$$

Thus

$$\in - P = \left[\frac{2N-2n-1}{3n}\right]\frac{K}{T^2} + \frac{[2N(1+3n) - 2n - 1]\wedge}{24\pi\, N\, nT^{(1-n)}}$$

$$\in + P = \left[\frac{3-6n-2N}{3n}\right]\frac{K}{T^2} + \frac{[(61+3n) - 2N(1+3n)]\wedge}{24\pi\, N\, nT^{(1-n)}}$$

The string energy conditions given by Hawking and Ellis [22] as $\in - P > 0$ and $\in + P > 0$ are satisfied when $n > 0$.

The expansion (θ) is given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$$

$$\theta = \frac{-nf}{\mu} + \frac{f}{\mu}$$

$$\theta = \frac{(1-n)}{\mu}$$

$$\theta = \frac{(1-n)}{T}\left[LT^2 - \frac{32\pi NK}{3n(n+1)T^{n-1}} + \frac{4\wedge T^2 \log T}{3n}\right]^{\frac{1}{2}}$$

$$\theta = (1-n)\left[\frac{LT^2}{T^2} - \frac{32\pi NK}{3n(n+1)T^{n-1}T^2} + \frac{4\wedge T^2 \log T}{3nT^2}\right]^{\frac{1}{2}}$$

$$\theta = (1-n)\left[L - \frac{32\pi NK}{3n(n+1)T^{n+1}} + \frac{4\wedge \log T}{3n}\right]^{\frac{1}{2}}$$
(35)

The components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right] \quad (36)$$

$$\sigma_1^1 = \frac{1}{3}\left[\frac{-2nf}{\mu} - \frac{f}{\mu}\right] \quad (37)$$

$$\sigma_1^1 = -\frac{(2n+1)}{3}\left[\frac{f}{\mu}\right]$$
(38)

$$\sigma_1^1 = -\frac{(2n+1)}{3T} f$$

$$\sigma_1^1 = -\frac{(2n+1)}{3}\left[L - \frac{32\pi NK}{3n(n+1)T^{n+1}} + \frac{4\wedge \log T}{3n}\right]^{\frac{1}{2}}$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right]$$

$$\sigma_2^2 = \frac{1}{3}\left[\left(\frac{\mu_4}{\mu} + \frac{v_4}{v}\right) + \frac{nf}{\mu} - \frac{1}{2}\left(\frac{\mu_4}{\mu} - \frac{v_4}{v}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}\frac{\mu_4}{\mu} + \frac{nf}{\mu} + \frac{3}{2}\frac{v_4}{v}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2} + n\right)\frac{f}{\mu} + \frac{3}{2}l\right]$$

$$= \frac{1}{3}\left[\left(\frac{(2n+1)}{2}\right)\frac{f}{\mu} + \frac{3}{2}l\right]$$

$$\sigma_2^2 = \frac{1}{3}\left[\frac{(2n+1)}{3}\left[L - \frac{32\pi NK}{3n(n+1)T^{n+1}} + \frac{4\wedge \log T}{3n}\right]^{\frac{1}{2}} + \frac{3}{2}l\right] \quad (41)$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{2C_4}{C} - \frac{B_4}{B} - \frac{A_4}{A}\right]$$

$$\sigma_3^3 = \frac{1}{3}\left[2\frac{1}{2}\left(\frac{\mu_4}{\mu} - \frac{v_4}{v}\right) - \frac{1}{2}\left(\frac{\mu_4}{\mu} + \frac{v_4}{v}\right) + \frac{nf}{\mu}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}\frac{\mu_4}{\mu} - \frac{3}{2}\frac{v_4}{v} + \frac{nf}{\mu}\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{2} + n\right)\frac{f}{\mu} - \frac{3}{2}l\right]$$

$$\sigma_3^3 = \frac{1}{3}\left[\frac{(2n+1)}{3}\left[L - \frac{32\pi NK}{3n(n+1)T^{n+1}} + \frac{4\wedge \log T}{3n}\right]^{\frac{1}{2}} - \frac{3}{2}l\right] \quad (42)$$

$$\sigma_4^4 = 0 \quad (43)$$

V. Discussion

The expansion in the model (33) decreases as time increases. When $T \to \infty$, $\theta \to (1-n)L$. When $T \to 0$, $\in = \lambda = P \to \infty$ and when $T \to \infty$, $\in = \lambda = P \to 0$. Since $\lim_{T\to\infty} \frac{\sigma_1^1}{\theta} = \frac{(2n+1)}{3(n-1)} \neq 0$, hence the model does not isotropize for large value of T, but for $n = -\frac{1}{2}$ the model isotropizes. The model (33) has point type singularity at T=0 (Mac Callum 1971).

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