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Axial Inhomogeneous Ponderomotive Plasma Dynamics in Entire Spatial Intense Laser Fields

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ABSTRACT
This study investigates the effects of axial inhomogeneity on the ponderomotive force within a plasma subject to intense laser fields. The nonlinearity in the dielectric constant arises because the self-focusing problem of nonlinear interaction of intense laser beams has been analysed considering the entire spatial characteristics of the laser beam without any paraxial ray approximation and Taylor series expansion of the dielectric constant—the effect of the axial inhomogeneity on the ponderomotive force for an arbitrary magnitude of intensity. The propagation characteristics of laser beams have been discussed. An appropriate expression for the nonlinear dielectric constant has been used considering the entire spatial characteristic of plasma in the analysis of laser-beam propagation in the non-paraxial approximation for a circularly polarised wave. Various types of inhomogeneity are discussed for plasma. The variations of the beam width parameter with the propagation distance, the self-trapping condition and the critical power have been evaluated. The saturating nature of the nonlinearity for the critical power for beam self-focusing. It is seen that the laser beam width tends to attain a constant value depending on the plasma inhomogeneity and the initial laser intensity. Numerical estimates are made for typical values of the laser-plasma interaction applicable for under-dense and over-dense plasmas, and the results are compared with the paraxial ray approximation method.

Keywords : Laser-Matter Interaction, Axial Inhomogeneity Self-Focusing, Self-Trapping.

I. INTRODUCTION

1. Introduction

Wave propagation in nonlinear medium has been a subject of intensive theoretical and experimental research [1–10]. The self-generated lens effect of the intense laser beam in a nonlinear medium such as plasma has been a prominent subject of many numerical, theoretical as well as experimental investigations [1,3to7]

Direct and indirect experimental evidence reveals that the smooth-looking laser beams have strong intensity spikes. The growth of nonuniform, radially symmetrical ripple superimposed on a Gaussian beam propagating in plasma has been studied [8 to 15].

Most of the studies are limited to various approximations, such as paraxial ray approximation. Due to this reason, the results of these analyses are far from the observed experimental results [18].

An alternate method (i.e. non-paraxial technique), developed by the author, has been used in the present study, where the entire spatial propagation characteristics of the laser beam are considered in place of the paraxial region. Here, analysis is done for homogeneous and inhomogeneous plasmas as a nonlinear medium. A detailed study of different types of inhomogeneities in plasma has been carried out. Thus, the results obtained by the new non-paraxial approach developed by the author are compared with the work of other researchers based on the paraxial method and the available experimental observations.

The motion of particles in plasma develops local concentrations of positively and negatively charged particles. These charge concentrations create long-range coulombic fields, and the charged particles in plasma move along a path in the field.

Plasma physics is one of the advanced disciplines of physics. In the present study, the self-generated lens effect in plasma, which can be used in the thermonuclear fusion mechanism, has been discussed.

The self-generated lens effect mechanism in a medium show that an intense electromagnetic beam creates a refractive index gradient across its own intensity distribution profile. [14 -17]The refractive index gradient depends on many factors, such as the wave profile of the electromagnetic beam, the nature of nonlinearity, the power level of the propagating beam, charge distribution in the medium, etc. The efficiency of the processes occurring in plasma and their reaction rates are, in general, directly dependent on the density distribution of the charged particles.

2. Ponderomotive Nonlinearity

Ponderomotive nonlinearity is a concept used in nonlinear optics to describe the effects of the ponderomotive force on the motion of electrons in the presence of an oscillating electromagnetic field. This force is particularly relevant when dealing with high-intensity laser fields.

Origin of pondermotive nonlinearity

When an intense laser beam interacts with a nonlinear medium, the oscillating electric field exerts a force on the electrons in the medium. This force can be divided into two components: one oscillating at the laser frequency and a slower, time-averaged component known as the ponderomotive force. The latter is responsible for pushing electrons away from regions of high intensity to lower intensity regions, effectively creating a modification in the local refractive index of the medium.

The ponderomotive force

The ponderomotive force is a nonlinear force that acts on charged particles in an oscillating electromagnetic field, such as a laser beam. This force tends to push the particles away from regions of high-field intensity.

The ponderomotive force Fp experienced by a charged particle in an electromagnetic field is given by:

$$F_p = -\nabla U_p = -\nabla\left(\frac{e^2E^2}{4m\omega^2}\right) \quad (1)$$

In the context of a laser beam, this force depends on the spatial gradient of the laser intensity. The ponderomotive nonlinearity arises due to the interaction of the drift velocity with the magnetic field of the wave and the motion of electrons in a uniform field.

The nonlinearity is due to ponderomotive force dominated in plasma, where the duration of the electromagnetic beam is much smaller than the energy relaxation time ($t < \tau$) of the electrons; such situations occur when the fast pulse of the laser beam interacts with strongly ionised collision less plasma.

For such type of collision with less plasma, the drift velocity of electrons can be obtained by the Ginzburg equation of motion [10], which is written as

$$\mathrm{m}\left(\frac{dv}{dt} + (\mathrm{v}.\nabla)\,\mathrm{v}\right) = -e((\mathrm{V} * \mathrm{B}/\mathrm{c}) - eE - \frac{\nabla Pe}{Ne} \quad (2)$$

Where v is the drift velocity of electrons, B represents the magnetic field of the interacting electromagnetic beam, and E is the total electric field in plasma, which is equal to the sum of space charge electric field and wave field. N and e denote the electronic concentration and velocity of light, respectively. P represents the force on electrons due to the gradient in the partial pressure of electrons.

In the above equation, the first two terms on R.H.S. represent the force on electrons due to magnetic and electric fields, respectively, and the third term indicates the force on electrons due to the gradient of the partial pressure. The equation (2) can be rewritten as

$$m\left(\frac{dv}{dt}\right) = \mathrm{Fp} - eE \; - \frac{\nabla Pe}{Ne} \quad (3)$$

Where

$$\mathrm{F_p} = -m\big((\mathrm{v}.\nabla)\,\mathrm{v}\big) - e((V * B/\mathrm{c}) \quad (4)$$

It is usually termed as the ponderomotive force.

3. Nonlinear Dielectric Constant

The Ponderomotive force is a fundamental characteristic of plasma. This is responsible for the redistribution of electronic concentration in plasma. This effect makes the dielectric constant of the plasma vary nonlinearly and causes a self-generated lens effect.

But the equation for the effective dielectric constant of the plasma in such a case can be written as

$$\varepsilon = \varepsilon_L + \varepsilon_{NL} < (EE^*) > \qquad (5)$$

Hence, the nonlinear part of the dielectric constant which arises due to laser beam-plasma interaction

$$\varepsilon_{NL} = \frac{\omega_p^2}{\omega^2}\left[1 - \exp\left[-\frac{3m}{4M}\alpha E_0^2 exp\left\{-\frac{r^2}{r_0^2}\right\}\right]\right] \qquad (6)$$

In the absence of the propagating beam, the medium behaves like a linear medium, and the corresponding dielectric constant is given as

$$\varepsilon_L = 1 - \frac{\omega_p^2}{\omega^2} \qquad (7)$$

These equations (5 to 7) represent that the nonlinearity arises due to ponderomotive force and depends on plasma frequency, beam frequency and intensity of the beam interacting with plasma.

The nonlinear part of the dielectric constant (ε_{NL}) of plasma for ponderomotive nonlinearity for different values of beam intensity (βE_0^2) are calculated and tabulated as follows

Table 1

Intensity parameter βE_0^2	**Nonlinear part of the dielectric constant ε_{NL}*10^-2**
1	**3.95**
2	**5.50**
3	**5.94**
4	**6.14**
5	**6.21**
6	**6.23**
7	**6.25**
8	**6.25**
9	**6.25**
10	**6.25**

Here, ωp =2.5 x 1013 rad/sec, ω= 1 x 1014rad/sec, N0 = 9.5 x 10 cm³ and r0= 30 μm.
The equation (6) represents that the nonlinearity arises due to ponderomotive force and depends on plasma frequency, frequency of beam and intensity of the beam interacting with plasma.

4. Equation For Self-Generated Lens Effect For Homogeneous Ponderomotive Nonlinearity In Plasma

For the ponderomotive nonlinearity in plasma, the equations for the self-generated lens effect (i.e. self-focusing)corresponding to a new technique of the non-paraxial approach[19] can be obtained by substituting the expression for dielectric constant from equation(6)

$$\frac{d^2f(z)}{dz^2}=\frac{2}{\kappa^2r_0^2r^2f(z)}-\frac{1}{\kappa^2r_0^4f^3(z)}\quad-\frac{f(z)}{r^2\epsilon_L}\,\frac{\omega_p^2}{\omega^2}\left[1-\exp\left[-\frac{3m}{4M}\alpha E_0^2exp\left\{-\frac{r^2}{r_0^2}\right\}\right]\right]\qquad(8)$$

Where $\frac{2}{\kappa^2r_0^2r^2f(z)}-\frac{1}{\kappa^2r_0^Af^3(z)}$ These terms represent diffraction term which depends on several factors: Characteristic scales related to the wave number and initial beam width or curvature.r^2 denotes a radial coordinate or some measure related to beam radius. f(z) in the denominator suggests that this effect weakens as f(z) increases.

$\frac{f(z)}{r^2\epsilon_L}\frac{\omega_p^2}{\omega^2}\left[1-\exp\left[-\frac{3m}{4M}\alpha E_0^2exp\left\{-\frac{r^2}{r_0^2}\right\}\right]\right]$ This represents a nonlinear effect related to a self-focusing or defocusing process, such as a pondermotive relativistic. Where ω_p^2 is square of\ plasma\ frequency, ω frequency of incident beam and E_0^2 related to the intensity of the incident laser beam

The nonlinearity term depends on types of nonlinearity as well as on charge distribution within plasma.

5. Classification of Plasma Based On Homogeneity

The nature of plasma can be classified according to the charge distribution within the plasma, such as uniform or nonuniform distribution[1 to 3].

(i) *Homogeneous Plasma*

If the charge distribution in plasma is uniform throughout, then plasma is termed as homogeneous plasma. In such a type of plasma, the charge density (N) has the same value at any time and space within the plasma.

$$N(x, y, z, t) = N_0(x, y, z, t) \qquad (9)$$

Here N_0 represents the density of plasma at x = 0, y = 0,z= 0 and t = 0

(ii*) Inhomogeneous Plasma*

Inhomogeneous plasma is in which the charge density distribution is nonuniform in space, where laser-plasma interaction is considered.

In real life, this variation in the charge density within plasma is quite complex. It depends upon the nature of the plasma, man-made or natural (ionospheric, etc.). For the study of the lens effect of the laser beam in plasma, some simple models for variation of the charge density are considered for the present non-paraxial approach [19].

The charge density of the plasma at any time and space can be represented by a mathematical function as

$$N(x, y, z, t) = N_0 * G(x, y, z, t) \qquad (10)$$

Here, N_0 represents plasma density at x = 0, y = 0, Z=0 and t = 0, and G(x,y,z,t) is the charge density profile function and has different shapes for different types of inhomogeneity variation in space.

The G(x,y,z,t) charge density profile function is responsible for the change in plasma frequency. The expression for the frequency with inhomogeneity is written as

$$\omega_p^2 = \frac{4\pi N_0 e^2}{m} G(z)$$

or

$$\omega_p^2 = \omega_{p0}^2 G(z) \qquad (11)$$

Where $\omega_{p0}^2 = \frac{4\pi N_0 e^2}{m}$ Is the homogeneous plasma frequency

The effective dielectric constant of the medium which depends on the plasma frequency of plasma and it can be linearly as well as nonlinearly dependent on the electric field. This nonlinear part of the dielectric constant is usually a measure of nonlinearity in the medium. Different mechanisms are responsible for the nonlinearity, which depends on the type of medium as well as the nature of the interacting field.

(iii) ***Inhomogeneous Plasma Models***

In the present analysis, it is assumed that the wave is propagating in the z-direction. For axially inhomogeneous plasma, the electron charge density varies along the z direction only, i.e. the charge distribution is nonuniform only along the direction of propagation. However, along the x-axis and y-axis, the charge density is supposed to be uniform in axially inhomogeneous plasma. For a steady state system, plasma inhomogeneity in the axially inhomogeneous plasma can be rewritten using equation (10) as

$$N(z) = N_0 * G(z) . \qquad (12)$$

Where G(z) is the density profile function. It is only a function of the z coordinate. Variations of charge density in axial direction can be different for different systems. Function G(z) have different forms for different types of axially inhomogeneous plasma. In the present study, a few variations for axially inhomogeneous plasma have been considered, which are found to be of practical importance and are discussed below

(iv) ***Linearly Increasing Axially Inhomogeneous***

Plasma, where charge density increases linearly with the propagation distance, is termed linearly increasing axially inhomogeneous plasma. For such type of axially inhomogeneous plasma, the charge density profile function is given as

$$G(z) = 1 + \frac{Z}{L} \qquad (13)$$

where L. is the characteristic scale length on inhomogeneity. It is clear from the function that when L >> z, the inhomogeneous plasma tends to be homogeneous [see Figure 1].

(v) *Linearly Decreasing Axially Inhomogeneous*

The charge density profile function for such type of plasma is given as

$$G(z) = 1 - \frac{Z}{L} \tag{14}$$

The charge density profile function G(z), for linearly increasing as well as linearly decreasing axially inhomogeneous plasmas at different axial distances of propagation, using equations (12) and (13), have been calculated for two values of characteristic scale length(L) depends on inhomogeneity and plotted in Figure 1.

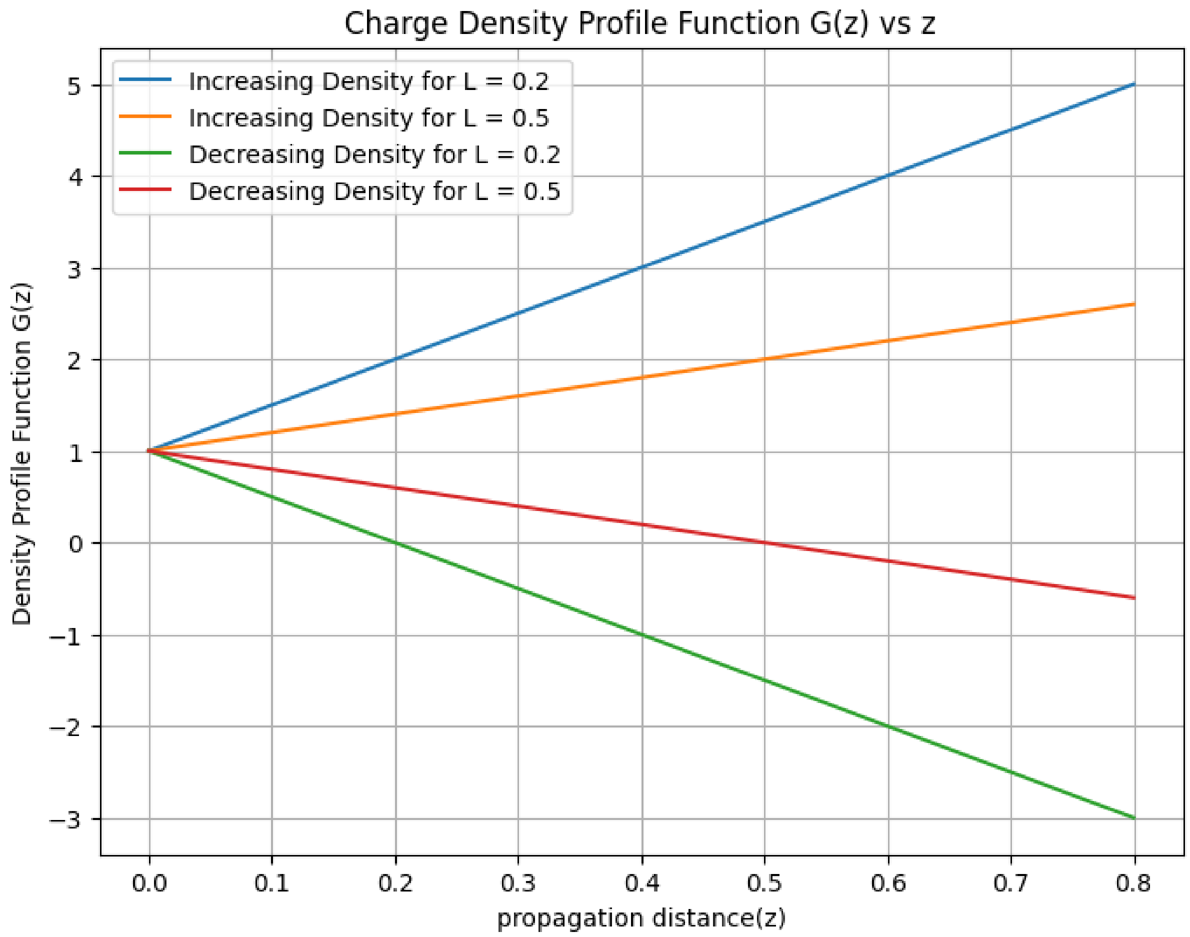

Figure 1

Variation of the charge density profile function G(z) for axially inhomogeneous plasma with propagation distance (z) for two values of characteristic scale length of inhomogeneity(L).

First two Curves: linearly increasing inhomogeneous plasma with L = 0.2 and 0.5, respectively,

The last two curves are linearly decreasing inhomogeneous plasma with L=0.2 and 0.5, respectively.

Exciting results are obtained, i.e. For a given value of L=0.2 and 0.5, the inhomogeneity of the plasma increases as the beam propagates in linearly increasing inhomogeneous plasma, and it decreases for linearly decreasing inhomogeneous plasma.

For inhomogeneous plasma, G (z) has a positive value up to z = L, after which it has a negative value. This change in the density profile function is responsible for the behaviour of the inhomogeneous plasma.
It is also observed that for both types of inhomogeneity, the plasma acts like a homogeneous medium for higher values of L.
It is also observed that for both types of inhomogeneity, the plasma acts like a homogeneous medium for higher values of L.

6. Equation For Self-Generated Lens Effect For Ponderomotive Nonlinearity In Inhomogeneous Plasma

For the ponderomotive nonlinearity in the equations for the self-generated lens effect (i.e. self-focusing) corresponding to the new technique of the non-paraxial approach can be obtained by substituting the expression for dielectric constant for axial increasing inhomogeneous plasma from equation (6) into (8), one gets

$$\frac{d^2f(z)}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^2 f(z)} - \frac{1}{\kappa^2 r_0^4 f^3(z)} - \frac{f(z)}{r^2 \epsilon_L}\left(1+\frac{Z}{L}\right)\frac{\omega_p^2}{\omega^2}\left[1-\exp\left[-\frac{3m}{4M}\alpha E_0^2 exp\left\{-\frac{r^2}{r_0^2}\right\}\right]\right] \tag{15}$$

For axial decreasing inhomogeneous plasma, the equation is converted as follows.

$$\frac{d^2f(z)}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^2 f(z)} - \frac{1}{\kappa^2 r_0^4 f^3(z)} - \frac{f(z)}{r^2 \epsilon_L}\left(1-\frac{Z}{L}\right)\frac{\omega_p^2}{\omega^2}\left[1-\exp\left[-\frac{3m}{4M}\alpha E_0^2 exp\left\{-\frac{r^2}{r_0^2}\right\}\right]\right] \tag{16}$$

It is a second-order differential equation for the dimensionless beam width parameter (i.e. normalised self-focusing parameter), a function of both r and z. This equation obtains the entire spatial propagation characteristics of the laser beam in the plasma due to ponderomotive nonlinearity.
It is not easy to solve this equation analytically. Hence, it is solved numerically [12] using Runge-Kutta method. For this purpose, typical sample plasma with the following parameters is considered in the present analysis

- ω_p(Frequency of incident beam) = 1 $*10^{14}$ rad/sec,
- ω (Plasma frequency) = 2.5 * 10^{13} rad/sec,
- T_0 (Temperature of plasma) = 10^5 k
- r_0 (Initial size of the laser beam) = 30 μm.
- N_0 Electron number density) = 9.5 * 10^{19}
- β E_0^2 (Initial intensity parameter) = ((3m)/(4M) * α * E_0^2 = 0.55

The results for homogeneous and Inhomogeneous plasma considering self-focusing due to ponderomotive nonlinearity have been calculated and plotted in Figure (2).

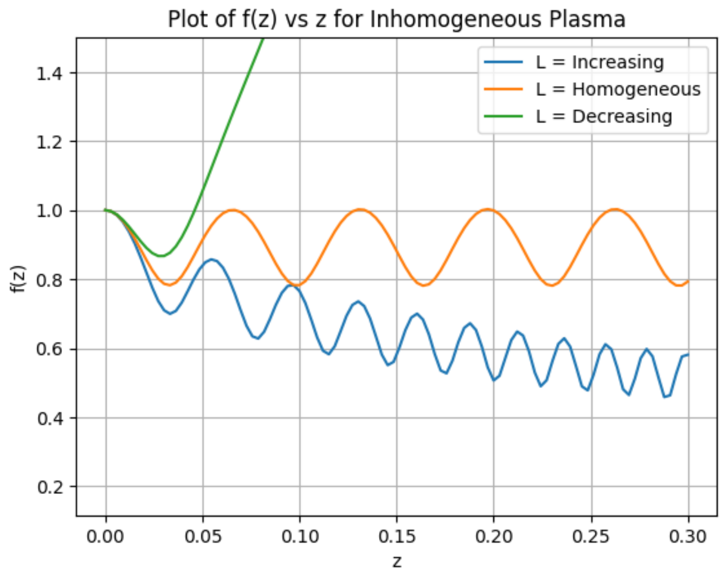

Figure (2)

The graph between self-focusing parameter f(z) and axial distance(z)

The variations of focusing parameter f with axial distance z demonstrate oscillatory behaviour, indicating that during propagation, the laser beam aperture in plasma first decreases, attains a minimum value, and then increases. This process repeats again and again, providing oscillatory behaviour. The self-focusing equations for homogeneous (8) and inhomogeneous plasma (9 and 10) represent the beam behaviour in plasma. One concludes that only nonlinear term changes due to inhomogeneity. This indicates that due to nonlinear terms, the entire shape of the beam in plasma may change.

These results are also presented in Figure (2) for comparison; careful observation of these results indicates that for linearly axially increasing inhomogeneous plasma, the minimum value of the normalised self-focusing parameter (f_{min}) for the second and higher orders decreases continuously, while it remains constant in the homogeneous plasma. This is because when electron charge density increases, nonlinearity increases. Hence, refraction dominates over the diffraction effects, which yields less value for f_{min}.

These curves also indicate that for an axial distance less than the characteristic length of inhomogeneity (i.e. $z < L$), linearly increasing inhomogeneous plasma has a low value of f min. But at x = L, both the curves have the same value of f min .

This is because for axial distance z to be less than the characteristic length L, the variation of electron charge density with z is more effective for linearly increasing the electron charge density profile.

Mathematically, these results show that at z=0, the value of f(z) and

df/dz= 0, i.e. the beam width has no initial divergence. With the increase in the value of z in the vicinity of z = 0, df/dz becomes negative, f starts decreasing, and its value becomes less than unity.

The first two terms in the R.H.S. of equation (11) decrease more rapidly than the third term. At this point, the axial intensity of the focused beam is considerably enhanced, and thus d^2f/dz^2 becomes positive.

Oscillatory behaviour of self-focusing beam width parameter (f) with axial distance (z) for axial inhomogeneous ponderomotive nonlinearity.

The observed oscillatory behaviour of beam aperture during propagation of laser beam in a nonlinear plasma medium, in the axial direction, maybe because in the vicinity of vacuum-plasma interface, i.e. at z=0, with the increasing value of z. diffraction divergence decreases more rapidly than nonlinearity-based convergence or focusing effect, consequently decreasing beam aperture. Due to the continuous decrease in beam aperture, at a particular value of z, the beam's intensity is considerably enhanced and diffraction divergence starts dominating over the focusing convergence effect. Thus, after attaining the minimum value, the beam aperture increases beyond z, i.e Z> L. After propagating to a certain length in plasma, the beam aperture increases up to the maximum value (f_{min}). Then, the focusing effect starts dominating the defocusing diffraction effect, and the beam aperture again starts decreasing.

Because of these two-diffraction and nonlinearity-related self-focusing effects and their dominance over one another during the propagation of the laser beam in the axial direction, the medium acts as an oscillatory waveguide.

7. Uniform Waveguide Propagation

From the study of the self-generated equations (8,15, and 16), one can conclude that the diffraction terms are responsible for divergence while refraction terms are responsible for the convergence of the beam and the inhomogeneity factor is combined with the refraction term.

On applying the boundary conditions for the vacuum-plasma interface, i.e., at z =0; f=1, df/dz= 0; and d^2f/dz^2; hence, for uniform waveguide propagation mode from equation (8), one gets

$$\frac{\omega_p \rho}{c} = (2r_0^2 - \rho^2)^{1/2} \frac{\rho}{r_0^2}\left[1 - \exp\left[-\beta E_0^2 exp\left\{-\frac{r^2}{r_0^2}\right\}\right]\right]^{-1/2} \tag{17}$$

For all types of axially inhomogeneous plasma, the self-trapping equations are the same for homogeneous; this is because electron charge density at the vacuum-plasma interface (i.e. z =0) equals 1. Hence, inhomogeneity has no effect on self-trapping conditions at the vacuum-plasma interface.

This equation is again numerically solved considering the parameters as referred to above.

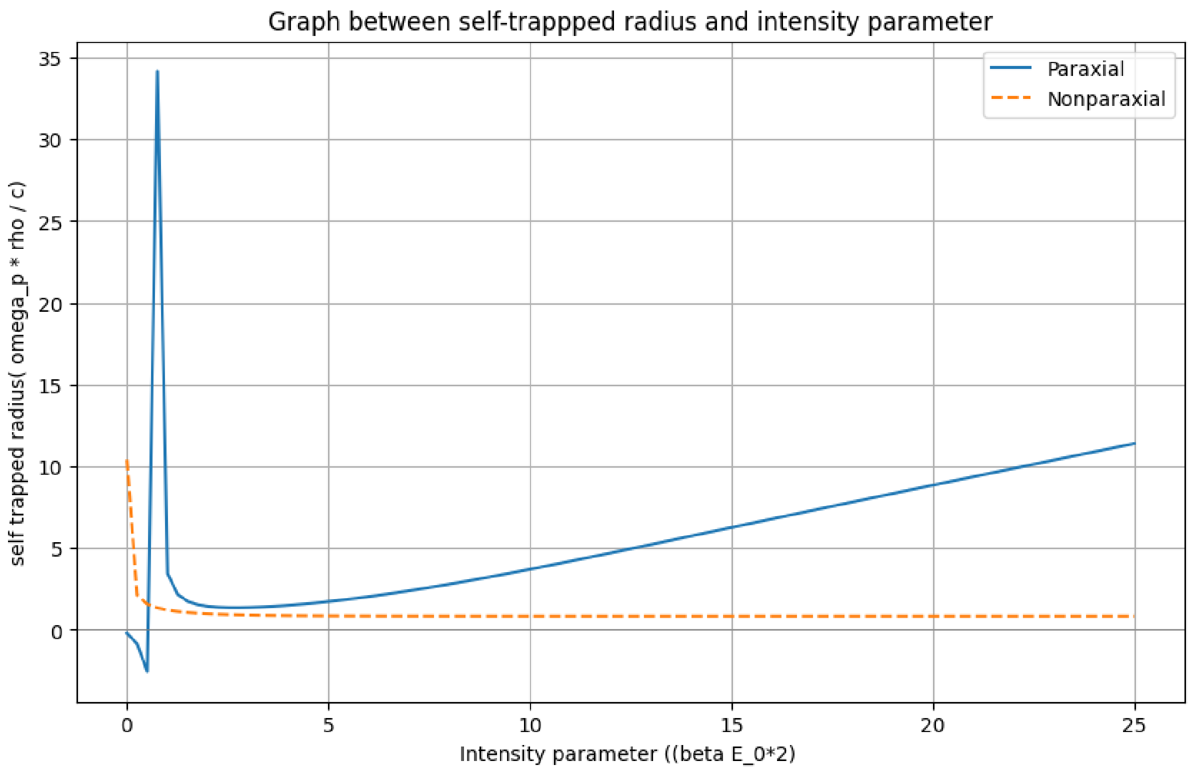

Fig 03

The graph between self-trapped radius and intensity parameter

The self-trapping behaviour of the beam is shown in Figure 3, where $\frac{\omega_p \rho}{c}$ plotted with (βE_0^2) For the present non-paraxial technique and compared with the results of paraxial ray approximation, it is observed that only one value of the self-trapped radius is observed, which matches with the practical result, but it has two values for any given value of the intensity of laser beam Results of the paraxial ray methods are obtained using relation [9,18]

8. Conclusion

The results obtained by the present study of axial inhomogeneity in shaping the ponderomotive dynamics of laser-driven plasmas, like self-focusing and self-trapping of the laser beam in axial inhomogeneous plasma, have good agreements with practical results. Simulations corroborated these findings, showing that axial inhomogeneity enhances the ponderomotive force, leading to more efficient energy transfer from the laser to the plasma. The simulations revealed localised regions of increased electron density, aligning with the experimental observations.

These findings using a new paraxial technique have implications for optimising laser-plasma interaction conditions in applications such as particle acceleration and magnetic field generation [12,19]. Future research will focus on exploring different inhomogeneity profiles and their effects on plasma behaviour.

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