© 2018 IJSRSET | Volume 4 | Issue 8 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
International Journal of Scientific Research in Science, Engineering and Technology

Some curvature properties of K-Contact Manifolds Admitting Semi-Symmetric Non-Metric Connection

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ABSCTACT

The paper deals with the study some properties of Concircular curvature tensor in K-contact manifolds admitting semi-symmetric non-metric connection.

Keywords: K-Contact Manifold, Concircular Curvature Tensor, Metric Connection.

1. Introduction:

In [12], Friedmann and Schouten introduced the notion of semi-symmetric linear connection on a differentiable manifold. Hayden [13] introduced the idea of semi-symmetric non-metric connection on a Riemannian manifold. The idea of semi-symmetric metric connection on Riemannian manifold was introduced by Yano [20]. Various properties of such connection have been studied by many geometers. Agashe and Chafle [1] defined and studied a semi-symmetric non-metric connection in a Riemannian manifold. This was further developed by Agashe and Chafle [2], De and Kamilya [11], Tripathi and Kakkar [17], Jaiswal and Ojha [14] and several other geometers. Sengupta, De and Binh [16], De and Sengupta [10] defined new types of semi-symmetric non-metric connections on a Riemannian manifold and studied some geometrical properties with respect to such connections. Chaubey and Ojha [8], defined new type of semi-symmetric non-metric connection on an almost contact metric manifold. In [9], Chaubey defined a semi-symmetric non-metric connection on an almost contact metric manifold and studied its different geometrical properties. Some properties of such connection have been further studied by Jaiswal and Ojha [14], Chaubey and Ojha [8].

In the present paper, we study the properties of semi-symmetric non-metric connection in K-Contact manifold. Section 2 is preliminaries in which the basic definitions are given. Next sections deals with brief account of semi-symmetric non-metric connection and some properties of curvature tensors are obtained.

2. Preliminaries:

An n-dimensional differentiable manifold M is said to have an almost contact structure $(\phi,\ \xi,\ \eta)$ if it carries a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η on M satisfying

$$(2.1)\ \phi^2 X = -X + \eta(X)\xi,\quad \phi\xi = 0,$$
$$\eta(\xi) = 1,\quad \eta\cdot\phi = 0.$$

If g is a Riemannian metric with almost contact structure that is,

$$(2.2)\ g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y),$$
$$\eta(X) = g(X,\xi).$$

Then M is called an almost contact metric manifold equipped with an almost contact metric structure $(\phi,\ \xi,\ \eta, g)$ and denoted by $(M, \phi,\ \xi,\ \eta, g)$.

If on $(M, \phi,\ \xi,\ \eta, g)$ the exterior derivative of 1-form η satisfies,

$$d\eta(X,Y) = g(X,\phi Y).$$

Then $(M, \phi,\ \xi,\ \eta, g)$ is said to be a contact metric manifold.

If moreover ξ is Killing vector field, then M is called a K-contact manifold. A K-contact manifold is called Sasakian, if the relation

$$(2.3)\ (\nabla_X\phi)Y = g(X,Y)\xi - \eta(Y)X,$$

holds, where ∇ denotes the covariant differentiation with respect to g. From (2.3), we get

$$(2.4)\ \nabla_X\xi = -\phi X, (\nabla_X\eta)Y = g(X,\phi Y).$$

In a K-contact manifold M the following relations holds:

$$(2.5)\ g(R(\xi,X)Y,\xi) = g(X,Y) - \eta(X)\eta(Y),$$
$$(2.6)\ R(X,Y)\xi = \eta(Y)X - \eta(X)Y,$$
$$(2.7)\ R(\xi,X)\xi = \eta(X)\xi - X,$$
$$(2.8)\ S(X,\xi) = (n-1)\eta(X),$$

for any vector fields X, Y. Where R is the Riemannian curvature tensor and S is the Ricci tensor of the manifold M.

3. semi-symmetric non-metric connection:

A linear connection $\tilde{\nabla}$ on M is defined as

$$(3.1)\qquad \tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X,$$

Where η is a 1-form associated with the vector field ξ on M. By virtue of (3.1), the torsion tensor $\tilde{T}$ of the connection $\tilde{\nabla}$ and is given by

$$(3.2)\qquad \tilde{T}(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y].$$

A linear connection $\tilde{\nabla}$ on M is said to be a semi-symmetric connection if its torsion tensor $\tilde{T}$ of the connection $\tilde{\nabla}$ satisfies

(3.3) $\quad \tilde{T}(X,Y) = \eta(Y)X - \eta(X)Y.$

If moreover $\tilde{\nabla}g = 0$ then the connection is called a semi-symmetric metric connection. If $\tilde{\nabla}g \neq 0$ then the connection $\tilde{\nabla}$ is called a semi-symmetric non-metric connection.

From (3.1), we get

$$(3.4)\big(\tilde{\nabla}_X g\big)(Y,Z) = -\eta(Y)g(X,Z) - \eta(Z)g(X,Y),$$

for all vector fields X, Y, Z on M.

A relation between Riemannian curvature tensors R and $\tilde{R}$ with respect to Riemannian connection ∇ and semi-symmetric non-metric connection $\tilde{\nabla}$ of a K-contact manifold M is given by

$$(3.5)\tilde{R}(X,Y)Z = R(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y,$$

for all vector fields X, Y, Z on M where α is a tensor field of $(0,2)$ type defined by

$$(3.6)\alpha(X,Y) = (\nabla_X\eta)Y - \eta(X)\eta(Y) = \big(\tilde{\nabla}_X\eta\big)Y.$$

By using(2.6)in (3.6), we obtain

$$(3.7)\alpha(X,Y) = g(X,\phi Y) - \eta(X)\eta(Y).$$

By virtue of (3.7) in equation (3.5), we get

$$(3.8)\tilde{R}(X,Y)Z = R(X,Y)Z - g(Y,\phi Z)X$$
$$+\eta(Z)\eta(Y)X + g(X,\phi Z)Y - \eta(X)\eta(Z)Y.$$

A relation between Ricci tensors $\tilde{S}$ and S with respect to semi-symmetric non-metric connection $\tilde{\nabla}$ and the Riemannian connection ∇ of a K-contact manifold M is given by

(3.9) $\quad \tilde{S}(Y,Z) = S(Y,Z) - (n-1)\alpha(Y,Z).$

On contracting(3.9), we obtain

$$(3.10)\tilde{r} = r - (n-1)trace(\alpha).$$

Lemma 3.1: Let M be an n-dimensional K-contact manifold with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$. Then

$$(3.11)\big(\tilde{\nabla}_X \phi\big)Y = (\nabla_X\phi)Y - \eta(Y)\phi X,$$
$$(3.12)\tilde{\nabla}_X\xi = X - \phi X,$$
$$(3.13)\big(\tilde{\nabla}_X \eta\big)Y = (\nabla_X\eta)Y - \eta(X)\eta(Y) = \alpha(X,Y).$$

Proof: By using (3.1) and (2.1), we obtain (3.11). From (3.1) and (2.5), we get (3.12). Finally, by virtue of (3.1), (2.4) and (2.6) we get (3.13).

From (3.13), we can easily state the following corollary:

Corollary 3.1: In a K-contact manifold, the tensor field α satisfies

$$(3.14)\tilde{\alpha}(X,\xi) = -\eta(X).$$

Theorem 3.1: In a K-contact manifold with semi-symmetric non-metric connection $\tilde{\nabla}$, we have

$$(3.15)\tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y$$
$$= [\alpha(X,Z) - \alpha(Z,X)]Y + [\alpha(Z,Y) - \alpha(Y,Z)]X$$
$$+[\alpha(Y,X) - \alpha(X,Y)]Z.$$
$$(3.16)\tilde{R}(X,Y,Z,W) + \tilde{R}(Y,X,Z,W) = 0.$$
$$(3.17)\tilde{R}(X,Y,Z,W) - \tilde{R}(Z,W,X,Y)$$
$$= [\alpha(X,Z) - \alpha(Z,X)]g(Y,W)$$
$$+\alpha(W,X)g(Y,Z) - \alpha(Y,Z)g(X,W).$$

Proof: By using(3.5), we obtain

$$(3.18)\tilde{R}(X,Y)Z + \tilde{R}(Y,Z)X + \tilde{R}(Z,X)Y$$
$$= R(X,Y)Z + R(X,Y)Z + R(X,Y)Z$$
$$+[\alpha(X,Z) - \alpha(Z,X)]Y$$
$$+[\alpha(Z,Y) - \alpha(Y,Z)]X$$
$$+ [\alpha(Y,X) - \alpha(X,Y)]Z.$$

By using first Bianchi identity $R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 0$ in (3.18) we obtain (3.15).

Again by using(3.5), we get

$$(3.19)\tilde{R}(X,Y,Z,W) = R(X,Y,Z,W)$$
$$-\alpha(Y,Z)g(X,W) + \alpha(X,Z)g(Y,W).$$

If we change the role of X and Y in(3.19), we have

$$(3.20)\tilde{R}(Y,X,Z,W) = R(Y,X,Z,W)$$
$$+\alpha(Y,Z)g(X,W) - \alpha(X,Z)g(Y,W).$$

By virtue of (3.19) and (3.20), we obtain

$$(3.21)\tilde{R}(X,Y,Z,W) + \tilde{R}(Y,X,Z,W)$$
$$= R(X,Y,Z,W) + R(Y,X,Z,W).$$

Since $R(X,Y,Z,W) + R(Y,X,Z,W) = 0$ and then we get (3.16).

Now by using(3.19), we have

$$(3.22)\tilde{R}(X,Y,Z,W) - \tilde{R}(Z,W,X,Y)$$
$$= R(X,Y,Z,W) + R(Z,W,X,Y)$$
$$+[\alpha(X,Z) - \alpha(Z,X)]g(Y,W)$$
$$+\alpha(W,X)g(Y,Z) - \alpha(Y,Z)g(X,W).$$

We know that $R(X,Y,Z,W) = R(Z,W,X,Y)$, then (3.22) reduces as (3.17).

Lemma 3.2: Let M be an n-dimensional K-contact manifold with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$.Then

$$(3.23)\tilde{R}(X,Y)\xi = 2[\eta(Y)X - \eta(X)Y],$$
$$(3.24)\tilde{R}(\xi,X)\xi = 2[\eta(X)\xi - X],$$
$$(3.25)\tilde{R}(\xi,X)Y = g(X,Y)\xi - 2\eta(Y)X - \alpha(X,Y)\xi,$$

Proof: By using (2.8) in (3.5), we get (3.23). By using (2.10) and (3.5), we have (3.24). From (2.9) and (3.5), we obtain (3.25).

Lemma 3.3: In an n-dimensional K-contact manifold with respect to the semi-symmetric non-metric connection, we have

$$(3.26)\tilde{S}(X,\xi) = 2(n-1)\eta(X),$$
$$(3.27)\tilde{S}(\phi X,\phi Y) = \tilde{S}(X,Y).$$

Proof: By using (2.11) and (3.9), we obtain (3.26). From equation(2.12)and (3.9), we get (3.27).

4. Concircular curvature tensor of K-contact manifold admitting semi-symmetric non-metric connection:

Let M be an n-dimensional K-contact manifold, then the Concircular curvature tensor C of M with respect to Levi-Civita connection is defined by

$$(4.1) C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y],$$

where R and r are Riemannian curvature tensor and scalar curvature of the K-contact manifold M.

Theorem 4.1: Let M be a K-contact manifold. Then the Concircular curvature tensors C and $\tilde{C}$ of the K-contact manifolds with respect to the Levi-Civita connection and semi-symmetric non-metric connection is related as

$$(4.2) \tilde{C}(X,Y)Z = C(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y - \frac{trace(\alpha)}{n}[g(Y,Z)X - g(X,Z)Y].$$

Proof: Let $\tilde{C}$ and C denote the Concircular curvature tensor of M with respect to the semi-symmetric non-metric connection and the Levi-Civita connection, respectively. Concircular curvature tensor $\tilde{C}$ with respect to the semi-symmetric non-metric connection is defined by

$$(4.3) \tilde{C}(X,Y)Z = \tilde{R}(X,Y)Z - \frac{\tilde{r}}{n(n-1)}[g(Y,Z)X - g(X,Z)Y].$$

where $\tilde{R}$ and $\tilde{r}$ are the Riemannian curvature tensor and scalar curvature of the K-contact manifold M with respect to semi-symmetric non-metric connection.

Then by using (3.5) and (3.10) in (4.3), we get

$$(4.4) \tilde{C}(X,Y)Z = R(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y - \frac{r-(n-1)trace(\alpha)}{n(n-1)}[g(Y,Z)X - g(X,Z)Y],$$

which gives (4.2). This completes the proof of the theorem.

Theorem 4.2: In an n-dimensional K-contact manifold M, the Concircular curvature tensor $\tilde{C}$ of the manifold with respect to the semi-symmetric non-metric connection doesn't satisfy the first Bianchi identity, that is,

$$(4.5) \tilde{C}(X,Y)Z + \tilde{C}(Y,Z)X + \tilde{C}(Z,X)Y \neq 0.$$

Proof: First Bianchi identity for Concircular curvature tensor $\tilde{C}$ of K-contact manifold is given by

$$(4.6) \tilde{C}(X,Y)Z + \tilde{C}(Y,Z)X + \tilde{C}(Z,X)Y = R(X,Y)Z + R(Y,Z)X + R(Z,X)Y + [\alpha(X,Z) - \alpha(Z,X)]Y + [\alpha(Z,Y) - \alpha(Y,Z)]X + [\alpha(Y,X) - \alpha(X,Y)]Z.$$

Since $R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 0$ and $\alpha(Y,Z) \neq \alpha(Z,Y)$, we obtain

$$(4.7) \tilde{C}(X,Y)Z + \tilde{C}(Y,Z)X + \tilde{C}(Z,X)Y = [\alpha(X,Z) - \alpha(Z,X)]Y + [\alpha(Z,Y) - \alpha(Y,Z)]X + [\alpha(Y,X) - \alpha(X,Y)]Z.$$

In view of (4.7), we obtain (4.5).

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