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Application of Power Series method to Non-Linear Differential Equations

Bhavya B S

Post Graduate Department of Mathematics & Research Centre in Applied Mathematics, MES College, Malleswaram, Bangalore, Karnataka, India

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ABSTRACT

We consider the nonlinear differential equation in classical form and look for power series solution and apply Pade approximation for the same. Radius of convergence is discussed by Domb Syke plot.

Keywords : Power Series Method, Pade Approximation, Dom Syke Plot

1. Introduction

A mathematical theory for water waves that would admit a wave solution that did not disperse with time was proposed by Joseph Boussinesq [1872]. By assuming the wave amplitude to be small compared to the canal depth, he arrived at the well known Boussinesq equation.

Conte et. al. [1994] have explained property of fusion and fission of a class of solitons for CB equation. Maria et. al. [1998] have obtained new exact solutions for generalized boussinesq equation, which has CB equation as a special case. Conte et. al. [1995] have derived Lax pair, Darboux transformation and hence the auto Backlund transformation for CB equation by Painleve analysis.

2. Power Series

Let us consider the classical Boussinesq equation

$$\eta_t = -\left((1+\eta)u + \frac{1}{3}u_{xx} \right)_x , \qquad (1)$$

$$u_t = -\left(\eta + \frac{1}{2}u^2\right)_x . \quad (2)$$

Traveling wave solutions can be found by assuming $z = x - \lambda t.$ Using this transformation in equations (1) and (2) and integrating once we get

$$2u_{zz} = 3u^3 - 9\lambda u^2 + 6\left(\lambda^2 - 1\right)u. \quad (3)$$

Let us consider series solution of (5.3) as follows

$$u = \sum_{n=0}^{\infty} a_n z^n . \quad (4)$$

Substituting (4) in (3) and comparing the terms of same power we get recurrence relation for $\lambda = 0.5$ as,

$$a_{n+2} = \frac{1}{2(n+1)(n+2)}\left\{3\sum_{i=0}^{n} b_i a_{n-i} - 9\times 0.5 b_n - 6\times 0.75 a_n\right\}, \quad (5)$$

where

$$b_n = \sum_{i=0}^{n} a_i a_{n-i} . \quad (6)$$

From the solution obtained by Dong (2002) suggests that we can choose initial condition for equations (5.1) and (5.2) as

$$\left.\begin{array}{l} u(0) = -0.37 \\ u'(0) = 0 \end{array}\right\} \quad (7)$$

Using initial conditions (7) in (5) we get series solution as given below

$$u = -0.37 + 0.2242z^2 + 0.00058z^4 - 0.00656z^6 + 0.00028z^8 + 0.000129z^{10}$$
$$-0.0000147z^{12} - 1.87947\times 10^{-6} z^{14} + 4.695\times 10^{-7} z^{16} + 9.6527\times 10^{-9} z^{18}$$
$$-1.14956\times 10^{-8} z^{20} - 5.90567\times 10^{-10} z^{22} + 2.2292\times 10^{-10} z^{24} + \cdots . \quad (8)$$

The Pade approximation of equation (8) upto ten terms is given in equation (9)

$$u_{pade}[8/8] = \frac{\left(-0.37 + 0.19386z^2 + 0.0095z^4 - 0.00107z^6 - 0.0000908z^8\right)}{\left(1 + 0.0821z^2 + 0.0255z^4 + 0.00079z^6 + 0.0000688z^8\right)} . \quad (9)$$

The series solution given in (8) represents double soliton solution TABLE 5.1

A comparision of series (8) and Pade approximants at z=0

C (no. of terms)	Power series	Pade Approximation
1	-0.37	-0.37
2	-0.145752	-0.37
3	-0.145752	-0.230375
4	-0.145172	-0.145171
5	-0.145172	-0.145171
6	-0.151734	-0.145171
7	-0.151734	-0.151294
8	-0.151454	-0.151295
9	-0.151454	-0.151295
10	-0.151454	-0.151295
11	-0.151324	-0.151295
12	-0.151339	-0.15134
13	-0.151339	-0.15134
14	-0.151341	-0.15134
15	-0.151341	-0.15134
16	-0.15134	-0.15134

We have shown that radius of convergence is very small by Domb-Sykes plot. This plot is typical for the analysis of the series in that the curves show an establishment of a linear relationship between the ratio of coefficients as the number of terms increases.

5.4 Phase – Plane Analysis

We consider the reduced differential equation for value $\lambda = 0.5$

$$2u_{zz} = 3u^3 - 4.5u^2 - 4.5u. \tag{10}$$

This differential equation may be studied in phase plane. Putting

$$\frac{du}{dz} = v, \tag{11}$$

$$\frac{dv}{dz} = \frac{3}{2}u^3 - \frac{4.5}{2}u^2 - \frac{4.5}{2}u. \tag{12}$$

The singularities of the system (11) and (12) are

$$v = 0, \tag{13}$$

$$u = \frac{-\frac{4.5}{2} \pm \sqrt{\left(\frac{4.5}{2}\right)^2 - 4\left(-\frac{3}{2}\right)\left(\frac{4.5}{2}\right)}}{2\left(-\frac{3}{2}\right)}. \tag{14}$$

Thus we get the following singular points

$(0,0)$ is a centre.

$(2.1,0)$ is a saddle point.

$(-0.68,0)$ is a saddle point.

Power series solution is a powerful and easiest tool to solve complicated nonlinear coupled differential equations. In this chapter power series solutions for nonlinear coupled differential equations of classical Boussinesq system are obtained by power series method and DTM. Pade is applied to find singularities and convergence is discussed by Domb Syke plot.

5.6 Graphs

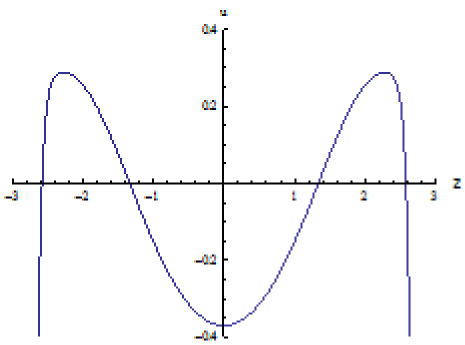


Fig 1.

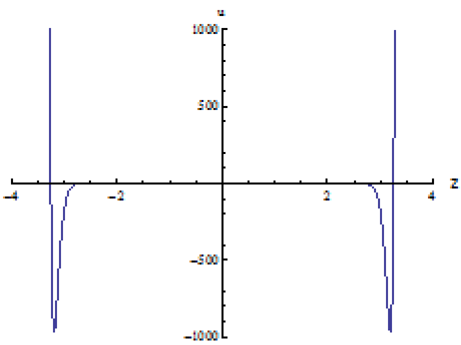
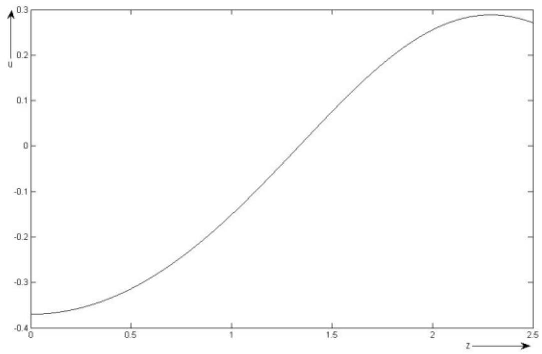
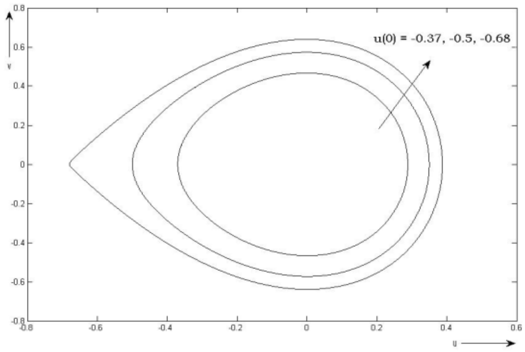


Fig 2





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