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Study on Estimation of Hydraulic Conductivity of Porous Media Using Drag Force Model

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ABSTRACT

The resistance is offered when the fluid flows through a porous medium which acts tangentially and perpendicularly to the surface of the media. This resistance offered by the porous mass can be analyzed by evaluating the ease with which water can flow through the porous media, and is expressed in terms of hydraulic conductivity or permeability of the porous medium. It involves a large number of variables such as particle size, its size distribution, shape, porosity and extent of medium. In the present study, experiments were conducted on four different course materials namely dolomite, river gravel, marble chips and crushed stones of varying sizes to find out the hydraulic conductivity in the laboratory and also utilizing a semi empirical approach based on the drag force model. Based on this experimental data, a relationship is developed between hydraulic conductivity (k), shape factor (z) and the size of the materials (d_g) keeping the porosity constant at 40%. Furthermore, it was observed that hydraulic conductivity obtained from the experimental results tends to give a higher value than those obtained from this relationship, whereas, those obtained from the model are conservative and tend to approach the experimental values as the size of the material increases.

Keywords: Coefficient of Drag, Drag force model, Hydraulic Conductivity

I. INTRODUCTION

Any material containing continuously connected pores will permit the passage of fluid through it. This property is indeed one of the most important soil properties and has a great significance in many fields of Civil Engineering such as irrigation, aquifers, storage reservoir, ground water, etc.

Depleting ground water is a major concern in an agricultural and drought prone country like India. Therefore, it has become vital to study the rate of flow with which water percolates through any porous medium or the resistance which is offered during this movement. And hence, it is predominant that one should have an intimate knowledge of replenishment of groundwater.

The present study is an experimental work involving a study on the effect of the resistance offered to laminar flow through porous media. It also includes the effect of shape and size of different materials on the hydraulic conductivity of the porous media. Moreover, a comparison is done to study the usefulness of the drag force model given by Rumer and Drinker [1966].

II. METHODS AND MATERIAL

2. Drag Force Model

Rumer and Drinker [1966] studied the resistance offered during flow by using drag force model in which, the drag force on individual particles comprising the strata is integrated to obtain the relationship between the superficial velocity of flow and piezometric gradient (fig. 1)

Considering the steady flow of a fluid through a cylindrical element of area dA and length ds , filled with uniform sediment of characteristic size d_g, the summation of forces acting in the s-direction of the fluid in an element results in a zero net force.

$$p\,n\,dA - \left(p + \left(\frac{\partial p}{\partial s}\right) ds\right) n\,dA - (\rho\,g\,n\,dA\,ds)\cos\Theta - F_r = 0$$

This can be further simplified to:

$$-\frac{\partial h}{\partial s} - \frac{Fr}{\rho\, g\, n\, dA\, ds} = 0 \qquad (1)$$

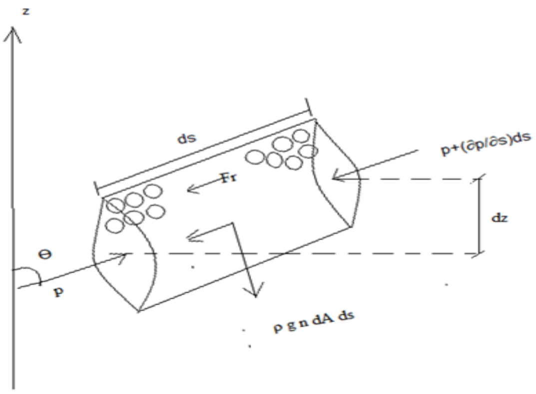

Figure 1: Drag Force Model

The resisting force offered by a single particle is governed by Stoke's law which can be expressed in generalized form as

$$f_p = \lambda\, \mu\, d\, v_s \qquad (2)$$

Where, f_p is the resistance or drag of a single particle, μ represents the dynamic viscosity of the fluid, v_s denotes the local average velocity of flow around the particle in s-direction, and λ represent a coefficient that takes into account the effects of neighbouring particles. λ will depend upon the porosity, the shape of the particles, and the distribution of the sizes of the particles.

The total resistance, F_r offered by all of the particles in an element will thus be

$$F_r = N\, f_p \qquad (3)$$

In which N represents the number of particles in the element. With the aid of (3) and (2), (1) can be written as

$$q_s = -\frac{\beta n^2}{\lambda(1-n)}\, d^2\ \frac{g\,\rho}{\mu}\frac{\partial h}{\partial s} \qquad (4)$$

Where, q_s is the specific discharge given by nv_s and the quantity $\frac{\beta n^2}{\lambda(1-n)}$ is dependent only on the pore system geometry and can be represented by constant C

Thus (4) becomes,

$$q_s = -Cd^2\ \frac{g\,\rho}{\mu}\frac{\partial h}{\partial s} \qquad (5)$$

Here, the product $C\,d^2$ is called as the intrinsic permeability k, hence

$$k = \frac{\beta n^2}{\lambda\,(1-n)} d^2 \qquad (6)$$

On observing the drag force model, it is also found that ratio $\frac{\lambda}{\beta}$ can be represented as shape factor z of the individual particle. Thus,

$$k = \frac{n^2 d^2}{(n-1)z} \qquad (7)$$

2.1. Relationship between Shape Factor and Hydraulic Conductivity

Hydraulic Conductivity of any material depends on the shape of the particles present in the media. This shape is taken into account by the aforementioned factor z which is defined as the ratio of the particle surface area to the surface area of the sphere of equal volume. The expression was worked out by Gupta, R.D. et al [1985] defined the shape factor as

$$z = \frac{S_p}{V_p^{\frac{2}{3}}} \qquad (8)$$

Where, S_p is the surface area of the particle and V_p is the volume of the particle. Also,

$$k = f(z, d) \qquad (9)$$

To study the functional relationship between the hydraulic conductivity, shape factor and size of particles, tests are conducted on various sizes of different materials and the results are analysed mentioned in eq. (26).

2.2. Relationship between Friction Factor and Reynolds Number

The permeation of fluid through porous media depends on a number of variables. According to Rose [1948], head loss across the porous bed, H may be expressed as,

$$H = F_1\,(v, l, d, D, \rho, \mu, g, e, n, z, U) \qquad (10)$$

Where,

H = head loss across porous bed;
v = superficial velocity of flow
l = length of bed
d = size of particle
D = diameter of the permeameter
ρ = mass density of fluid
μ = viscosity of fluid
g = acceleration due to gravity
e = surface roughness of particles
n = porosity of the bed
z = dimensionless shape factor
U = dimensionless factor for size distribution.

By dimensional analysis the above equation reduces to,

$$\frac{H}{d} = F_2\left(\left(\frac{\rho v d}{\mu}\right), \frac{gd}{v^2}, \frac{L}{D}, \frac{D}{n}, n, z, \frac{e}{D}, U\right) \quad (11)$$

If bed is composed of uniform sizes are studied and it is assumed that surface roughness will have comparatively small effect on porous media flow, Eq. 11 can be written as,

$$\frac{H}{d} = F_3\left(\left(\frac{\rho v d}{\mu}\right), \frac{gd}{v^2}, \frac{L}{D}, n, z\right) \qquad (12)$$

Parameters H/d, L/d, and gd/v^2 can be combined to give a single dimensionless factor, conventionally known as the friction factor i.e.

$$F_3\left(\left(\frac{\rho v d}{\mu}\right), \frac{gd}{v^2}, \frac{L}{D}, \frac{D}{d}, n, z, \frac{H}{d}\right) = 0 \qquad (13)$$

Eq. 2.32 may be further solved as,

$$F_4\left(\left(\frac{\rho v d}{\mu}\right), \frac{igd}{v^2}, \frac{D}{d}, n, z\right) = 0 \qquad (14)$$

Where,
i = hydraulic gradient
$\frac{2igd}{v^2}$ = Friction factor
$\frac{\rho v d}{\mu}$ = Reynold's number.

If all the experiments are conducted at a constant porosity and if it is assumed that the D/d ratios involved in this study will have negligible effect on percolation. Eq.14 simplifies to

$$F_5\left(\left(\frac{\rho v d}{\mu}\right), \frac{igd}{v^2}, n, z\right) = 0 \qquad (15)$$

This approach has been used to develop an empirical relationship between k, d_g and z.

2.3. Materials Used

- Dolomite: 2cm, 2.24cm, 2.64 cm
- Marble Chips: 0.63cm, 1.12 cm, 1.32cm
- River Gravel: 1.6cm, 2cm, 2.24cm
- Crushed Stones: 0.4cm, 0.63cm, 0.8cm

2.4. Experimental Equipment

The equipment's used were:

- Constant head permeameter as shown in Fig.2
- Discharge measuring device
- Weighing Balance
- Pycnometer
- Manometer
- Source of supply
- Oven

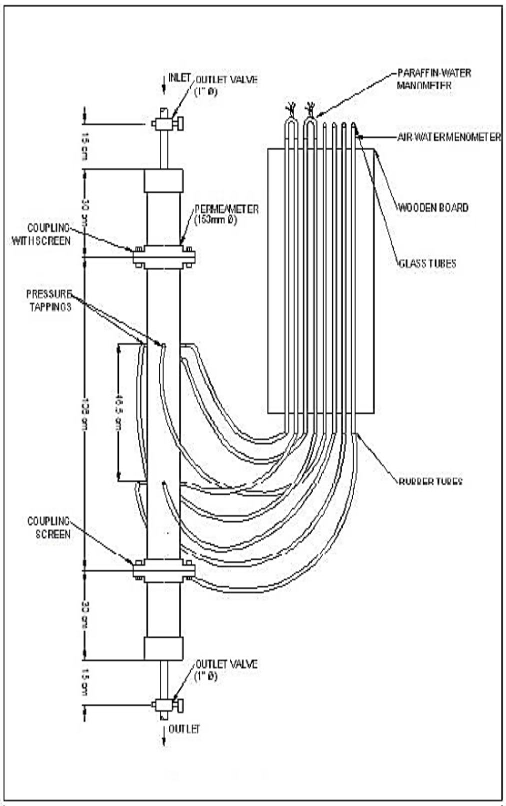

Figure 2: Constant head Permeameter

2.5. Experimental Procedure

The various tests conducted during the itinerary of this study are explained below.

2.5.1. Sieve analysis tests

All the four samples were sieved mechanically through the various sieve sizes. Percentage finer was calculated and thereafter a plot between sieve size and percentage finer by weight on semi log graph paper was prepared as shown in fig. 3 to find the mean size of the particles.

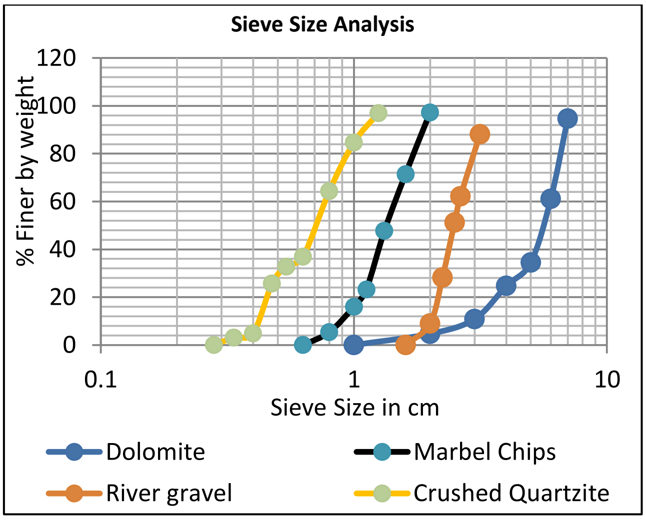

Figure 3: Grain size distribution graphs of granular samples

2.5.2. Hydraulic tests

The hydraulic tests are very important to study the effect of resistance to flow of water in a given sample of material. The weight of the material needed to fill the permeameter was calculated by using the following formula:

$$W_s = (1 - n)\, V_t \;\; G_s \gamma_w \qquad (16)$$

Where,

W_s is weight of the material, n is porosity, V_t is volume of the tube, G_s is the specific gravity of the material and γ_w is the specific weight of water

Readings were taken in descending order of magnitude of discharge and hydraulic gradient, i, to find the hydraulic conductivity, k.

III. RESULTS AND DISCUSSION

Present study investigates a relationship between the hydraulic conductivity (k) of the materials used, their shape factor (z) and size of the particles (d_g- geometrical mean diameter of particle). To study the relationship between shape factor (z) and size of particles (d_g), curve of logarithm of shape factor (log z) and hydraulic conductivity (log k) of particles is drawn. The results of the experimental investigation on the different materials used in the present study are presented in the form of Friction Factor (F_r) vs Reynold's number (R_e) on log-log scale to accertain the laminar flow condition.

The results are discussed in relation to the different aspects of the problem studied is given below.

3. Relationship between Shape Factor (z) and Hydraulic Conductivity (k)

Experimental results for shape factor tests for these materials are shown in figure 3.

From figure 3, it is clear that shape factor and hydraulic conductivity are related by the equation:

$$k = C^{'} z^{m} \qquad (17)$$

The value of index m in this equation was found to be constant at -1.35 (fig.3) for all the materials. The value of C' varies with shape of particles and its value, obtained from Fig. 3, for various materials is shown in Table 1.

The Equation 11 can therefore, be written as:

$$C' = k\, z^{1.35} \qquad (18)$$

Where, C' = parameter defining the shape of the particles.

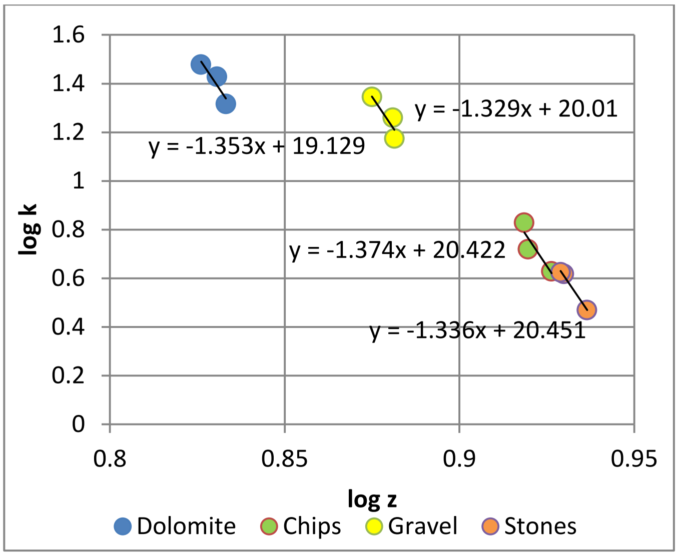

Figure 4: Curve of log k vs log z

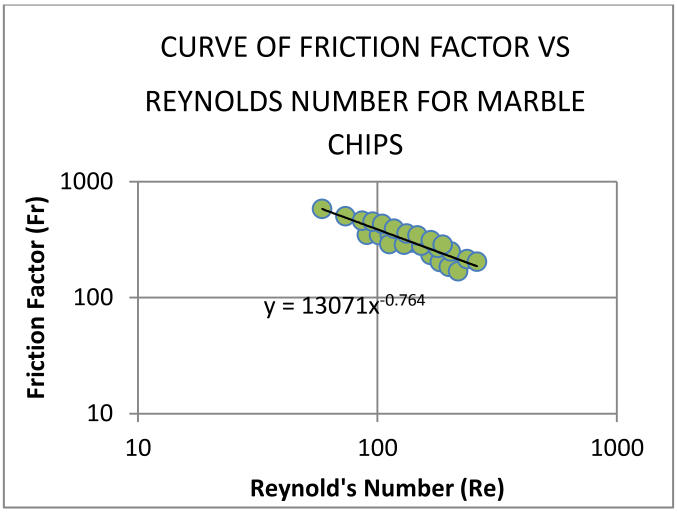

Figure 6: F_r-R_e Curve for Marble Chips

3.1. Relationship Between Hydraulic Conductivity (k), mean diameter of the material (d_g) and Shape Factor (z)

Equation 15 was used to develop a relationship between hydraulic conductivity and shape factor. The results of experimental investigation for different materials can therefore be plotted on log-log graph as Friction Factor (F_r) vs. Reynold's number (R_e) as shown in Figure 5-8.

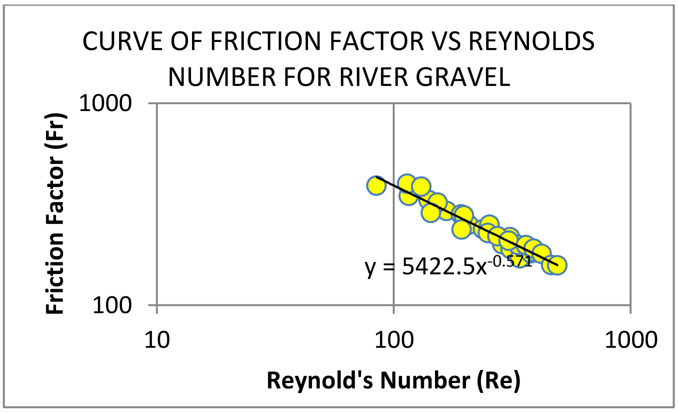

Figure 7: F_r -R_e Curve for River Gravel

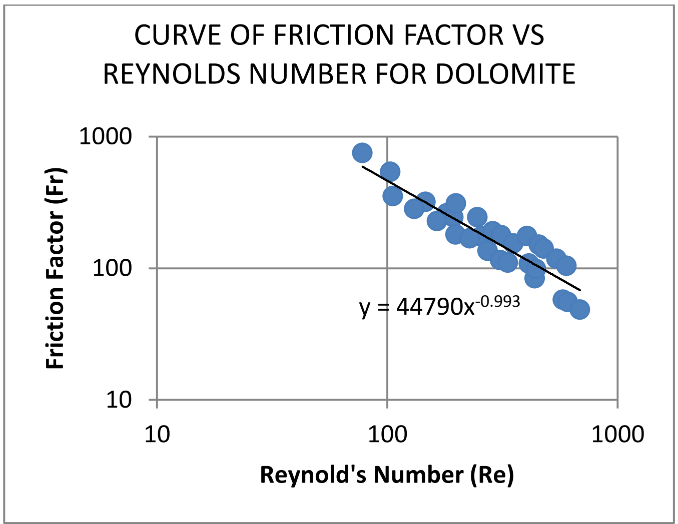

Figure 5: F_r -R_e Curve for Dolomite

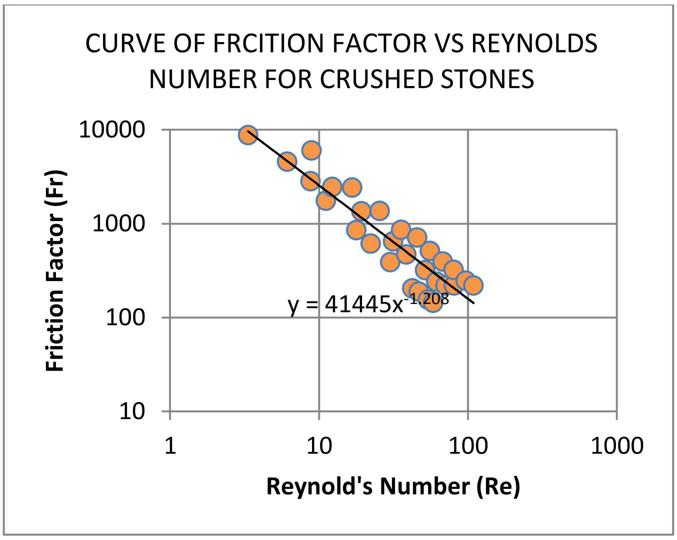

Figure 8: F_r -R_e Curve for Crushed Stones

The variation of all the graphs is found to be linear which signifies a laminar range and represented by,

$$F_r = \frac{C_1}{Re} \qquad (19)$$

Substituting values of F_r $\left(2\,i\,g\frac{d_g}{v^2}\right)$, R_e $\left(v\frac{\rho d}{\mu}\right)$ and $v = ki$, in eq. (21),

$$\frac{k}{z\,d_g^2} = \frac{2g}{C_1\,v} = C \qquad (20)$$

Where, C_1 is the constant which for a given porosity depends only on the shape of particles; C, parameter defining the hydraulic conductivity of the material; d_g, geometric mean diameter of particles; ν, kinematic viscosity of the fluid, other parameters being as mentioned earlier. For a constant porosity, the value of 'C' varies with shape of particles. The values of C_1 obtained from Figure 5-8 are shown in Table 1.

As both C and C' are dependent on the shape of materials, it is clear that they are function of one another. Mathematically, this can be expressed as:

$$C' = f(C) \qquad (21)$$

To study the relationship between C' and C, values of C' and C obtained from experimental results are plotted as log C vs log C' (Fig. 9).

TABLE I

Values of C and C_1

S. NO.	Materials	C'	Log C'	C_1	C	logC
1	Dolomite	309.05	2.5	44790	50.11	1.7
2	Marble Chips	102.81	2.01	13071	16.9	1.22
3	River Gravel	281.8	2.45	5422.5	40.88	1.611
4	Crushed Stones	65.75	1.81	41445	5.34	0.727

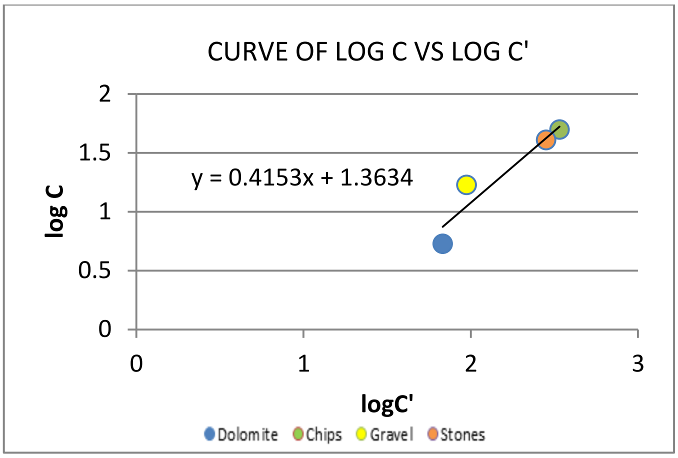

Figure 9 : log C vs log C' at 40% porosity

All the points in Figure.9 followed a straight line given by:

$$C' = BC^m \qquad (22)$$

The values of B and m in the above equation were obtained with the help of fig. 9

The final equation can be written as,

$$C^{-0.4153} = \frac{2.6019}{C'} \qquad (23)$$

Putting the expressions for C and C' from equations (19) and (21) we get,

$$k_t = \left(\frac{2.6019\,d^{0.9806}}{z^{1.35}}\right)^{\left(\frac{1}{0.47}\right)} \qquad (24)$$

Where k_t is the hydraulic conductivity of soil; d_g, geometric mean diameter of particles (cm); and z is the shape factor of the particles.

From the above equation, it is obvious that by conducting shape factor tests and sieve analysis, an idea about the hydraulic conductivity of a given material can be obtained. Since Equation 24 is based on experimental results at 40% porosity, the above value of hydraulic conductivity is applicable to this porosity only. For other porosity a suitable porosity function can be used.

3.2. Comparitive Analysis

A comparison is made between the k values obtained from equation (7), (24) by taking mean diameter (d_g) for every material used and k values obtained from the experiments conducted in laboratory and converting all the units to cm^2. The values are shown in Table. 3.

TABLE III
Comparison between k_{exp}, k_t and k_{model}

S.NO.	MATERIAL	EXPERIMENTAL ($x10^{-6}$ cm^2)	THEORETICAL ($x10^{-6}$ cm^2)	DRAG FORCE MODEL ($x10^{-6}$ cm^2)
1	DOLOMITE	2.82	2.9	2.338
2	MARBLE CHIPS	0.7753	0.5366	0.387
3	RIVER GRAVEL	1.775	1.25	1.32
4	CRUSHED STONES	0.4353	0.175	0.126

IV. CONCLUSION

Analysis of results reflects that shape factor of the particle increases as the mean diameter of material decreases.

Furthermore, it was observed that the hydraulic conductivity increases as the size of the material increases. Further, the graph between K and d_g is a straight line with R^2 value of 0.86 which gives a good approximation of the intrinsic permeability of media composed of relatively uniform size particles, whether they are composed of spherical or angular in shape (fig.10).

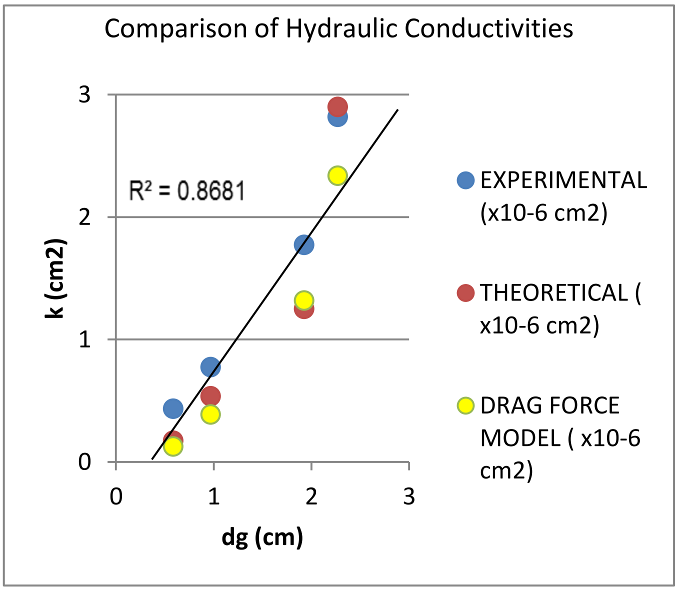

Figure 10: Comparison of Hydraulic Conductivities

V. REFERENCES

[1] Boadu, F. K., 2000. Hydraulic Conductivity of Soils from Grain-Size Distribution: New Models. Journal of Geotechnical and Geoenvironmental Engineering.

[2] Gupta.R. D., Md. Jamil, Md. Afaq Alam., 1991. Study of shape effect on sedimentation of regular and irregular shaped particles. I.E. (I) J. vol. 72, pp-146

[3] Kozeny, J., 1927. Uber Kapillare Leitung Des Wassers in Boden. Sitzungsber Akad. Wiss.Wien Math.Naturwiss.Kl. (In German), Volume 2, pp. 136,271-306.

[4] Leva, M. et al., 1957. Fluid flow through packed and fluidised systems. U.S. Bureau of Mines, Volume 504, p. 149.

[5] R R Rumer Jr and Drinker Philip.A .,1966 ,Resistance to laminar flow through porous media , Journal of hydraulic division proceeding of ASCE vol 92.

[6] Rose, H. E., 1948. Futher research in fluid flow through beds of granular material. Proceedings of the Institution of Mechanical Engineering, Volume 158, p. 327.

[7] Rose, H. E., 1953. The measurement of particle size in very fine powders. Constable.

[8] Scheidegger A E.,1957, Physics of flow through porous media , University of Toronto Press

[9] Wadell, H., 1934. Shape determination of large sedimental rock fragments. The Pan-American Geologist, Volume 61, pp. 187-220.