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Solution of Fuzzy Algebraic Equations for New Fuzzy Number

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ABSTRACT

The use of new concept of fuzzy number as an equivalence class on R for solving fuzzy equation is proposed. This approach of solving fuzzy equations by new concept of fuzzy number has a considerable advantage. Through theoretical analysis, an illustrative example and computational results the paper shows that the proposed approach is more general and straight-forward for solving fuzzy algebraic equations.
Keywords: Fuzzy Number, Fuzzy Arithmetic and Fuzzy Algebraic Equations.

I. INTRODUCTION

Fuzzy equations are one of the important areas of fuzzy set theory in which fuzzy numbers and arithmetic operations on fuzzy number play a fundamental role. These are the equations in which co-efficient and unknowns are fuzzy numbers and formulas are constructed by operations of fuzzy arithmetic. Such equations have a great potential applicability. Fuzzy equations were investigated by Dubois and Prade[2]. Sanchez [3] put towards a solution of fuzzy equation by using extended operations. Also various researchers have proposed different methods for solving the fuzzy equations [see Buckley [1], Wasowski [4].Interval arithmetic was suggested by Dwyer in 1995. Several authors such as Herzberger[5], Neumaier etc have studied interval arithmetic. The notion of fuzzy number as being convex and normal fuzzy set of some referential set was introduced by Zadeh. In the literature various operations were defined on fuzzy numbers but these operations do not explicitly make it clear.

In this paper, we have defined new definition of fuzzy numbers and their fuzzy arithmetic operations on new fuzzy numbers and finally solved fuzzy some fuzzy equations. In this paper In section 2 we discuss preliminary definitions and the existing function principle operations are given. In section 3, the new definition of fuzzy number and its properties along with arithmetic are discussed and In section 4, we solved fuzzy equation for new fuzzy numbers. Concluding remark is given in section 5.

II. METHODS AND MATERIAL

2. Preliminaries:

We first review certain standard definitions.

2.1 Fuzzy set: Let E be a set and let x be an element in E. Then a Fuzzy subset of A is characterized by

$$A = \{(x, \mu_A(x))/\, x \in E\}$$

Where $\mu_A(x)$ is the membership of x in A. $\mu_A(x)$ is commonly called the fuzzy membership function of the fuzzy set A. For an ordinary set $\mu_A(x)$ $is\ either$ 0 or 1 while for a fuzzy set $\mu_A(x) \in [0,1]$.

2.2 Normal fuzzy set: A fuzzy set A is said to be normal if its membership function $\mu_A(x)$ is unity.
i.e. $\mu_A(x) = 1$.

$2.3\ \alpha - cut$: An $\alpha - cut$ α_A of a fuzzy set A is an ordinary set of elements with membership not less than α for $0 \le \alpha \le 1$. $This\ means$ $\alpha_A = \{x \in E / \mu_A(x) \ge \alpha\}$.

2.4 Support of fuzzy set: The support of fuzzy set A is denoted by sup $p(A)$ $and\ is$ defined as the set of elements with membership nonzero.

i.e. sup $p(A) = \{x \in E / \mu_A(x) > 0\}$.

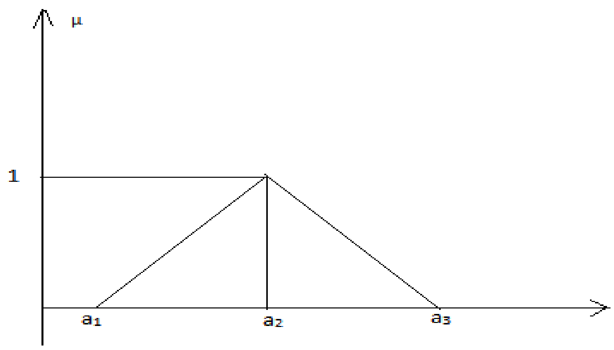
2.5 Convex: A fuzzy set A is convex if $\mu_A(tx_1 + (1 - t)x_2) \geq min(\mu_A(x_1), \mu_A(x_2))$, x_1 , $x_2 \in E$ and $t \in [0,1]$. More a fuzzy set is convex if all $\alpha - level$ sets are convex.

2.6 Fuzzy Number: A fuzzy set A on R must possess the following three properties to qualify as a fuzzy number,

(1) A must be a fuzzy normal set
(2) α_A must be closed interval for every $\alpha \in [0,1]$
(3) $The\ support\ of\ A$ Must be bounded.

2.7 Triangular Fuzzy number: A fuzzy set A is called triangular fuzzy number with peak a_2 , left width $a_2 - a_1 > 0$ and right width $a_3 - a_2 > 0$ if its membership function has the following form:

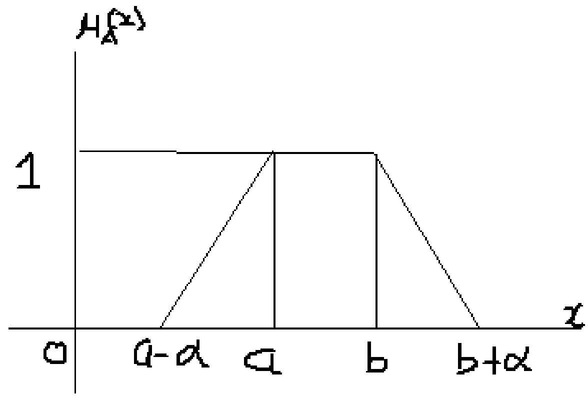
$$\mu_A(x) = \begin{cases} 0\ for\ x < a_1\ and\ x > a_3 \\ \dfrac{x - a_1}{a_2 - a_1}\ for\ a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}\ for\ a_2 \leq x \leq a_3 \end{cases}$$



Traangular fuzzy number $\boldsymbol{A = (a_1, a_2, a_3)}$

2.8 : Trapezoidal Fuzzy Number: A Fuzzy set A is called trapezoidal fuzzy number with tolerance interval $[a, b]$, left width α and right width β if its membership function has the following form:

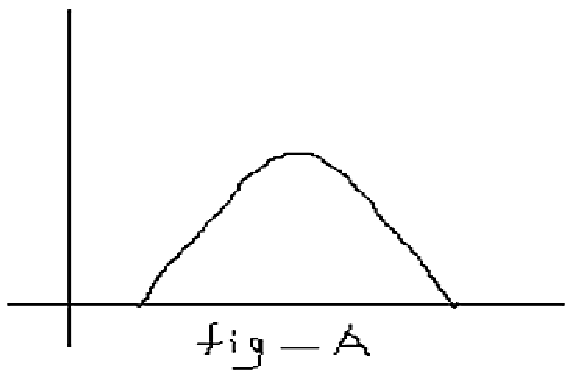
$$\mu_A(x) = \begin{cases} 1 - \dfrac{(a - x)}{\alpha}\ ; a - \alpha \leq x \leq a \\ 1\ ; a \leq x \leq b \\ 1 - \dfrac{(x - b)}{\beta}\ ; b \leq x \leq b + \beta \\ 0\ ; otherwise \end{cases}$$

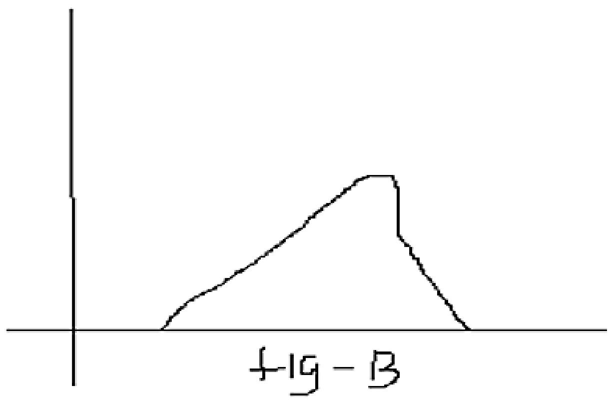


2.9 : $\boldsymbol{\alpha - cut}$ of a triangular fuzzy number: We get a crisp interval by $\alpha - cut$ operation, interval A_α shall be obtained as $A_\alpha = [\alpha_1{}^\alpha, \alpha_3{}^\alpha] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]$.

Remark:

1. Membership function of fuzzy numbers need not be symmetric
2. Fuzzy numbers can also be represented by "bell-shaped" membership functions as given in below figure-A which is symmetrical and figure-B which is asymmetric.





III. RESULTS AND DISCUSSION

3. New definition of fuzzy numbers and their arithmetic:

In this section our objective is to define a new definition of fuzzy numbers their representation, arithmetic and its properties.

3.1 Hypothesis: A Fuzzy number is simply an ordinary numbers whose precise value is somewhat uncertain. Fuzzy- n means near n and not very away from n, because otherwise it must be quite different from n that is what the essentiality the word fuzzy means and that is what we would like to interpret it. How near or how much is the affordable "awayness" of fuzzy- n from exact n? Generally we should define this as less than 0.5. Every ϵ should satisfy $|\epsilon| < 0.5$

3.2 Definition : Fuzzy Number

We define new fuzzy number as for $n \in R, fuzzy - n$ we mean the collection of numbers very close to n or n itself and is denoted by $\bar{n}$. we define fuzzy number $-n$ as follows:
$\bar{n} = [n - \epsilon, n + \delta]\ for\ |\varepsilon| < \alpha$ & $|\delta| < \alpha$ Where α is small non negative number we shall take $\alpha = 0.5$

All $\varepsilon's\ and\ \delta's$ are positive numbers.

3.3 Arithmetic operators on fuzzy numbers:

3.3.1 Addition: Let $\bar{n} = [n - \epsilon_1, n + \delta_1]$ and $\bar{m} = [m - \epsilon_2, m + \delta_2]$ are any two fuzzy numbers then
$\bar{m} + \bar{n} = [(m+n) - \epsilon, (m+n) + \delta]$
where $\epsilon = \min\{\epsilon_1 + \epsilon_2, 0.5\}$ &
$\delta = min\ \{\delta_1 + \delta_2, 0.5\}$.

3.3.2 Properties for addition of fuzzy numbers:

Commutative Property:

For $n, m \in R$ Let $\bar{n} = [n - \epsilon_1, n + \delta_1]$ and $\bar{m} = [m - \epsilon_2, m + \delta_2]$ then
$\bar{m} + \bar{n} = [(m+n) - \epsilon, (m+n) + \delta]$ where $\epsilon = \min\{\epsilon_1 + \epsilon_2, 0.5\}$ &
$\delta = min\ \{\delta_1 + \delta_2, 0.5\}$.
$= [(n+m) - \epsilon, (n+m) + \delta]$
$= \bar{n} + \bar{m}$.

Associative Property: For $n, m, r \in R$ Let $\bar{n} = [n - \epsilon_1, n + \delta_1]$, $\bar{m} = [m - \epsilon_2, m + \delta_2]$ and

$\bar{r} = [r - \epsilon_3, r + \delta_3]$
$(\bar{n} + \bar{m}) + \bar{r} = [(n+m) - \epsilon, (n+m) + \delta] + [r - \epsilon_3, r + \delta_3]$
Where $\epsilon = \min\{\epsilon_1 + \epsilon_2, 0.5\}$ &
$\delta = min\ \{\delta_1 + \delta_2, 0.5\}$

$= [(n+m) + r - \varepsilon', (n+m) + r + \delta']$
Where $\epsilon' = \min\{\epsilon + \epsilon_3, 0.5\}$ &
$\delta' = min\ \{\delta + \delta_3, 0.5\}$
$= [\,n + (m+r) - \varepsilon', n + (m+r) + \delta']$
$= \bar{n} + (\bar{m} + \bar{r})$.

Additive Identity: For $0 \in R$ we define fuzzy-0 by $\bar{0} = [0 - \epsilon_2, 0 + \delta_2]$

Now $\bar{n} + \bar{0} = [n - \epsilon_1, n + \delta_1] + [0 - \epsilon_2, 0 + \delta_2]$

$= [n+0 - \epsilon, n+0 + \delta]$

Where $\epsilon = \min\{\epsilon_1 + \epsilon_2, 0.5\}$ &

$\delta = min\,\{\delta_1 + \delta_2, 0.5\}$

$= [n - \epsilon, n + \delta]$

$= \bar{n}.$

Thus $\bar{0}$ is the additive identity for fuzzy numbers.

Additive Inverse : For each fuzzy-n ($\bar{n}$) we define fuzzy-(-n) denoted by $\overline{-n}$ and given by

$\overline{-n} = [-n - \epsilon_2, -n + \delta_2]$ Such that

$\bar{n} + \overline{-n} = [n - \epsilon_1, n + \delta_1] + [-n - \epsilon_2, -n + \delta_2]$

$= [n + (-n) - \epsilon, n + (-n) + \delta]$

$= [0 - \epsilon, 0 + \delta]$ Where $\epsilon = \min\{\epsilon_1 + \epsilon_2, 0.5\}$ &

$= \bar{0}.$ $\delta = min\,\{\delta_1 + \delta_2, 0.5\}$

i.e. $\overline{-n}$ is the additive inverse of $\bar{n}$.

3.3.3 Multiplication: Let $\bar{n} = [n - \epsilon_1, n + \delta_1]$ and $\bar{m} = [m - \epsilon_2, m + \delta_2]$ are any two fuzzy numbers then their multiplication is defined by

$\bar{n}\,\bar{m} = [n - \epsilon_1, n + \delta_1]\,[m - \epsilon_2, m + \delta_2]$

$= [nm - \epsilon, nm + \delta]$ where $\epsilon = \epsilon_1\epsilon_2$ and $\delta = \delta_1\delta_2$.

3.3.4 Properties for multiplication of fuzzy numbers:

Commutative Property:

For $n, m \in R$ Let $\bar{n} = [n - \epsilon_1, n + \delta_1]$ and $\bar{m} = [m - \epsilon_2, m + \delta_2]$ then

$\bar{n}\,\bar{m} = [n - \epsilon_1, n + \delta_1]\,[m - \epsilon_2, m + \delta_2]$

$= [nm - \epsilon, nm + \delta]$ where $\epsilon = \epsilon_1\epsilon_2$ and $\delta = \delta_1\delta_2$

$= [mn - \epsilon, mn + \delta]$

$= [m - \epsilon_2, m + \delta_2][n - \epsilon_1, n + \delta_1]$

$= \bar{m}\,\bar{n}.$

Associative Property: For $n, m, r \in R$ Let $\bar{n} = [n - \epsilon_1, n + \delta_1]$, $\bar{m} = [m - \epsilon_2, m + \delta_2]$ and

$\bar{r} = [r - \epsilon_3, r + \delta_3]$ are any three fuzzy numbers then

$(\bar{n}\,\bar{m})\bar{r} = [nm - \epsilon', nm + \delta']\,[r - \epsilon_3, r + \delta_3]$ where $\epsilon' = \epsilon_1\epsilon_2$ and $\delta' = \delta_1\delta_2$

$= [(nm)r - \epsilon, (nm)r + \delta]$ where $\epsilon = \epsilon'\epsilon_3 = \epsilon_1\epsilon_2\epsilon_3$ and $\delta = \delta'\delta_3 = \delta_1\delta_2\delta_3$

$= [n(mr) - \epsilon, n(mr) + \delta]$

$= \bar{n}\,(\bar{m}\,\bar{r})$

Multiplicative Identity: For $1 \in R$ we define fuzzy-1 as $\bar{1} = [1 - \epsilon_2, 1 + \delta_2]$ such that for any fuzzy number $\bar{n} = [n - \epsilon_1, n + \delta_1]$ we have

$\bar{n}\,\bar{1} = [n - \epsilon_1, n + \delta_1][1 - \epsilon_2, 1 + \delta_2]$

$= [n1 - \epsilon, n1 + \delta]$ where $\epsilon = \epsilon_1\epsilon_2$ and $\delta = \delta_1\delta_2$
$= [n - \epsilon, n + \delta]$
$= \bar{n}$

Thus $\bar{1}$ is the multiplicative identity for fuzzy numbers.

Multiplicative Inverse: For each non-zero $n \in R$ we define fuzzy-$\frac{1}{n}$ denoted by $\overline{(\frac{1}{n})} = \frac{1}{\bar{n}}$ and defined as $\frac{1}{\bar{n}} = \left[\frac{1}{n} - \epsilon_2, \frac{1}{n} + \delta_2\right]$ such that

$$\bar{n}\ \frac{1}{\bar{n}} = [n - \epsilon_1, n + \delta_1]\left[\frac{1}{n} - \epsilon_2, \frac{1}{n} + \delta_2\right]$$
$= [n\frac{1}{n} - \epsilon, n\frac{1}{n} + \delta]$ where $\epsilon = \epsilon_1\epsilon_2$ and $\delta = \delta_1\delta_2$
$= [1 - \epsilon, 1 + \delta]$
$= \bar{1}$

i.e. $\frac{1}{\bar{n}}$ is the multiplicative inverse of $\bar{n}$.

3.3.5 Fuzzy multiplication is distributive over fuzzy addition:

For three fuzzy numbers $\bar{n} = [n - \epsilon_1, n + \delta_1]$, $\bar{m} = [m - \epsilon_2, m + \delta_2]$ and $\bar{r} = [r - \epsilon_3, r + \delta_3]$ we have
$\bar{n}\,(\bar{m} + \bar{r}) = \bar{n}\bar{m} + \bar{n}\,\bar{r}$.

3.3.6 The collection of all $fuzzy - n$, where $n \in R$ has an algebraic structure with the usual fuzzy addition and fuzzy multiplication.

However the usual " uniqueness " properties of addition and multiplication do not hold. But if the fact that

$$[n - \epsilon_1, n + \delta_1] \sim [n - \epsilon_2, n + \delta_2]$$

where '$\sim$' is a homeomorphism between two intervals such that the image of n is n, is taken in to account, then $[n - \epsilon_1, n + \delta_1]$ can be treated as the same as $[n - \epsilon_2, n + \delta_2]$ just having different but identical form.

In this sense $[n - \epsilon, n + \delta] = [n, n]$

Thus each $fuzzy - n$ is an equivalence class in collection of all fuzzy number which are partitioned by the equivalence relation '$\sim$'.

In this sense the equivalence class of fuzzy numbers forms a field with induced addition and multiplication operations.

3.3.7 Theorem: The function h_n: $[n - \epsilon_1, n + \delta_1] \to [n - \epsilon_2, n + \delta_2]$ defined by

$$h_n(x) = \begin{cases} \frac{\epsilon_2}{\epsilon_1}(n - x) + n\,;\ n - \epsilon_1 \le x \le n \\ \qquad n\ \ ; x = n \\ \frac{\delta_2}{\delta_1}(x - n) + n\,; n \le x \le n + \delta_1 \end{cases}$$

is homeomorphism between $[n - \epsilon_1, n + \delta_1]$ to $[n - \epsilon_2, n + \delta_2]$.

3.3.8 For any real r we define an equivalence relation $'\sim_\epsilon'$

By $[r-\epsilon_1, r+\delta_1] \sim_\epsilon [r-\epsilon_2, r+\delta_2]$ if and only if there exists a homeomorphism h_n from $[r-\epsilon_1, r+\delta_1]\ to\ [r-\epsilon_2, r+\delta_2]$.

Consider the collection of all equivalence classes $R_\varepsilon = \{[r_\epsilon] = \overline{r} / \ r \in R\}$.
Now we define fuzzy operation $+_\epsilon, -_\varepsilon, \times_\epsilon\ and\ /_\epsilon$ as follows:

1. $[r_1-\epsilon_1, r_1+\epsilon_2] +_\epsilon [r_2-\delta_1, r_2+\delta_2] = [r_1+r_2-(\epsilon_1+\delta_1),\ r_1+r_2+(\epsilon_2+\delta_2)]$
2. $[r_1-\epsilon_1, r_1+\epsilon_2] -_\varepsilon [r_2-\delta_1, r_2+\delta_2] = [r_1-r_2-(\epsilon_1+\delta_1),\ r_1-r_2+(\epsilon_2+\delta_2)]$
3. $[r_1-\epsilon_1, r_1+\epsilon_2] \times_\epsilon [r_2-\delta_1, r_2+\delta_2] = [r_1r_2-(\epsilon_1\delta_1),\ r_1r_2+(\epsilon_2\delta_2)]$
4. $[r_1-\epsilon_1, r_1+\epsilon_2] /_\epsilon [r_2-\delta_1, r_2+\delta_2] = [\frac{r_1}{r_2}-(\epsilon_1\delta_1), \frac{r_1}{r_2}+(\epsilon_2\delta_2)]\ (r_2 \neq 0)$

For $r_1, r_2 \in R$ we define

$[\{r_1\}_\epsilon] +_\epsilon [\{r_2\}_\epsilon] = [\{r_1+r_2\}_\epsilon]$ &

------------------ (*)

$[\{r_1\}_\epsilon] \times_\epsilon [\{r_2\}_\epsilon] = [\{r_1r_2\}_\epsilon]$

3.3.9 Theorem: R_ε is a commutative semi ring with zero $\overline{0}$ and unity $\overline{1}$ under the operations $+_\epsilon$ & $\times_\epsilon$.

3.3.10 Theorem: $R_\varepsilon = \{\bar{r} = [r]_\varepsilon \ /\ r \in R\ \}$ is a field under fuzzy addition and fuzzy multiplication defined in (*).

3.3.11 Theorem: For any fuzzy $[r]_\epsilon \in R_\varepsilon$ and any $n \in$ N we have

(a) $\{[r]_\epsilon\}^n = [r^n]_\epsilon$

(b) $\{[r]_\epsilon\}^{\frac{1}{n}} = \left[r^{\frac{1}{n}}\right]_\epsilon$ *and*

(c) For any $k \in R$, $k[r]_\epsilon = [kr]_\epsilon$.

4. **Solution of Fuzzy Algebraic Equations:** In this section we concentrate on solving general fuzzy algebraic equation like $\overline{a_0}\ \bar{x}^n + \overline{a_1}\ \bar{x}^{n-1} + \cdots\cdots + \overline{a_n} = \overline{0}$; $n \in$ N where $\overline{a_0}, \overline{a_1}, \ldots \overline{a_n}$ & $\overline{0}$ are fuzzy numbers and $\bar{x}$ is fuzzy variable in R_ε .

4.1 First we give illustration of solution of fuzzy linear equation $\bar{a} + \bar{x} = \bar{b}$ where $\bar{a} = \overline{1}, \bar{b} = \overline{3}$ and $\bar{x}$ is the fuzzy variable.

Here $\overline{1} + \bar{x} = \overline{3}$

By adding $\overline{-1}$ on both sides

$\Rightarrow\ \overline{-1} + (\overline{1} + \bar{x}) = \overline{-1} + \overline{3}$

$\Rightarrow\ (\overline{-1} + \overline{1}) + \bar{x} = (\overline{-1} + \overline{3})$

$\Rightarrow\ \overline{0} + \bar{x} = \overline{(-1+3)}$

$\Rightarrow\ \bar{x} = \overline{2}$.

4.2 Now we give illustration of solution of fuzzy quadratic equation

$\bar{x}^2 - \bar{5}\bar{x} + \bar{2} = \bar{0}$

By comparing with $\bar{a}\bar{x}^2 + \bar{b}\,\bar{x} + \bar{c} = \bar{0}$ we have $\bar{a} = \bar{1}, \bar{b} = \overline{-5}$ & $\bar{c} = \bar{2}$

Now $\Delta = \bar{b}^2 - \bar{4}\,\bar{a}\,\bar{c}$

$= \overline{25} - \bar{4} \times \bar{1} \times \bar{2}$

$= \overline{25} - \bar{8}$

$= \overline{17}$

Now $\sqrt{\Delta} = \sqrt{\overline{17}} = \pm\overline{4.12}$

The solutions are given by

$\overline{x_1} = \frac{\overline{-b} + \sqrt{\Delta}}{\bar{2}\,\bar{a}}$ and $\overline{x_2} = \frac{\overline{-b} - \sqrt{\Delta}}{\bar{2}\,\bar{a}}$

$= \frac{\bar{5} + \overline{4.12}}{\bar{2}\,\bar{1}}$ $= \frac{\bar{5} - \overline{4.12}}{\bar{2}\,\bar{1}}$

$= \overline{4.56}$ $= \overline{0.44}$

IV. CONCLUSION

Main aim of this paper is to introduce a new definition of fuzzy numbers and their arithmetic to undergo the solution of fuzzy algebraic equations. This new concept of fuzzy numbers may help us to solve many fuzzy algebraic equations for various fuzzy numbers. Also in future we discuss about the solutions for system of fuzzy linear equations and numeric solution fuzzy algebraic equation.

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