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Pythagorean Triangle With (One leg of right triangle $p^2 - q^2$) $-2\ \frac{area}{perimeter} = \alpha^3 - \beta^3$

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ABSTRACT

Pattern of Pythagorean Triangle for each of which, the ratio $(One\ leg\ of\ right\ trinagle\ (p^2 - q^2)) - 2\left(\frac{Area}{Perimeter}\right)$ may be expressed as the difference to two cubes as integers. A few interesting relations are also given.

Keywords : Pythagorean triangle, Ratio (Area/perimeter) as Difference of Two Cubic Integers.

I. INTRODUCTION

It is well known that Pythagoras triangle is a treasure house which contains many interesting results for an extensive review of the literature. The method of obtaining three non-zero integers x, y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians. One may refer [1-7]. In [8-11] special Pythagoras problems are studies. In this communication we, present yet another interest Pythagorean Triangles, where in each of which, $(One\ leg\ of\ right\ trinagle\ (p^2 - q^2)) - 2\left(\frac{Area}{Perimeter}\right)$ may be expressed as thedifference to two cubes as integers. A few interesting relations are also given. In addition, the recurrence relations for the sides of the triangle are presented.

Notation

$$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right] = poligonal\ \text{number}\ of\ rank\ n\ with\ \text{sides}\ m$$

II. METHODS AND MATERIAL

Method of Analysis

The most cited solution of the Pythagorean equation $x^2 + y^2 = z^2$ is

$$\begin{aligned} x &= 2pq \\ y &= p^2 - q^2 \text{ where } p > q > 0 \\ z &= p^2 + q^2 \end{aligned} \tag{1}$$

Denoting the Area and the Perimeter of the above Pythagorean triangle by A and P respectively, the assumption that the ratio $\frac{A}{P}$ can be expressed as the difference of two cubic non zero integers leads to the equation

$$p(p-q) = \alpha^3 - \beta^3 \tag{2}$$

Where α, β are the non zero integer.($\alpha > \beta > 0$)
Choosing

$$p = \alpha^2 + \beta^2 + \alpha\beta, \quad p - q = \alpha - \beta \tag{3}$$

in (2), and solving we get, $p = \alpha^2 + \beta^2 + \alpha\beta$ and

$$q = \alpha^2 + \beta^2 + \alpha\beta - \alpha + \beta \tag{4}$$

In which follows, we obtain the values of x , y , z

$$x(\alpha,\beta) = 2pq \quad \Longrightarrow x = 2\alpha^4 + 2\beta^4 + 6\alpha^2\beta^2 + 4\alpha\beta^3 + 4\alpha^3\beta + 2\beta^3 - 2\alpha^3 \quad (5)$$

$$y(\alpha,\beta) = p^2 - q^2 \Longrightarrow y = 2\alpha^3 - 2\beta^3 + 2\alpha\beta - \alpha^2 - \beta^2 \quad (6)$$

$$z(\alpha,\beta) = p^2 + q^2 \Longrightarrow x = 2\alpha^4 + 2\beta^4 + 6\alpha^2\beta^2 + 4\alpha\beta^3 + 4\alpha^3\beta + 2\beta^3 - 2\alpha^3 + 2\beta^3 - 2\alpha\beta + \alpha^2 + \beta^2, \text{ where } \alpha > \beta \quad (7)$$

Few examples are given

α	β	p	q	x	y	z
2	1	7	6	88	13	85
4	3	37	36	2664	73	2665
5	4	61	60	7320	121	7321
9	7	193	191	73726	768	73730
3	2	19	18	684	37	685
7	3	79	75	11850	616	11816
5	2	39	36	2808	225	2817
7	4	93	90	16740	549	16749
8	5	129	126	32508	765	32517
7	2	67	62	8308	645	8333

Properties

1. $x(\alpha,\beta) \equiv 0 \ (mod\ 2)$
2. $x(\alpha,1) - 4\alpha Pen_\alpha - Obl_\alpha + 2\alpha$ is a perfect square.
3. $6(x(\alpha,1) - 4\alpha Pen_\alpha - Obl_\alpha + 2\alpha)$ is a nasty number.
4. $(x - z) + (\alpha^2 + \beta^2) \equiv 0(mod2)$
5. $6(z - x)$ is a nasty number.
6. $y(\alpha,\beta) + (\alpha^2 + \beta^2) \equiv 0(mod2)$
7. $z(\alpha,1) = (T_{4,\alpha} + 1)(4T_{3,\alpha} + 5)$
8. $z(\alpha,1) = 4(T_{4,\alpha})(T_{3,\alpha}) + 2T_{9,\alpha} + 7\alpha + 5$
9. $y(\alpha,1) - (\alpha^2 + 1) \equiv 0(mod2)$
10. $z(\alpha,\beta) - (\alpha^2 + \beta^2) \equiv 0 \ (mod\ 2)$
11. $z(\alpha,1) - 4\alpha Pen_\alpha - 12Tri_\alpha - 5$ is a perfect square.
12. $x(\alpha,\beta) - (y(\alpha,\beta) + z(\alpha,\beta)) \equiv 0(mod2)$
13. $x(\alpha,\beta) - (y(\alpha,\beta) + z(\alpha,\beta)) + 2(\alpha^3 - \beta^3)$ is a perfect square
14. $y(\alpha,\beta) + \ (\alpha^2 + \beta^2) \equiv 0 \ (mod\ 2)$
15. $y + z = 0 \ (mod\ 2)$
16. $x - (y + z) = 0 \ (mod\ 2)$

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