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Vibration Analysis of moderately thick symmetric cross-ply laminated composite Plate using FEM

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ABSTRACT

This paper presents the vibration analysis of orthotropic laminated composite plate using finite element method. A suitable finite element model is proposed and developed based on first order shear deformation theory using ANSYS parametric design language (APDL) code. The model has been discretized using an appropriate eight noded element (SHELL 281) from the ANSYS element library. The free vibrations are computed using Block-Lanczos algorithm, initially the model response is verified for orthotropic plate with the available literature. The convergence study has been done for the developed model. A number of examples concerning various thickness-to-span ratios, material properties and boundary conditions are considered. The boundary conditions of the plate play an important role in the free vibrations of the plate. The author proved that non-dimensional frequencies are higher for fully clamped boundary condition in comparison to other boundary conditions.

Keywords: Finite Element Method; Orthotropic Plate; Free Vibration; ANSYS.

I. INTRODUCTION

Composite materials find a wide range of application, especially in weight sensitive structures like aircraft, spacecraft due to their high strength-to-weight ratio and high stiffness-to-weight ratio. Laminated plates used in these structures are often subjected to dynamic loads which create vibration. When a system operates at the system natural frequency, resonance can happen causing large deformations and even catastrophic failure in improperly constructed structures. Careful designs can minimize those unwanted vibrations. This necessitates the study of harmonic and vibrational characteristics of these plates.

Laminated composite plate structures have found numerous importance in structural elements used in various fields like aerospace, military, automotive industries etc. Bending, buckling and dynamic issues of laminated plates of various shapes subjected to various patterns of boundary conditions have been the area of study of many researchers [1 – 36]. It has been observed that the boundary conditions play a explicate role in vibration analysis. Most of the researches have focused their work on restrained boundary conditions [6 – 13], which involves some suitable correlation between a displacement component and the corresponding force being more realistic. These more realistic edge conditions are being explored by several researchers [14 – 24) with the help of a model being termed as `elastic edges.' Liew et al. [14] presented apparently the first known outcome of free vibration analysis of symmetrically laminated cross-ply rectangular plates with edges carring uniform elastic restraints - translational as well as rotational. Shu and Wang [16] applied generalized differential quadrature methodology for the vibration analysis of thin isotropic plates with mixed and non-uniform boundary conditions. Gorman [15] applied superposition-Galerkin method and Zhou [17] applied the Rayleigh-Ritz method along with static Timoshenko beam functions for acquiring the natural frequencies of isotropic Mindlin rectangular plates. Ashour [18] did the vibration analysis of isotropic plates having irregular thickness in one direction and edges elastically restrained against both rotation and translation using the finite strip transition matrix methodology. Karami et al. [19] studied the natural frequencies of moderately thick symmetric laminated plates with elastically restrained edges using the Differential Quadrature Method (DQM). Ohya et al. [20] presented the natural frequencies and mode shapes of the rectangular isotropic Mindlin plates with internal columns resting on uniform elastic edge supports using the superposition method. They achieved the compatibility between the plate and the column by requiring that the column and plate rotations be equal. Using one and two dimensional Fourier series expansions for the implicit spatial Discretization Li et al. [21] presented an exact series of results for the transverse vibration of isotropic thin rectangular plates with elastic boundary supports. Li and Yu [22] developed an empirical formula based on the analytical results obtained from the Rayleigh-Ritz method for predicting natural frequencies of a thin orthotropic rectangular plate with uniformly restrained edges. Zhang and Li [23] studied the vibration of thin isotropic rectangular plates with arbitrary non-uniform elastic edge restraints, again, using two dimensional Fourier series expansions. Hsu [24] presented the free vibration analysis of orthotropic rectangular plates resting on nonlinear elastic foundations and having linearly elastic edge supports using DQM. A detailed study of the available literature was conducted to know the present state of knowledge available in the open literature, which can assist in achieving the present goals effectively. Several methods have been used to study such types of problems. DQM is one of the newer techniques being developed to study the problems whose mathematical model is a set of differential equation(s) - linear or nonlinear, ordinary or partial. Shu and Richards [25] applied the generalized differential quadrature method to solve two-dimensional incompressible Navier-Stokes equations. Karami and Malekzadeh [28] did the static and stability analysis of arbitrary straight-sided quadrilateral thin plates using DQM. Wang and Wang [29] studied the free vibration of thin sector plates by a new version of Differential Quadrature method. . Shu [31] presents a good study of the Differential Quadrature technique and its various applications in engineering problems like the ones of Navier - Stokes equation, structural analysis and chemical engineering. Ngo-Cong et al. [33] presents a new effective radial basis function (RBF) collocation technique for the free vibration analysis of laminated composite plates using the first order shear deformation theory (FSDT). Maithry and Rao [36] investigated the dynamic response of laminated composite plates to excitations, varying arbitrarily with time using ANSYS 13.0 software and suggested the most robust fibre orientation with respect to various response parameters. However, the aim of the work is to study the effect of composite laminated materials on the vibration behavior by doing modal and harmonic analysis of the models.

II. Methodology

2.1 Background

ANSYS mechanical APDL is used to analyse the orthotropic laminated composite plate. The analysis was done on orthotropic plates with various combinations of different boundary conditions. It consisted of building geometry of the model and distributing the material properties along the thickness of model. Meshing the model with a proper smart sized mesh types, applying loads on the model, setting boundary conditions on the model, and finally running and solving the model. Modal analysis is done to find the system vibration parameters (i.e. natural frequencies, and mode shapes). And then for all these combination harmonic analysis is done to find out the frequency response of plate.

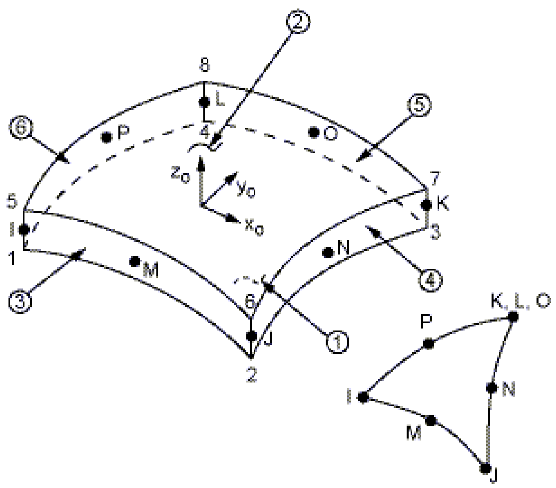
2.2 Modelling

According to the default parameters such as $E_x / E_y = 40,\ \nu_{xy} = 0.25,\ G_{xy} = G_{xz} = 0.6\,E_y, G_{yz} = 0.5\,E_y$, and $\rho = 1$ kg/m^3 and using ANSYS mechanical APDL capabilities, a modal analysis problem has been solved for Orthotropic models. In order to compare with the published results of Ngo-Cong at el [], the same shear correction factors $K_S = \pi^2/12$ and nondimensionalised natural frequencies $\varpi = \omega\left(b^2/\pi^2\right)\sqrt{\rho h / D_0}$ with $D_0 = E_2 h^3 / 12\left(1 - \nu_{12}\nu_{21}\right)$ are also employed.The element SHELL281 was chosen to mesh the model.

SHELL281 is suitable for analyzing thin to moderately-thick shell structures. It is an 8-node element with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes.

SHELL281 may be used for layered applications for modeling laminated composite shells or sandwich construction. The accuracy in modeling composite shells is governed by the first order shear deformation theory.

The degenerate triangular option should only be used as filler elements in mesh generation. This makes the comparison between the different orthotropic models easy.



x_o = Element x-axis if ESYS is not provided.

x = Element x-axis if ESYS is provided.

Figure 1. SHELL281 Geometry

2.3 Governing Equations

Figure 2 shows the geometry of a laminated rectangular plate made up of orthotropic layers. Considering the first order shear deformation theory, the displacement fields are expressed as follows:

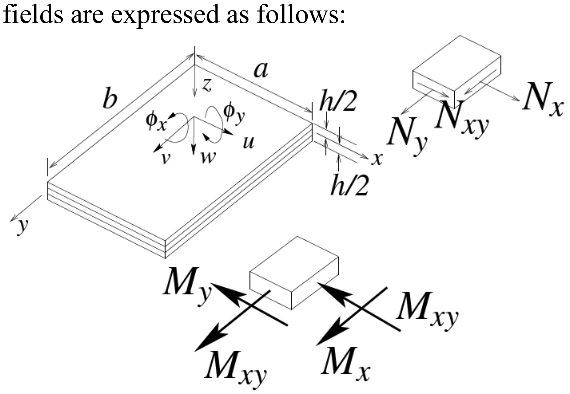


Figure 2. Geometry of the problem

$$\left.\begin{aligned} u^*(x,y,z,t) &= u(x,y,t) + z\phi_x(x,y,t) \\ v^*(x,y,z,t) &= v(x,y,t) + z\phi_y(x,y,t) \\ w^*(x,y,z,t) &= w(x,y,t) \end{aligned}\right\} \quad (1)$$

The strain displacement relations can be expressed as follows.

In-plane strains at the mid-plane are:

$$\in_x^0 = \frac{\partial u}{\partial x} \quad (2)$$

$$\in_y^0 = \frac{\partial v}{\partial y} \quad (3)$$

$$\in_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (4)$$

The curvatures are:

$$K_x^0 = \frac{\partial \phi_x}{\partial x} \quad (5)$$

$$K_y^0 = \frac{\partial \phi_y}{\partial y} \quad (6)$$

$$K_{xy}^0 = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \quad (7)$$

The shear strains in xz and yz planes are:

$$e_{xz} = \phi_x + \frac{\partial w}{\partial x} \quad (8)$$

$$e_{yz} = \phi_y + \frac{\partial w}{\partial y} \quad (9)$$

The constitutive equations are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \in_x^0 \\ \in_y^0 \\ \in_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ A_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} K_x^0 \\ K_y^0 \\ K_{xy}^0 \end{Bmatrix} \quad (10)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ A_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \in_x^0 \\ \in_y^0 \\ \in_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x^0 \\ K_y^0 \\ K_{xy}^0 \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = k^2 \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} e_{yz} \\ e_{xz} \end{Bmatrix} \tag{12}$$

Where, for an n-layered (k = 1, 2, …n) plate,

$$\left. \begin{array}{l} A_{ij} = \sum_{k=1}^{n} (z_{k+1} - z_k) Q_{ij}^{(k)}, \qquad Bij = \sum_{k=1}^{n} (z_{k+1}^2 - z_k^2) Q_{ij}^{(k)}, \\ D_{ij} = \sum_{k=1}^{n} (z_{k+1}^3 - z_k^3) Q_{ij}^{(k)} \quad \forall \ i,j = 1,2,6 \end{array} \right\} \tag{13}$$

$$A_{ij} = \sum_{k=1}^{n} (z_{k+1} - z_k) Q_{ij}^{(k)}, \quad \forall \ i,j = 4,5 \tag{14}$$

The stiffness constants $Q_{ij}^{(k)}$ for the kth layer are

$$Q_{11}^{(k)} = \frac{E_x^{(k)}}{\left[1 - v_{xy}^{(k)^2} \left(E_y^k \big/ E_x^{(k)}\right)\right]}, \quad Q_{22}^{(k)} = \frac{E_y^{(k)}}{\left[1 - v_{xy}^{(k)^2} \left(E_y^k \big/ E_x^{(k)}\right)\right]}, \tag{15}$$
$$Q_{12}^{(k)} = \frac{v_{xy}^{(k)} E_y^{(k)}}{\left[1 - v_{xy}^{(k)^2} \left(E_y^{(k)} \big/ E_x^{(k)}\right)\right]}$$

For the cases of symmetric and antisymmetric cross-ply laminated plates made up of orthotropic layers

$$Q_{66}^{(k)} = G_{xy}^{(k)}, Q_{16}^{(k)} = Q_{26}^{(k)} = 0, A_{16} = A_{26} = 0, B_{16} = B_{26} = 0, D_{16} = D_{26} = 0 \tag{16}$$

$$Q_{44}^{(k)} = G_{yz}^{(k)}, Q_{55}^{(k)} = G_{zx}^{(k)}, Q_{45}^{(k)} = 0, A_{45} = 0 \tag{17}$$

Equations of motion in terms of stress resultants and non-dimensional coordinates can thus be derived using the principle of virtual work or the equilibrium considerations as, [32]

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial_y} = \alpha_{uv} \left(I_0 \frac{d^2 u}{dt^2} + I_1 \frac{d^2 \phi_x}{dt^2} \right) \tag{18}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial_y} = \alpha_{uv} \left(I_0 \frac{d^2 u}{dt^2} + I_1 \frac{d^2 \phi_y}{dt^2} \right) \tag{19}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial_y} + q(t) = I_0 \frac{d^2 w}{dt^2} \tag{20}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial_y} - Q_z = I_1 \frac{d^2 u}{dt^2} + I_1 \frac{d^2 \phi_x}{dt^2} \tag{21}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial_y} - Q_y = I_1 \frac{d^2 v}{dt^2} + I_1 \frac{d^2 \phi_y}{dt^2} \tag{22}$$

The inertias are defined as follows

$$(I_0, I_1, I_2) = \sum_{k=1}^{n} \int_{zk}^{zk+1} p^{(k)}, (1, z, z^2) dz \tag{23}$$

Since every laminated plate considered in this work has symmetry in terms of density (ρ) about the mid-plane (z = 0), the inertia component I1 is always zero. In equations 18 & 19, by putting the parameter α_{uv} = 0 or 1, the presence of in-plane inertia in the formulation is controlled. Using equations 2- 17, the equations 18- 22 can be expressed in terms of the five displacement components (u, v, w, ϕ_x, ϕ_y), defined in equation 1.

III. Numerical Results and discussion

3.1 Modal Analysis

Modal Analysis is done to compute the natural frequencies for each model. The solver used for modal analysis is Block Lanczos. For the plates with the CCCC, SSSS and CFFF boundary condition, the frequency decreases, as the plate thickness increases.

3.1.1 Convergence study

In this section, the convergence study was done for free vibration analysis of laminated composite orthotropic plates for determining mesh size. Table 1 shows the convergence study of non-dimensionalised natural frequencies for three ply [0°/90°/0°] square laminated clamped plates for thickness to span ratios t/b = 0.2.

Table 1: convergence study of non-dimensionalised fundamental frequencies with three-ply [0°/90°/0°] of rectangular laminated clamped plate with t/b = 0.2

Mesh Size	$\varpi = \omega\left(b^2/\pi^2\right)\sqrt{\rho h/D_0}$							
	1	2	3	4	5	6	7	8
7×7	4.1488	5.6754	7.4319	7.9268	8.4120	10.0844	10.5125	11.1462
9×9	4.1484	5.6720	7.4279	7.9059	8.4062	10.0642	10.4305	11.1172
11×11	4.1481	5.6710	7.4265	7.8988	8.4046	10.0578	10.4031	11.1077
13×13	**4.1481**	5.6707	7.4258	7.8948	8.4035	10.0541	10.3866	11.1020
15×15	4.1481	5.6710	7.4265	7.8988	8.4046	10.0578	10.4031	11.1077
17×17	4.1481	5.6703	7.4258	7.8941	8.4032	10.0534	10.3839	11.1013
19×19	4.1481	5.6703	7.4255	7.8938	8.4032	10.0531	10.3822	11.1006

Thus, the mesh-divisions are finalized as 13x13.

3.1.2 Verification of Modal analysis,

Table 2 shows the comparison study for clamped three ply [0°/90°/0°] laminated orthotropic plate for different thickness-to-span ratio on the non-dimensionalised fundamental frequency with other published results, using a grid of 13×13. As seen from Table 2, the results obtained using present ANSYS formulations are in good agreement with those of D. Ngo-Cong et al.

Table 2: Comparison of non-dimensionalised natural frequencies of Three-ply [0°/90°/0°] square and rectangular laminated plates with various thickness-to-length ratio for clamped Boundary Condition.

a/b	t/b	$\varpi = \omega\left(b^2/\pi^2\right)\sqrt{\rho h/D_0}$									
		Mode 1		Mode 2		Mode 3		Mode 4		Mode 5	
		Present	D. Ngo-Cong	Present	D. Ngo-Cong	Present	D. Ngo-Cong	Present	D. Ngo-Cong	Present	D. Ngo-Cong
1	0.001	14.0840	14.6791	16.9941	17.6539	23.8089	24.3897	34.7465	34.7431	37.6201	39.1978
	0.050	10.6652	10.9532	13.4123	14.0298	18.9343	20.3988	22.6846	23.1977	24.2559	24.9817
	0.100	7.1504	7.4108	9.4866	10.3935	13.3460	13.9134	13.5708	15.4372	15.0117	15.8068
	0.150	5.2659	5.5482	7.1411	8.1470	9.6071	9.9044	9.9756	11.6223	10.7960	12.0305
	0.200	4.1508	4.4166	5.6747	6.6423	7.4289	7.6998	7.9015	9.1856	8.4079	9.7417
2	0.001	4.9180	5.1140	10.1780	10.5488	10.2293	10.6073	14.1542	14.3851	18.9596	19.4334
	0.050	4.5926	4.7791	8.5817	8.8414	9.2164	9.8490	11.8803	12.5142	14.3382	14.7127
	0.100	3.8603	4.1412	6.3453	6.6172	7.3117	8.3548	8.9423	9.8967	9.6689	9.9710
	0.150	3.1911	3.5397	4.8825	5.1819	5.7767	6.9271	6.8954	7.4270	7.1393	7.9371
	0.200	2.6686	3.0454	3.9384	4.2485	4.6964	5.7921	5.5451	5.9066	5.6295	6.5358

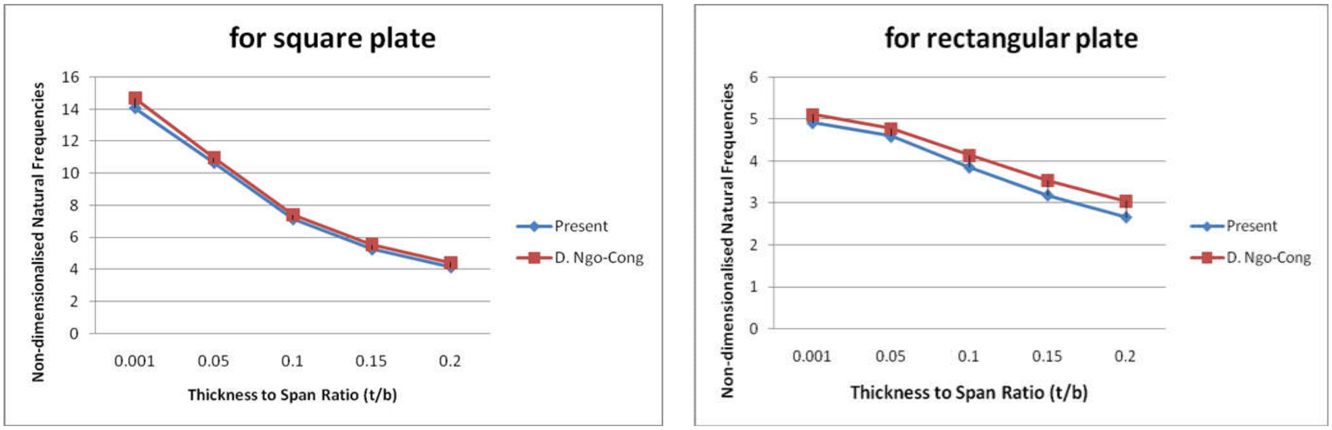


Figure 3. Comparison of Non-dimensionalised natural frequencies for first set

The natural frequency of the material with proposed 13x13 mesh size is observed comparatively less with D. Ngo cong. hence author suggest the mesh 13x13 has satisfactory results.

3.2 Present Study

A laminated composite orthotropic plate is considered and the natural frequencies are computed using a mesh size of 13×13. Also, the effect of boundary conditions, number of layers and aspect ratio (length-to width ratio) on the free vibration characteristics is analyzed. The results are presented through following sub-sections.

1. Effect of Boundary Conditions
2. Effect of number of Layers
3. Effect of Configuration
4. Effect of Modulus Ratio

3.2.1 Effect of Boundary Conditions

A laminated cross ply square plate (0°/90°/0°) with three different boundary conditions as given below is studied

1. All edges simply supported (SSSS)
2. All edges clamped (CCCC)
3. Cantilever plate (CFFF)

Table 3: First six natural frequencies (Hz) for a square cross-ply (0°/90°/0°) with different length to span ratio (t/b) for three different boundary conditions

t/b	**BC**	***Mode Number***					
		1	2	3	4	5	6
0.001	*SSSS*	95.055	135.56	232.72	360.45	380.43	383.65
	CCCC	210.69	254.20	356.10	519.65	562.78	589.69
	CFFF	31.742	35.717	68.193	151.29	198.91	203.52
0.05	*SSSS*	4393.0	6280.7	10491	13948	14817	16390
	CCCC	7885.3	9915.0	13997.0	16778.0	17939.0	19629.0
	CFFF	1527.6	1699.9	3257.0	5763.7	7079.8	7982.1
0.1	*SSSS*	7386	10702	17013	18860	20476	24449
	CCCC	10593	14055	19773	20102	22238	26323
	CFFF	2762.2	3023.7	5764.2	5922.5	11230	11506

From the above table, it can be observed that the natural frequency of vibration is highest for CCCC plate, whereas it is lowest for CFFF plate. Simply supported plates show intermediate values. The following graph in Figure 5.1 depicts the variation in natural frequencies with boundary conditions.

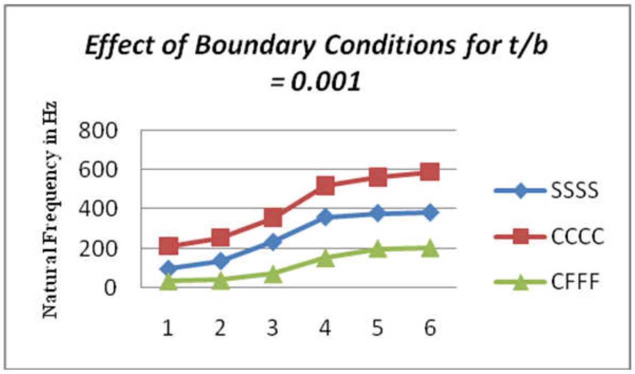


Fig. 5 (a)

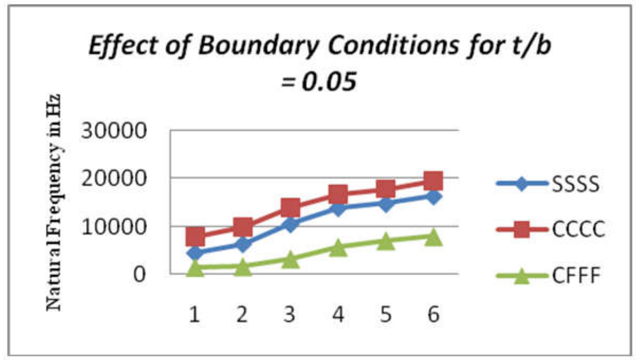


Fig. 5 (b)

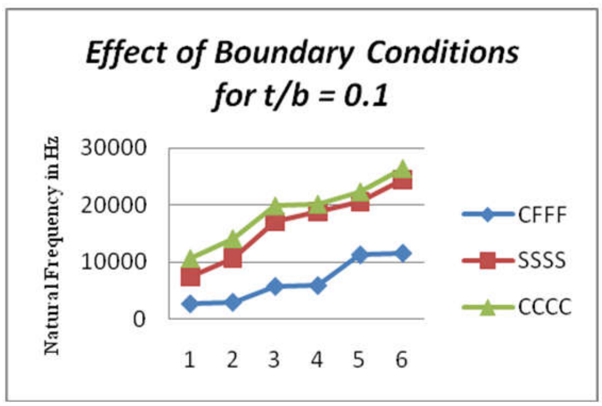


Fig. 5 (c)

Figure 5: Variation of natural frequencies (Hz) for a square cross-ply plate for three different boundary conditions taking thickness to span ratio (t/b) = 0.001, 0.05 and 0.1

3.2.2 Effect of number of Layers

The above results were obtained for a symmetric three layer cross-ply plate. In this section, the variation of natural frequencies with variation in number of layers of laminates was studied for a five layer and seven layer cross-ply plate keeping length to span ratio (t/b) of the square plate is 0.001.

Table 4: First six natural frequencies (Hz) for the boundary condition CCCC and varying number of layers

Number of layers	Mode Number					
	1	2	3	4	5	6
3 layers 0°/90°/0°	210.69	254.20	356.10	519.65	562.78	589.69
5 layers 0°/90°/0°/90°/0°	210.73	333.57	519.91	571.41	590.37	768.20
7 layers 0°/90°/0°/90°/0°/90°/0°	210.75	365.10	498.42	590.80	648.68	814.90

Table 5: First six natural frequencies (Hz) for the boundary condition SSSS and varying number of layers

Number of layers	Mode Number					
	1	2	3	4	5	6
3 layers 0°/90°/0°	95.055	135.56	232.72	360.45	380.43	383.65
5 layers 0°/90°/0°/90°/0°	95.056	197.43	330.66	380.46	405.14	528.80
7 layers 0°/90°/0°/90°/0°/90°/0°	95.056	220.67	315.63	380.47	464.76	567.29

Table 6: First six natural frequencies (Hz) for the boundary condition CFFF and varying number of layers

Number of layers	Mode Number					
	1	2	3	4	5	6
3 layers 0°/90°/0°	31.742	35.717	68.193	151.29	198.91	203.52
5 layers 0°/90°/0°/90°/0°	28.868	33.247	107.39	180.91	186.08	224.52
7 layers 0°/90°/0°/90°/0°/90°/0°	27.408	31.988	121.54	171.76	177.23	224.81

From the above three tables, it can be observed that with the increase in number of layers, there is an increase in the frequency of vibration. The variation in frequencies can be best understood from the following graphs.

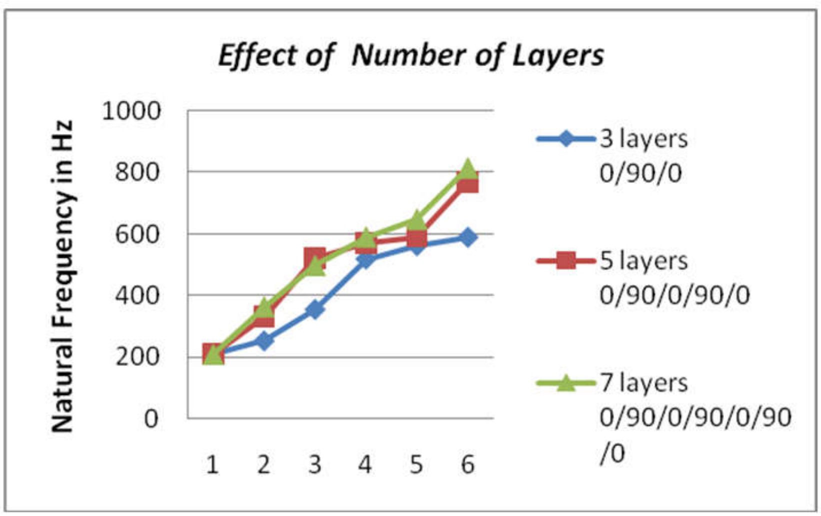


Figure 5.3(a): Variation of frequency (Hz) with number of layers for CCCC square cross-ply

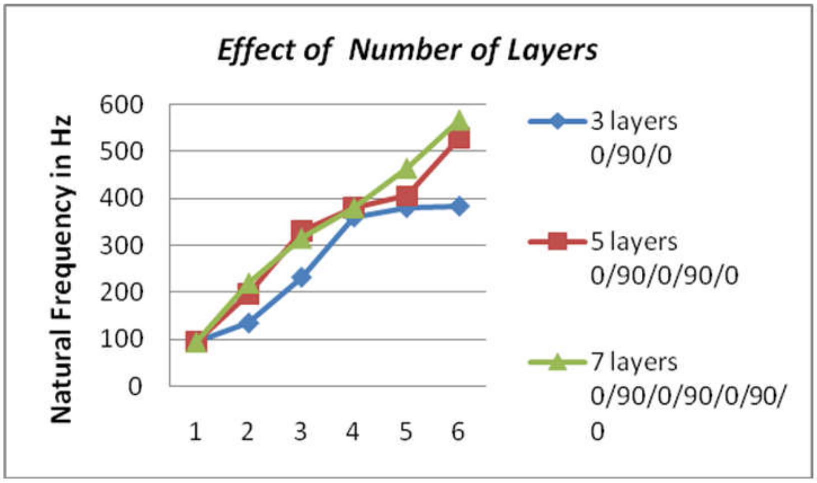


Figure 5.3(b): Variation of frequency (Hz) with number of layers for SSSS square cross-ply

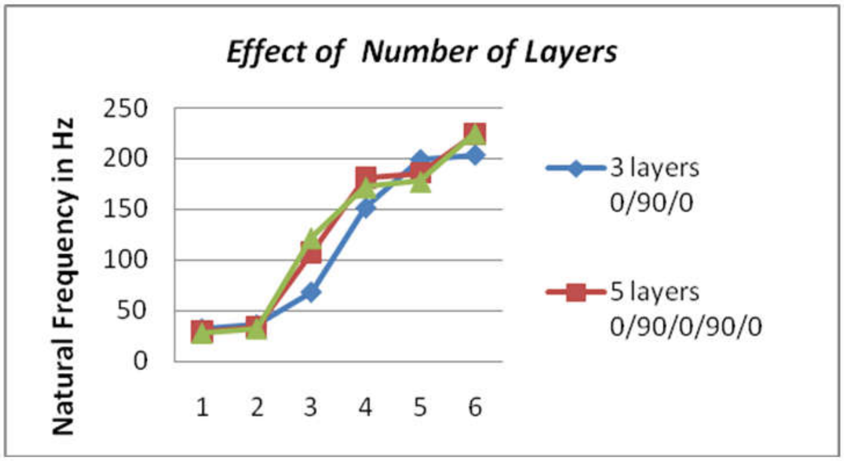


Figure 5.3(c): Variation of frequency (Hz) with number of layers for SSSS square cross-ply

3.2.3 Effect of Shapes

In this section, the variation of natural frequencies with variation in shape of the plate was studied for a three ply (0°/90°/0°) laminated plate keeping length to span ratio (t/b) is 0.001 with clamped boundary condition.

Table 7: First six natural frequencies (Hz) for the boundary condition CCCC and varying shapes.

Shape	Mode Number					
	1	2	3	4	5	6
Square (a/b = 1)	210.69	254.20	356.10	519.65	562.78	589.69
Rectangle (a/b = 2)	73.568	152.24	153.01	211.65	283.59	285.24
Circular (d = 1)	250.77	488.9	538.14	761.01	793.20	908.13

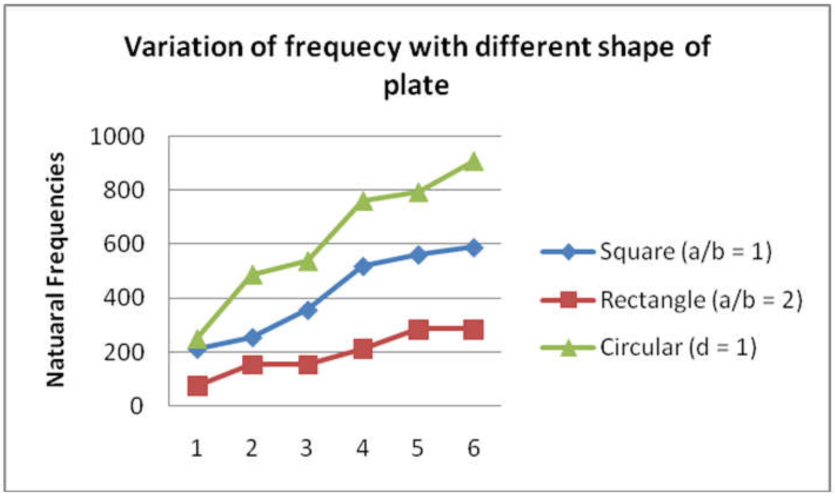


Figure 5.4: Variation of frequency (Hz) with different shapes for CCCC square cross-ply

3.2.4 Effect of Modulus Ratio

Table 8: First six natural frequencies (Hz) for the boundary condition CCCC with different modulus ratios.

Modulus Ratio	Mode Number					
	1	2	3	4	5	6
$E_1/E_2 = 40$	210.69	254.20	356.10	519.65	562.78	589.69
$E_1/E_2 = 30$	183.94	226.93	324.46	478.17	488.96	517.32
$E_1/E_2 = 20$	152.56	195.94	289.46	401.77	432.82	433.09

IV. Conclusion

The free vibration of a laminated composite plate analyzed for different boundary conditions with different number of layers were observed to increase, irrespective of the boundary condition applied. When the no. of layers is increased from 5 to 7, the frequency seems to be decreasing with all boundary condition CFFF. Thus proves less vibration when thickness of plate is increase.

V. Future Scope

Author believes that the composite plate with number of holes can be a matter of observation. Further, with the increase in length of plate, the change in value of natural frequency can also be observed.

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