

Finite Element Analysis of Two-Crack Shaft in a Rotor Disc-Bearing System

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ABSTRACT

The current analysis has bestowed a study of natural frequency characteristics of a shaft of various profiles. Structural steel and alloy 6061 system were used for this model analysis. The impact of diameter with totally different or completely different profiles of the Shaft without crack and shaft with two cracks on the natural frequency and modes of various materials and critical speed effects were analyzed on different profile and materials of shaft and distribution on the shaft was studied. The cluster of nodes and parts is understood as meshing this method is finished to determine convergence of solution the development convergence of solution may be a relation between accuracy, degree of freedom and no. of nodes and parts as the amount of nodes and parts are enhanced at variable iteration a convergence of solution is obtained. Meshing are of various varieties i.e. Tetrahedral, Quadrahedral, Hexahedral, square mesh and triangular mesh, tetrahedral mesh offers higher convergence throughout finite element simulation a stiffness matrix, damping matrix, stress matrix is resolved on ANSYS at each and every node and element by iteration strategies like runge-kutta etc. to determine convergence of solution.

Keywords: FEA, FEM, Structural Steel, Alloy 6061, Critical Speed, Natural Frequency

I. INTRODUCTION

The subject rotor dynamics is termed an individual branch of applied mechanics that deals with the performance and detection of spinning structures [1]. The predictions of the system dynamic facet are meticulously essential within the style of rotating structures. Typically it analyses the behavior of rotating structures that ranges from fans, gear trains to turbines and craft jet engines. Rotating systems usually develop instabilities that are excited by unbalance and therefore the internal makeup of the rotor system and should be corrected [2]. This is often the prime space of interest for the planning engineers who model the rotating systems. In figure 1 shows the essential diagram of rotor dynamics.

II. BACKGROUND AND SIGNIFICANCE

From ISO definition, rotor are often outlined as a body that is suspended through a group of cylindrical rest or bearings that grants the system to rotate freely concerning an axis secured in area [3-4]. Within the

basic level of rotor dynamics, it's connected with one or additional mechanical structures (rotors) supported by bearings that rotate around a novel axis. The non-spinning structure is named a stator.

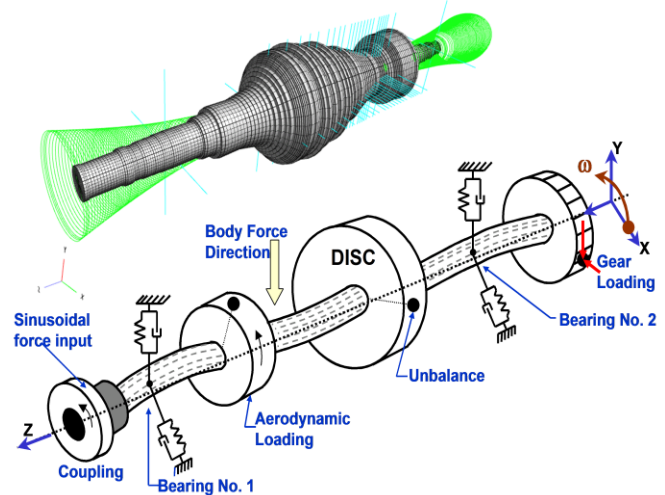


Figure 1. Basic Diagram of Rotor Dynamics

When the spin speed will increase the amplitude of vibration increases and is most at a speed referred to as critical speed. This amplitude is usually elevated by unbalance forces from disk of the spinning system [5].

Once the system reaches excessive amplitude of vibration at the critical speed, ruinous failure happens. Ordinarily turbo machineries often develop instabilities that are in the main attributable to the interior configuration, and will be corrected. Typically rotating structures originates vibrations relying upon the complexness of the mechanism concerned within the method [6-7]. Even a little misalignment within the machine will increase or excite the vibration signatures. System vibration behavior attributable to imbalance is that the main aspects of rotating machinery, and it should be measured well and reviewed while designing. Every object including rotating structures shows natural frequency counting on the complexness of the structure. The critical speed of those rotating structures arises once the motility speed meets with its natural frequency. The primary critical speed are often encountered at all-time low speed. But because the speed will increase additional critical speeds also can be noticed. It is very essential to cut back the motility unbalance and excessive external forces to attenuate the forces that actuate resonance. The most important concern of planning a rotating machine is, avoiding the vibration in resonance that creates a damaging energy [8]. Things involving rotation of shaft close to critical speed should be avoided. Once these aspects are unheeded it would leads to wear and tear of the instrumentality, failure of the machinery, human injury and someday price of lives.

III. RELATED WORK

The paper [9], modified harmonic balance technique planned to work out the wave properties of a cracked rotor disc-bearing system. The analytical model of the cracked rotor disc-bearing system is developed mistreatment the Timoshenko beam theory. The crack sculptures as a respiratory crack mistreatment Mayes associated Davies model and an alternate respiratory operate is additionally planned to model the respiratory behavior of the cracks. The impact of the crack parameters like depth, location and relative position were investigated on the crucial speeds, shaft center orbit and lateral response of the analytical model of the rotor disc-bearing system. The presence of the second crack intensifies the impact of the primary crack on the crucial speeds considering its depth and placement on the shaft. The tiny depth cracks are a lot of sensitive to the propagation of the second crack compared with comparatively deep cracks. The detection of the 2 cracks on a rod is a lot of possible considering the

frequency spectrum of the lateral vibration whereas the crack has been sculptures using the softly-clipped circular respiratory function. Using the circular function respiratory function yields smart estimations of the crucial speeds and shaft center orbits of the cracked rotor disc-bearing system.

The paper [10] impact of crack on the natural frequency of the rotor system with open crack has been studied and it's been ascertained that the crack had a big impact on increasing the natural frequency of the rotor system as compared to the un-cracked rotor system. This can be attributable to the buildup of the strain energy within the section of the crack. This impact may be utilized to review the dynamics behavior of the rotor through Joseph Campbell diagram analysis. It offers the crucial speed of the system that is probably going to occur throughout the runtime of the rotor. Through this analysis it's seen that the crucial speed of the cracked rotor will increase than that of the un-cracked rotor. This may be used as a tool to examine the doable presence of the crack within the rotor.

The paper [11] study proposes work the helpful effects of each movement and stationary damping on the dynamic vibration of a rotating rotor with a crosswise open crack on the surface of the shaft. Totally different from the previous study, within the method of modeling, special attention is paid to each the excellence between the 2 styles of damping and therefore the property in movement damping caused by the crack on the surface of the shaft. The motion equations embrace the non-inertial terms (such as consequence and force, etc.), and that they are shaped for rotating motion in moving frame instead of mechanical phenomenon reference system. Therefore, the anisotropic system with the multi periodical varied coefficients is simplified specified the soundness analysis primarily based root locus technique with may be applied. By the Laplacian chemist root locus technique, the Joseph Campbell diagram, decay rate plot and roots locus plot are derived. totally different from the previous numerical approach results, within which the resonance region are roughly delineated while not correct boundary for the cracked rotor, the correct crucial rotation speeds vary are given on the Joseph Campbell diagrams. Additionally, the secondary crucial speed, on that the resonance is found between the natural frequencies of the system and therefore the bending force, is additionally situated on the Joseph Campbell diagrams. The property of stiffness

attributable to the various crack depth is evidenced to play a decisive role on the length of crucial vary each in damping and non-damping system. Furthermore, the damping effects are divided into 2 varieties. The stationary damping is shown to own the distinct helpful impact. The rotating damping, however, is evidenced to be a conspicuous destabilizing issue which will cause the decay rate amendment to be positive particularly within the crucial or super crucial region, though it's going to cut back in price of the real a part of the chemist root within the begin of the rotation. Additionally, the property magnitude relation of the movement damping is additionally verified to own the sure unstable influence within the case of each low level and high level proportion of stationary damping. These results could offer sensible relevancy to crack detection and instability management of the serious loading turbo-machinery system. Furthermore, this study provides a proposal for the applied modeling of cracked rotor system that the results of the enhanced proportion and therefore the aggravated property of the movement damping, because the results of the crack on the surface of the fatigue rotor, ought to been taken into consideration.

The paper [12], Experimental investigations of vibration are with success conducted and valid through analytical equation of crack rotor system. The subsequent conclusions are drawn: Variation in stiffness has been ascertained clearly with the rise in crack depth whereas the stiffness of un-cracked rotor is optimum. However, once the depth of crack will increase, the stiffness of the shaft drastically reduces. When making the second crack, stiffness additionally reduces however the result of second crack on the stiffness is found marginal. Stress concentration for the shaft is marginally affected at the center crack. It's been ascertained that larger crack depth has the additional vital result on the shaft. Crack at middle of the axis has optimum result on the stiffness as compared to the second crack, created at the other position of the shaft. Position of crack plays a big parameter in multi-crack rotor detection. Since the damaged axis contains a part of nonlinearity, OROS system will even provide higher results once is also used for a true system. It's been clearly incontestable that the natural frequency is reduced with the reduction in stiffness which can additionally cause resonance at terribly low excitation speed of the motor. The employment of OROS system for crack detection are

often handily applied with nonlinear models of multi-cracked rotor system.

IV. FINITE ELEMENT ANALYSIS

The finite element methodology (FEM) (its exercise usually referred to as finite element analysis (FEA)) could be a numerical technique for locating approximate solution of partial equation (PDE) additionally as integral equation. the solution approach relies either on eliminating the differential equation completely (steady state problem), or rendering the PDE into an approximation system of standard equation, that are then numerically integrated victimization normal technique like Euler's technique, Runge-kutta, etc.

In determination partial differential equations, the first challenge is to form an equation that approximates the equation to be studied, however is numerically stable, that means that error within the input and intermediate calculation don't accumulate and cause the ensuing output to be insignificant. There are some ways of doing this, all with benefits and disadvantage. The finite element technique may be a sensible choice for finding partial equation over sophisticated domain (like cars and oil pipelines), once domain changes (as throughout a solid state reaction with a moving boundary), once and the specified exactness varies over the complete domain, once the answer lacks smoothness.

A. Finite element Analysis

FEA consists of a pc model of a material or design that's stressed and analyzed for specific results. It's utilized in new product style, and existing product refinement. A corporation is ready to verify a projected style are going to be able to perform to the client's specifications before producing or construction. Modifying an existing product or structure is used to qualify the product or structure for a replacement service condition. Just in case of structural failure, FEA could also be wont to facilitate confirm the look modifications to satisfy the new condition. There are typically 2 forms of analysis that are utilized in industry: 2-D modeling, and three-D modeling. Whereas 2-D modeling conserves simplicity and permits the analysis to be run on a comparatively traditional pc, it tends to yield less correct results. Three-D modeling, however, produces additional correct results whereas sacrificing the power to run on nearly the quickest computers effectively. Among every

of those modeling schemes, the applied scientist will insert various algorithms (functions) which can build the system behave linearly or non-linearly. Linear systems are way less advanced and usually don't take into consideration plastic deformation. Non-linear systems do account for plastic deformation, and lots of are capable of testing a material all the thanks to fracture.

B. Finite element Analysis Work

FEA uses a mesh system of points referred to as nodes that build a grid referred to as a mesh. This mesh is programmed to contain the material and structural properties that outline how the structure can react to sure loading conditions. Nodes are allotted at a particular density throughout the material depending on the anticipated stress levels of a specific space. Regions which can receive massive amounts of stress typically have the next node density than those that experience very little or no stress.

V. MODELLING METHODOLOGY

The procedure for solving the problem is

- Modeling of the geometry.
- Meshing of the domain.
- Defining the input parameters.
- Simulation of domain.

Finite Element - Analysis of Steel Shaft
Analysis Type- Modal analysis

Steps for the Stress and Deflection Analysis

Stress and deflection analysis have been carried out by using ANSYS fem solver.

1. First of all the model i.e the rotor with notch and without notch are modelled using Creo software. The model is then saved in *.iges/* .igs format (iges stands for initial graphics exchange specification, which is a neutral data format file that helps in digital exchange of data among various CAD software).
2. This model is then imported to the ANSYS.
3. Meshing of the solid model is generated by using the meshing function of the ANSYS finite element solver package.
4. Then various boundary conditions are applied to the model and solved for stress and deflection.
5. Crack is generated for studying the open crack or notch behavior of the rotor.
6. Modal analysis and open crack analysis is studied

for the model with and without crack.

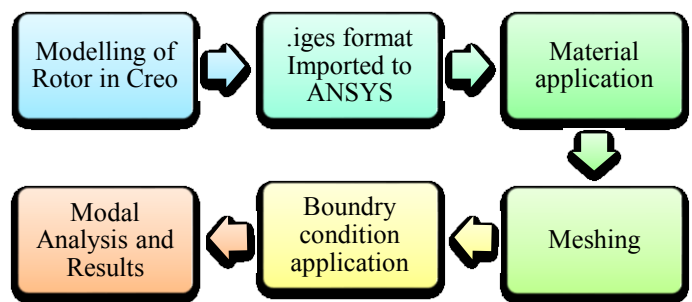


Figure 2 - Flow chart of Modal analysis performed on Rotor

Pre-processing

Preprocessing include CAD model, meshing and defining boundary conditions.

CAD Model

Table 1. Dimension of Shaft

Length of shaft.	1270mm
Diameter of shaft	19.05mm
Diameter of Disc	152.4mm
Thickness of Disc	25.4mm

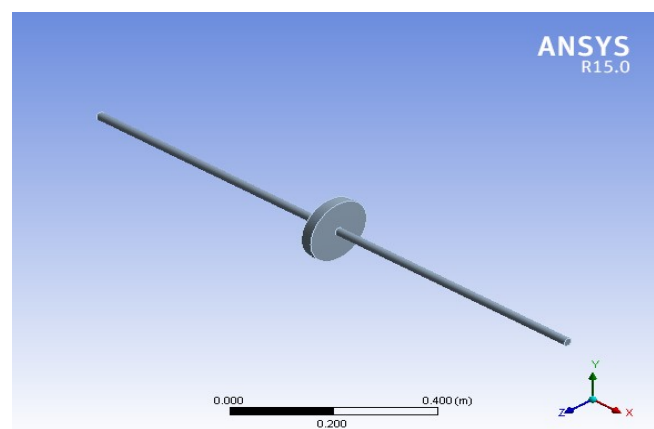


Figure 3. CAD Model of without Crack

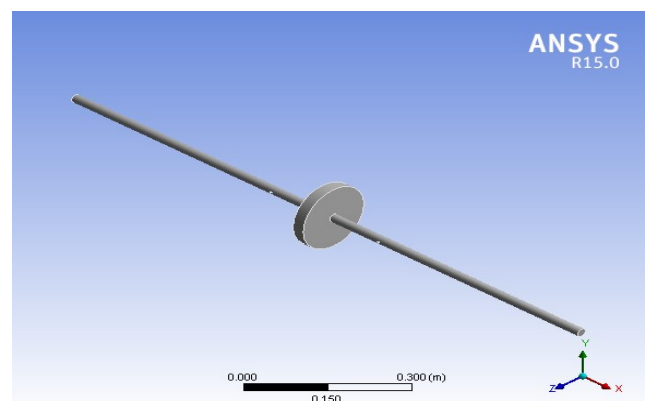


Figure 4. CAD Model with Crack

Meshing

The group of nodes and elements is known as meshing this process is done to determine convergence of solution the phenomenon convergence of solution is a relation between accuracy, degree of freedom and no. of nodes and elements as the quantity of nodes and elements are increased at variable iteration a convergence of solution is obtained. Meshing are of different types i.e. Tetrahedral, Quadrahedral, Hexahedral, Square mesh and triangular mesh, tetrahedral mesh gives better convergence during finite element simulation a stiffness matrix, damping matrix, stress matrix is solved on ANSYS at each and every node and element by iteration methods like runge-kutta etc. to determine convergence of solution.

Meshing data for without crack

NODES	3396
ELEMENTS	1628

Meshing data for with Crack

NODES	3067
ELEMENTS	1460

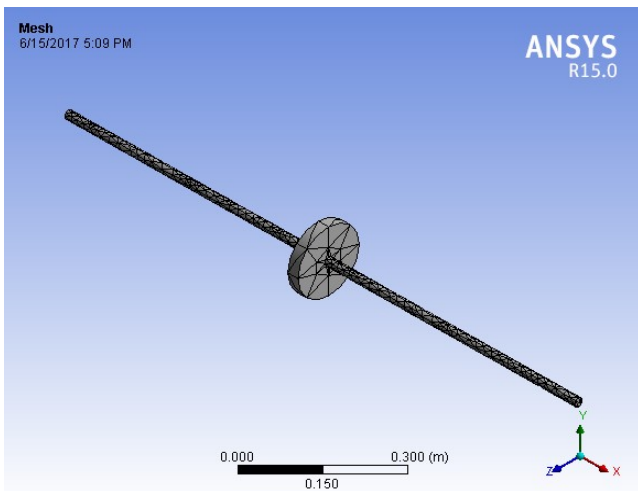


Figure 5. Mesh domain of without Crack Shaft

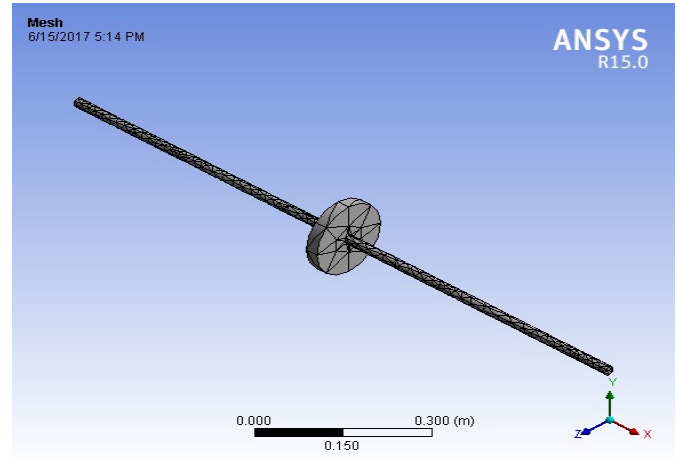


Figure 6. Mesh domain of Shaft-2

Boundary condition and Loading

The boundary conditions applied to the finite element model is as follows.

- The Bearing support parallel to the cross-section of shaft.
- Angular velocity of 25000rpm is applied on the cross section shaft.

Table 2. Properties of different a material

Material Properties	Stainless Steel	Alloy6061
Young's Modulus	1.93e11	69e9
Poisson's Ratio	0.31	0.33
Density	7750kg/m ³	2700 kg/m ³

Problem Formulation

The study of various literatures we find the natural frequency is lower as compared to present study. The purpose of this study is to predict critical speed and natural frequency with different material at constant angular velocity of 25000 rpm.

Solution

In this step victimization finite component technique by ANSYS 15.0 to solve the matter for the outlined material properties, boundaries conditions and mesh size.

Post-Processing

For viewing and interpretation of results of on top of solved downside. The result will be viewed in numerous formats graph, value, animation etc.

Definition of different modes

Mode -1 (Backward Whirling) – mode one represents a magnitude of frequency with respect to shaft rotation in unbalanced masses condition.

Mode -2 (Forward Whirling) - mode two represents a magnitude of frequency with respect to shaft rotation in balanced masses condition.

Mode -3 (Backward Whirling) - mode three represents a magnitude of frequency with respect to shaft rotation in unbalanced masses condition.

Mode -4 (Forward Whirling) - mode four represents a magnitude of frequency with respect to shaft rotation in balanced masses condition.

Mode -5 (Backward Whirling) - mode five represents a magnitude of frequency with respect to shaft rotation in unbalanced masses condition.

Mode -6 (Forward Whirling) - mode six represents a magnitude of frequency with respect to shaft rotation in balanced masses condition.

Mode -7 (Forward Whirling) - mode six represents a magnitude of frequency with respect to shaft rotation in balanced masses condition.

VI. SIMULATION RESULTS DISCUSSION

Shaft with Different Materials (Critical Speed Frequency and Stiffness)

A Modal - analysis was distributed to investigate critical speed of shaft with completely different material by exploitation Campbell diagram and relation between natural frequencies and spin speed and two sorts of materials of Alloy 6061 and steel with differing types of shaft model to work out the distribution on the Shaft of the various sorts of shaft. Distribution contours just in case of completely different or of various diameter for the two different profiles are shown in Figure, and also the result of completely different or of varied shaft profiles on the frequency and modes distribution for various different diameter and materials are diagrammatical within the Figure below.

Analysis of Shaft with Constant Diameter and Different Materials

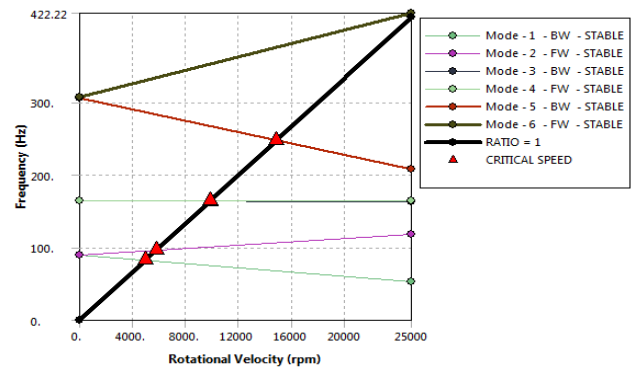


Figure 7. Result of Campbell diagrams of frequency and rotational velocity distributions along the structural steel without Crack

This diagram is show in colors line forward and backward whirling. Other inclined line is also called synchronous excitation or engine order line. This engine order line is intersecting in forward and backward whirling line. This intersecting point is show the critical speed. This is also known as Campbell diagram. This diagram shows the frequency and rotational velocity distributions along the structural steel without Crack.

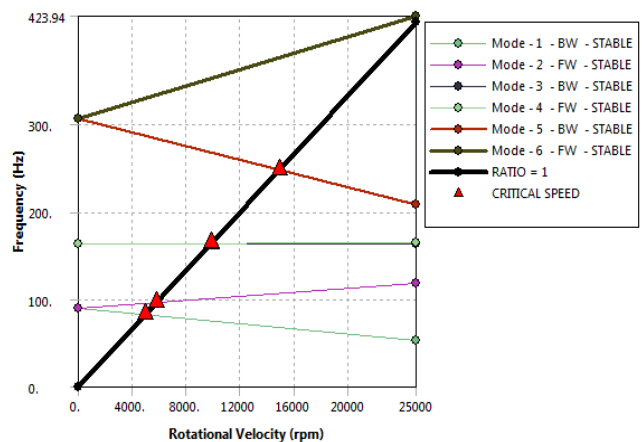


Figure 8. Result of Campbell diagram of frequency and rotational velocity distributions along the alloy 6061 without Crack

This diagram is show in colors line forward and backward whirling. Other inclined line is also called synchronous excitation or engine order line. This engine order line is intersecting in forward and backward whirling line. This intersecting point is show the critical speed. This is also known as Campbell diagram. This diagram shows the frequency and rotational velocity distributions along the alloy 6061 without Crack.

Table 3. Critical Speed of Shaft without Crack

Critical Speed		
Modes	Structural Steel	Alloy 6061
1(BW)	4947.5	4958
2(FW)	5768.2	5782.8
3(BW)	9811.2	9841
4 (FW)	9829	9860.7
5(BW)	14855	14900
6 (FW)	NONE	NONE

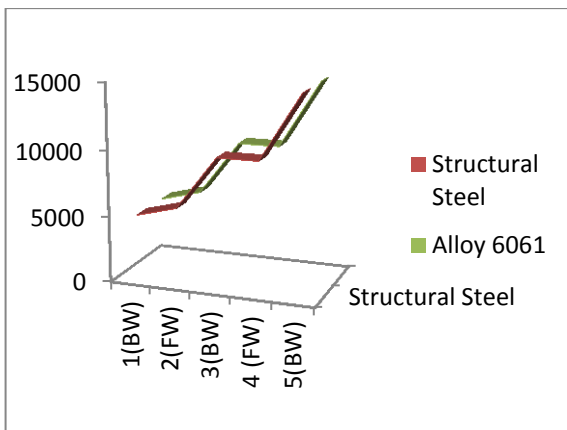


Figure 9. Graph shows comparison of critical speed of two different materials

The above table and graph shows the critical speed values and comparison of two materials with respect to modes on shaft without crack, the result shows that structural steel seems to be less critical speed due to its high stiffness and lesser deformation at different whirling modes.

Analysis of Shaft with Two Cracks

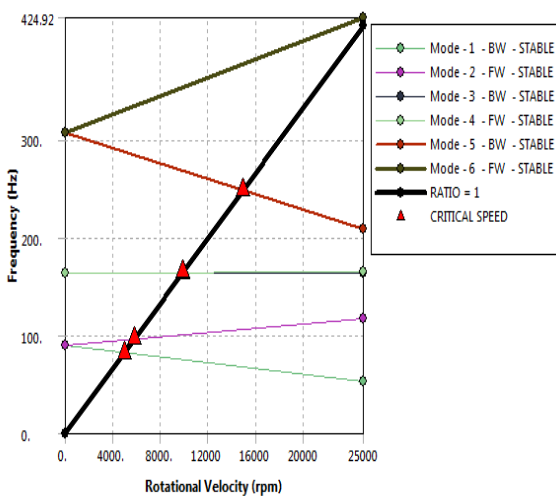


Figure 10. Result of Campbell diagrams of frequency and rotational velocity distributions along the structural steel shaft with two Cracks

This diagram is show in colors line forward and backward whirling. Other inclined line is also called synchronous excitation or engine order line. This engine order line is intersecting in forward and backward whirling line. This intersecting point is show the critical speed. This is also known as Campbell diagram. This diagram show the frequency and rotational velocity distributions along the structural steel shaft with two Cracks.

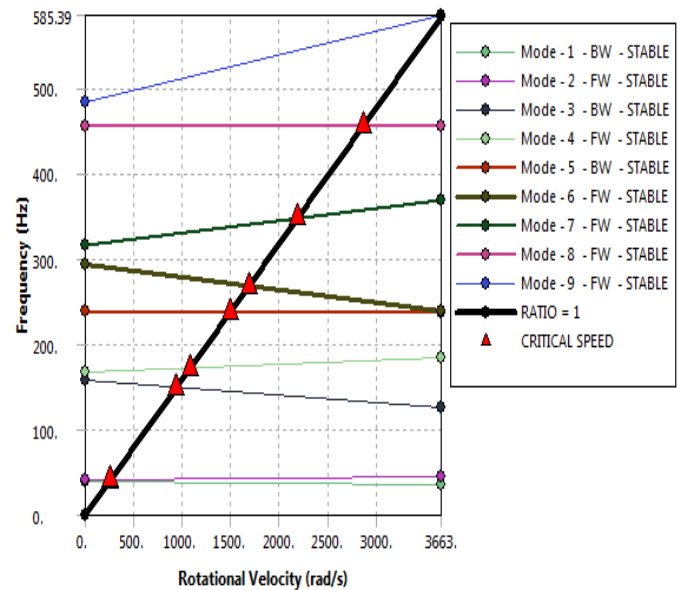


Figure 11. Result of Campbell diagram of frequency and rotational velocity distributions along the Alloy 6061 with two Cracks

This diagram is show in colors line forward and backward whirling. Other inclined line is also called synchronous excitation or engine order line. This engine order line is intersecting in forward and backward whirling line. This intersecting point is show the critical speed. This is also known as Campbell diagram. This diagram shows frequency and rotational velocity distributions along the Alloy 6061 with two Cracks.

Table 4. Critical Speed of Shaft with Two Crack.

Critical Speed		
Modes	Structural Steel	Alloy 6061
1(BW)	4957.7	4964.8
2(FW)	5784.3	5793
3(BW)	9855.6	9871.3
4 (FW)	9880.6	9898
5(BW)	14927	14949
6 (FW)	NONE	NONE

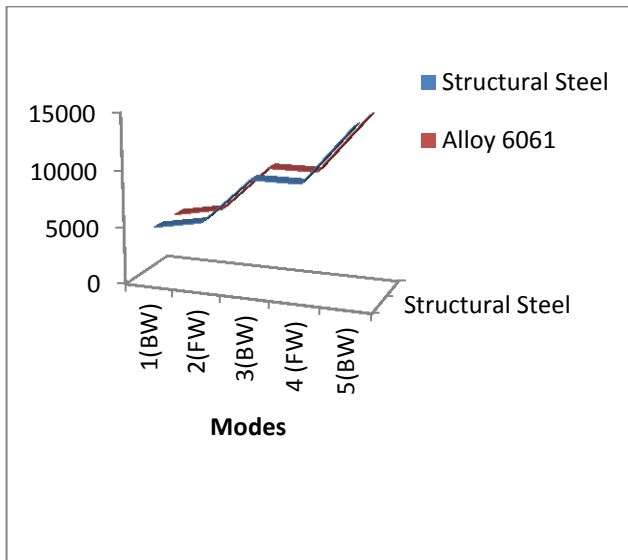


Figure 12. Graph shows comparison of critical speed of two different materials.

The above table and graph shows the critical speed values and comparison of two materials with respect to modes on shaft without crack, the result shows that structural steel seems to be less critical speed due to its high stiffness and lesser deformation at different whirling modes.

Analysis of Shaft stiffness without Crack

Red color - Red color represents that a deformation is maximum with respect to its maximum frequency.
 Blue color – Blue color represents that a deformation is maximum with respect to its minimum frequency.
 Green color – Green color represents that a deformation is intermediate with respect to its blue color frequency.
 Yellow color – Yellow color represents that a deformation is intermediate with respect to its red color frequency.

The deformations of shaft shown in contour plots are w.r.t. frequency at constant angular velocity of 25000rpm these deformation changes as per modes of frequency that at particular section the value of frequency is high and low, this frequency is damped natural frequency of shaft at synchronous speed which occurs due to mass imbalance also the modal damping ratio was considered, it is a ratio of damping constant to the damped natural frequency from below contour plots the values obtained are compared in graphs and was predicted that in every material damped natural frequency increases, Hence modal damping ratio

decreases from this effect logarithmic decrement increases that means decrease in amplitudes.

Variable colored contours of shafts represents a deformation of a rotating element with respect to its natural frequency blue color on contours represents a minimum effect of frequency, Green color represents that a frequency is more than blue color, Red color represents that a frequency is maximum at a particular that section of a rotating element.

Table 5. Natural frequency of shaft without Crack.

Natural frequency		
Mode	Structural steel	Alloy6061
1	89.672	89.867
2	89.678	89.892
3	163.63	164.12
4	163.71	164.25
5	306.00	307.05
6	306.17	307.17

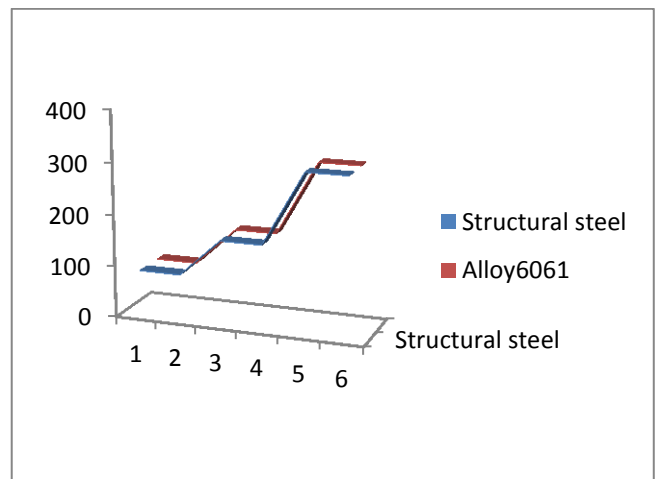


Figure 13. Graph shows modes and frequency of a shaft without Crack

The above table & graph shows the natural frequency values of two materials with respect to modes on shaft without crack, two materials were compared with respect to their modes.

In case of stepped shaft the diameter changes hence frequency increases the deformations of shaft shown in contour plots are w.r.t. frequency at constant angular velocity of 3663 rad/s these deformation changes as per modes of frequency that at particular section the value of frequency is high and low this frequency is damped natural frequency of shaft at synchronous speed which

occurs due to mass imbalance also the modal damping ratio was considered it is a ratio of damping constant to the damped natural frequency from below contour plots the values obtained are compared in graphs and was predicted that in every material damped natural frequency increases, Hence modal damping ratio decreases from this effect logarithmic decrement increases that means decrease in amplitudes.

Red color - Red color represents that a deformation is maximum with respect to its maximum frequency.
 Blue color – Blue color represents that a deformation is maximum with respect to its minimum frequency.
 Green color – Green color represents that a deformation is intermediate with respect to its blue color frequency.
 Yellow color – Yellow color represents that a deformation is intermediate with respect to its red color frequency.

Table 6. Natural frequency of shaft with two Crack.

Natural frequency		
Mode	Structural steel	Alloy6061
1	89.845	89.967
2	89.907	90.042
3	164.34	164.60
4	164.59	164.89
5	307.48	307.83
6	308.02	308.47

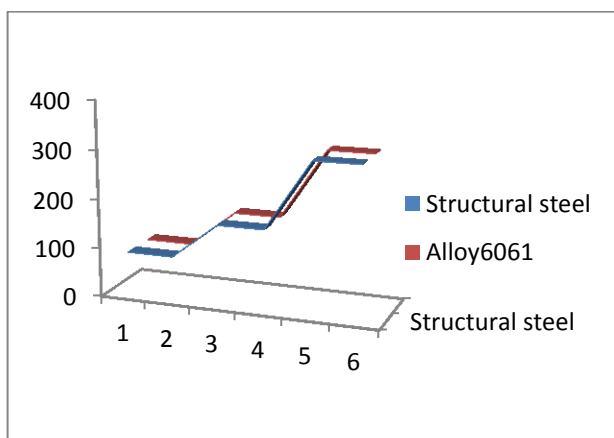


Figure 14. Graph shows modes and frequency of a shaft with two Cracks

The above table & graph shows the natural frequency values of two materials with respect to modes on shaft with two cracks, two materials were compared with respect to their modes.

Analysis of Shaft stiffness without Crack

Stiffness	
Structural Steel	Alloy6061
12.421	7.221
12.429	7.22
11.273	6.596
11.282	6.596
13.664	8.007
13.701	7.985

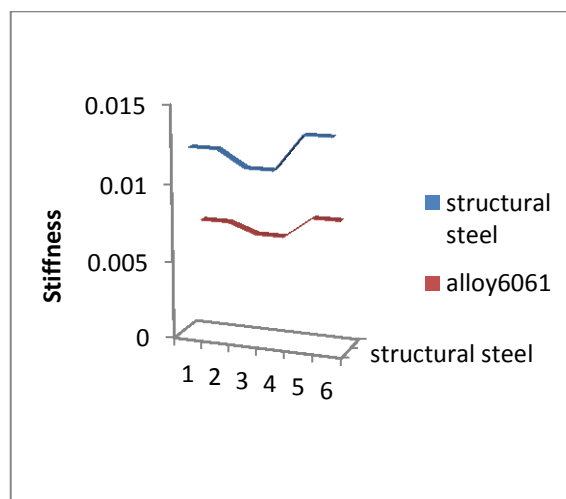


Figure 15. Graph shows modes and stiffness of a shaft without Cracks

The above table & graph shows the Stiffness values of two materials with respect to modes on shaft without cracks, two materials were compared with respect to their modes.

Analysis of Shaft stiffness with two Cracks

Stiffness	
Structural Steel	Alloy6061
12.307	7.219
12.306	7.217
11.239	6.591
11.236	6.59
13.572	7.952
13.579	7.957

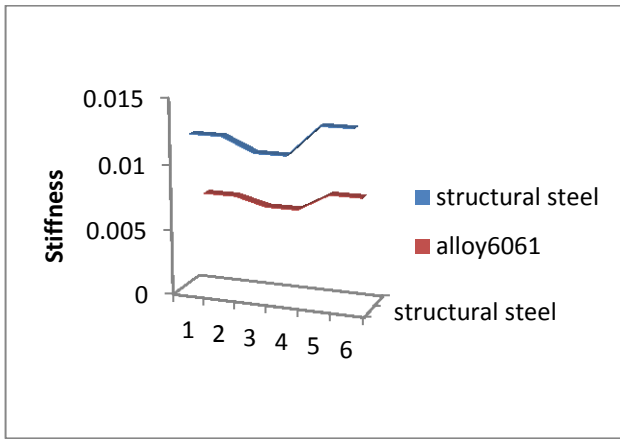


Figure 16. Graph shows modes and Stiffness of a shaft with two Cracks

The above table & graph shows the Stiffness values of two materials with respect to modes on shaft with two cracks, two materials were compared with respect to their modes.

Discussion

It is evident that there is an increase in the frequency along the shaft for all the two profiles with various different shaft. It is found that the alloy 6061 has maximum frequency in the case of different diameter profile and minimum for structure steel profile. It is also seen that the frequency distribution along the material structural steel of the shaft for all the two profiles decreases with an increase in the natural frequency modes. This is because different shaft value will lead to more frequency being transferred to the material and less vibration stored in the shaft, hence resulting in low vibration and higher frequency base. This reduced vibration will induce smaller resonance condition and the consequent increase in the frequency of different material with shaft diameter of different profiles.

VII. VII CONCLUSION

The natural frequency along the shaft profile is found to be maximum of the alloy 6061 material profile with shaft without crack and varies along the length up to the shaft for all the two profiles. The critical speed distribution along the shaft is maximum for alloy 6061 and minimum for structural steel of a shaft with different profiles. The magnitude of frequency is minimum in the case of structural steel material profile with shaft without crack. The nature of the natural frequency is maximum near its end in 3rd and 4th, 6th mode.

The nature of the critical speed is maximum near its masses and between the end of the shaft where masses are placed of shaft and changes with respect to shaft material with different profile towards the end and between masses of the shaft for the same RPM and different modes of natural frequency.

In a comparison with the structural steel and alloy 6061 material resulted in higher frequency characteristics close to the end of the shaft for a different shaft diameter profile. The critical speeds are maximum for alloy 6061 at high frequency and minimum for structural steel at less frequency on same RPM.

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