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Bayesian Multiple Deferred Sampling (0,2) Plan with Poisson Model Using Weighted Risks Method

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ABSTRACT
This paper deals with designing of Bayesian Multiple Deferred State sampling plan (BMDSSP) (0, 2) with gamma prior distribution and reduced producer and consumer risk. The operating procedure of Bayesian Multiple Deferred State sampling and the derivation of performance measures using gamma prior distribution are given. The designing of the Bayesian Multiple Deferred State sampling plan (BMDSSP) (0, 2) for two specified points on the OC curve and (AQL, LQL) of the developed plan are given. The Gamma prior distribution for given (μ_1,1-α) and (μ_2,β).
Keywords: Bayesian MDS-1(c1, c2), Gamma Poisson Distribution, Minimum Risks Plan, Acceptable Quality Level (AQL), Limiting Quality Level (LQL).

I. INTRODUCTION

Bayesian acceptance sampling considers both process and sample variations. Thus the distinction between the conventional and Bayesian approach is associated with utilization of prior process history or knowledge in selection of distribution to describe the random fluctuations involved in acceptance sampling. Calvin (1984) has introduced the procedures and constructed tables for using Bayesian sampling plans. Hald (1981) has studied comparison of traditional theory and Bayesian theory. Wetherill and Chiu (1975) have reviewed of Acceptance sampling Schemes with emphasis on the economic aspect. Wortham and Baker's (1976) have constructed MDS sampling plan a specified into four parameters such as n, m, c_1 and c_2.Single sampling plans involving a minimum sum of risks for the binomial model for the OC curve can be found from Golub's (1953) tables. Soundarajan (1981) has extended Golub's approach to a single sampling under the conditions of the Poisson model for the OC curve. The drawback of Golub's approach is that the sampling plan involving a minimum risks may still result in larger producer's and consumer's risks. Since minimizing the sum of risks is a desirable feature (as it leads to a better shouldered OC curve), one may attempt to design sampling plans involving smaller producer's and consumer's risks. Soundarajan and Govindaraju (1983) have, therefore, modified the Golub's approach of minimizing the sum of risks such that the producer's and consumer's risks are below the specified levels (e.g. 0.01, 0.05, etc.,) in the case of single sampling plans. For the selection of MDS- 1 (c_1 , c_2) sampling plans Soundararajan and Vijayaraghavan (1989) presented tables for the given AQL and LQL for the purpose of fixed values of α and β. As an extension of their work this presents a table and procedure in order to find the MDS- 1 (c_1, c_2) sampling plans of Wortham and Baker which is involving a minimum sum of risks for given AQL and LQL dividing the fixation of producer's and consumer's risk. The Bayesian operating characteristic curve by Schafer (1967) has considered for the selection of sampling plans. Suresh and Latha (2001) obtained the Procedures and Tables for Selection of Bayesian Single Sampling with Weighted Risks. Lauer (1978) analysed the influence of prior information in comparison with the acceptance probability of sampling plans where the proportion defective p, that follows a Beta distribution with the conventional Operating Characteristic (OC) values. Suresh and Latha (2001) have designed a procedure for average probability of acceptance function for single sampling plans with the Gamma prior distribution. Latha and Jeyabharathi (2012) have studied the performance measures for the Bayesian chain-sampling plan using Beta Binomial distribution. Latha and Arivazhagan (2015) have studied the Bayesian Chain sampling plan using Beta prior distribution. Latha and Subbiah (2014) have given Bayesian Multiple Deferred State (BMDS-1) sampling plan with the weighted Poisson distribution. Latha and Rajeswari (2013) have discussed the asymptotic property for Bayesian Chain sampling plan Soundarajan and Vijaraghavan (1990) extended this approach to multiple deferred sampling plan of type MDS-1(0,2) limiting to the acceptance number at 0 and 2. Subramani and Govindaraju (1990) have presented tables of the selection of multiple deferred state MDS – 1 sampling plan for given acceptable and limiting Quality using the Poisson distribution. The Govindaraju and Subramani have found a method of parameters of Multiple Deferred State (MDS) plan of type MDS-1 of Rembert Vaerst (1982) which involves minimum sum of producer's risk and consumer's risks for given AQL and LQL.

This paper deals with designing of Bayesian Multiple Deferred State sampling plan (BMDSSP) (0, 2) with gamma prior distribution and reduced producer and consumer risk. The operating procedure of Bayesian Multiple Deferred State sampling and the derivation of performance measures using gamma prior distribution are given.

II. METHODS AND MATERIAL

MDS - 1(c_1, c_2) Plan

In situations involving costly or destructive testing by attributes, a single sampling plan having acceptance number zero with a small sample size is often employed. The small size is warranted due to the costly nature of testing and a zero acceptance number arises out of a desire to maintain a steep OC curve. But a single sampling plan having a zero acceptance number has the following disadvantages:

(1) A single defect in the sample calls for rejection of the lot.
(2) The OC curves of all such plans have a uniquely poor shape, in that the probability of acceptance starts to drop rapidly for the smaller values of p.

In contrast, single sampling plans having c = 1 or more, as well as double and multiple sampling plans, lack these undesirable characteristics, but require a larger sample size. In such situations, the multiple dependent state sampling plan with acceptance numbers c_1 = 0 and c_2 = 1 MDS – (0, 1) is an appropriate choice to overcome these shortcomings. By the operation of this plan, the OC curve of the single sampling plan with an acceptance number of zero is improved, creating the swell in the upper portion of the OC curve. The result is an improved probability of acceptance for lower values of p.

The MDS – 1 plan is applicable to the case of Type B situations where lots expected to be of the same quality are submitted for inspection seriously in the lot production. MDS – 1 plans are extensions of chain sampling plans of Dodge's (1955) type ChSP – 1. Both the MDS – 1 and chain sampling plans achieve a similar reduction in sample size when compared to the unconditional plans, such as single and double sampling plans. Rembert Vaerst (1980) has developed Multiple Deferred State MDS-1(c_1, c_2) Sampling Plans in which the acceptance or rejection of a lot is based in not only on the results from the current lot but also on sample results of the past or future lots The Operating procedure of the MDS – 1 plan as given by,

Condition for Application of MDS-1(c_1, c_2)

1. Interest centers on an individual quality characteristic that involves destructive or costly tests such that normally only a small number of tests per lot can be justified.
2. The product to be inspected comprises a series of successive lots or batches (or material or of individual units) produced by an essentially continuing process.
3. Under normal conditions the lots are expected to be essentially of the same quality.

4. The product comes from a source in which the consumer has confidence.

Operating Procedure of MDS-1(c_1, c_2)

Step 1: For each lot, Select a Sample of n units and test each unit for conformance to the specified requirements.
Step 2: Accept the lot if d (the observed number of defectives) is less than or equal to c_1; reject the lot if d is greater than c_2.
Step 3: If $c_1 < d \le c_2$, accept the lot provided in each of the samples taken from the preceding or succeeding i lots, the number of defectives found is less than or equal to c_1; otherwise reject the lot.
The OC function of MDS-1(c_1, c_2) is given by,

$$P_a(p) = P_a(p,n,c_1) + [P_a(p,n,c_2) - P_a(p,n,c_1)][P_a(p,n,c_1]^m$$

Rembert Vaerst has presented certain tables giving minimum MDS-1(c_1,c_2) plans indexed by AQL and LQL and observes the following properties.

1. MDS-1(c_1,c_2) Plans are natural extension of ChSP-1 Plans of Dodge (1955).
2. MDS-1 (c_1,c_2) plans allows significant reduction in sample size as compared to single sampling plans.
3. The use of acceptance number c2 increases the chances of acceptance in the region of principal interest. Where the product percent defective is very low.
4. When m = 0, the plan becomes a single sampling plan with sample size n, and acceptance number c_2.
5. When m =∞, the plan becomes a single sampling plan with sample size n, and acceptance number c_1.

Bayesian Average Probability of Acceptance

The Poisson Model of the OC function of MDS-1(c_1,c_2) plan is given by

$$P_a(p) = \sum_{r_1=0}^{c_1} \frac{e^{-x}x^{r_1}}{r_1!} + \left(\sum_{r_2=0}^{c_2} \frac{e^{-x}x^{r_2}}{r_2!} - \sum_{r_1=0}^{c_1} \frac{e^{-x}x^{r_1}}{r_1!}\right)\left(\sum_{r_1=0}^{c_1} \frac{e^{-x}x^{r_1}}{r_1!}\right)^m \quad (1)$$

for x= np, from the past history it is observed that the process average p follows Gamma prior distribution with the parameter (s, t) and density function,

$$w(p) = \frac{e^{-pt}p^{s-1}t^s}{\Gamma s}\quad , \qquad 0 < p < 1,\; s,t > 0, q = 1-p \qquad (2)$$

Where $\mu = \frac{s}{t}$, Under the proposed Multiple Deferred State Sampling Plan, the Probability of Acceptance of Multiple Deferred State Sampling Plan of type MDS-1(c_1, c_2) plan based on the Gamma Poisson Distribution is given by,

$$\bar{P} = \int_0^\infty P_a(p)w(p)dp$$

$$\bar{P} = \int_0^\infty \sum_{r_1=0}^{c_1} \frac{e^{-x}x^{r_1}}{r_1!} + \left(\sum_{r_2=0}^{c_2} \frac{e^{-x}x^{r_2}}{r_2!} - \sum_{r_1=0}^{c_1} \frac{e^{-x}x^{r_1}}{r_1!}\right)\left(\sum_{r_1=0}^{c_1} \frac{e^{-x}x^{r_1}}{r_1!}\right)^m \left(\frac{e^{-pt}p^{s-1}t^s}{\Gamma s}\right) dp$$

$$\bar{P} \int_0^\infty \frac{e^{-x}e^{-pt}p^{s-1}t^s}{\Gamma s} dp + \int_0^\infty \frac{x\, e^{-(1+m)x}e^{-pt}p^{s-1}t^s}{\Gamma s} dp + \frac{1}{2}\int_0^\infty \frac{x^2 e^{(1+m)x}e^{-pt}p^{s-1}t^s}{\Gamma s} dp$$

When c_1=0, c_2=2,

$$\bar{P} = \frac{(s)^s}{(s+n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s+(m+1)n\mu)^{s+1}} + \frac{n^2\mu^2(s+1)\,s^{s+1}}{2(s+(m+1)n\mu)^{s+2}} \qquad (3)$$

Table 1 is constructed giving the value of μ for given $\bar{P}$, s and m. The above equation is mixed distribution of Gamma Poisson distribution.

Selecting a plan when the sample size is fixed

Table 2 and 3 are used to select a Bayesian MDS using Poisson distribution for the given AQL (μ_1) and LQL

(μ_2) which involves minimum sum of risks. Suppose that ν_1 and ν_2 are the weights considered such that $\nu_1 + \nu_2 = 1$, then $\nu_1 \alpha + \nu_2 \beta$ can be minimized for obtaining the parameters of the required plan. Instead of minimizing $\nu_1\alpha + \nu_2\beta$ the expression $\alpha + \nu\beta$ can be minimized, where $\nu = \frac{\nu_2}{\nu_1}$ is the index of relative importance given to the consumer's risk in comparison with the producer's risk. When $\nu > 1$, the plan obtained will be more favourable to the consumer compared to the equal weights plan. When $\nu < 1$, it will be more favourable to the producer than the equal weights plan.

Fixed Sample Size

The Acceptance Quality Level (AQL) and Limiting Quality Level (LQL) corresponding to APA curve are referred as $\mu_1$andμ_2, respectively. The AQL and LQL are usual quality levels in OC curve corresponding to the probability acceptance $1 - \alpha = 0.95$ and $\beta = 0.10$, respectively. When sample size n is fixed the minimum value of expression is obtained. Let $n\mu_{1=\lambda_1}$and$n\mu_{2=\lambda_2}$.

Minimizing $\alpha + \nu\,\beta = \overline{P_{\lambda_1}}(R) + \overline{P_{\lambda_2}}(A)$is equivalent to minimizing$\nu\overline{P_{\lambda_2}}(R) - \overline{P_{\lambda_1}}(A)$. (4)

When λ_1 and $\lambda_2$are given, expression (9) is considered as a function of i and minimized. The resulting value of m is obtained as

$$m = \nu\left(\frac{\lambda_2}{\lambda_1}\right)^2\left[\frac{s+(m+1)\lambda_1}{s(m+1)\lambda_2}\right]^{s+2} - \left[\frac{s+(m+1+\frac{s+2}{2})\lambda_1}{s(m+1+\frac{s+2}{2})\lambda_2}\right] \quad (5)$$

Example

It is given that, nAQL = 0.03 and nLQL = 4 and s=1, from this tables, it is obtained that the value of m with ν =0.05, 1, 1.5 and 2are 8,10,15,22 respectively. It is also obtained that for nAQL=0.01 and nLQL = 4 and ν =0.05 the value of m for s=1 is 182, for s=5, m = 1 for s=9, m =0.

Construction of Tables

The Poisson Model of the OC function of MDS-1(c_1, c_2) plan is given by

$$\bar{P} = \int_0^\infty \sum_{r_1=0}^{c_1}\frac{e^{-x}x^{r_1}}{r_1!} + \left(\sum_{r_2=0}^{c_2}\frac{e^{-x}x^{r_2}}{r_2!} - \sum_{r_1=0}^{c_1}\frac{e^{-x}x^{r_1}}{r_1!}\right)\left(\sum_{r_1=0}^{c_1}\frac{e^{-x}x^{r_1}}{r_1!}\right)^m\left(\frac{e^{-pt}p^{s-1}t^s}{\Gamma s}\right)\,dp \quad (6)$$

for x= np, from the past history it is observed that the process average p follows Gamma prior distribution with the parameter (s, t) and density function,

$$w(p) = \frac{e^{-pt}p^{s-1}t^s}{\Gamma s}, \quad 0 < p < 1,\ s,t > 0, q = 1 - p \quad (7)$$

The probability of acceptance of Multiple Deferred State Sampling Plan of type MDS-1(c_1 ,c_2) plan based on the Gamma Poisson Distribution is given by,

$$\bar{P} = \frac{(s)^s}{(s+n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s+(m+1)n\mu)^{s+1}} + \frac{n^2\mu^2(s+1)\,s^{s+1}}{2(s+(m+1)n\mu)^{s+2}} \quad (8)$$

Table 1 is constructed giving the value of μ for given$\bar{P}$, s and m. The above equation is mixed distribution of Gamma Poisson distribution.

Table 1: value of i Minimizing (α+ 0.5β) with s=5, for [BMDS (0,2)]

$\mu_1\rightarrow$ / $\downarrow \mu_2$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.15	0.2	0.25	0.3	0.35
1.00	4	3	2	2	2	2	2	1	1	1	1	1	0	0	0
1.25	3	2	2	2	1	1	1	1	0	0	-	-	-	-	-
1.50	3	2	2	1	1	1	1	1	1	1	0	-	-	-	-
1.75	2	2	1	1	1	1	1	1	0	-	-	-	-	-	-
2.00	2	1	1	1	1	1	1	-	-	-	-	-	-	-	-
2.25	2	1	1	1	1	1	0	-	-	-	-	-	-	-	-
2.50	2	1	1	1	1	1	0	-	-	-	-	-	-	-	-
2.75	1	1	1	1	1	0	-	-	-	-	-	-	-	-	-
3.00	1	1	1	1	0	0	-	-	-	-	-	-	-	-	-
3.25	1	1	1	0	0	0	-	-	-	-	-	-	-	-	-
3.50	1	1	1	0	0	-	-	-	-	-	-	-	-	-	-
3.75	1	1	0	0	-	-	-	-	-	-	-	-	-	-	-

4.00	1	1	0	0	-	-	-	-	-	-	-	-	-	-	-
4.25	1	1	0	-	-	-	-	-	-	-	-	-	-	-	-
4.50	1	0	0	-	-	-	-	-	-	-	-	-	-	-	-
4.75	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-
5.00	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-
5.25	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-
5.50	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-
5.75	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-
6.00	1	0	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 2 : Value of i Minimizing (α+ 2β) with s=5, for [BMDS (0, 2)]

$\mu_{1\rightarrow}$ / $\downarrow \mu_2$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.15	0.2	0.25	0.3	0.35
1.00	5	4	3	3	3	2	2	2	2	2	2	1	1	1	1
1.25	4	3	3	2	2	2	2	2	2	2	1	1	1	1	1
1.50	3	3	2	2	2	2	1	1	1	1	1	1	1	1	1
1.75	3	2	2	2	1	1	1	1	1	1	1	1	1	0	0
2.00	3	2	2	1	1	1	1	1	1	1	1	1	0	0	0
2.25	2	2	1	1	1	1	1	1	1	1	1	0	0	0	0
2.50	2	2	1	1	1	1	1	1	1	1	0	0	0	-	-
2.75	2	1	1	1	1	1	1	1	1	1	0	0	-	-	-
3.00	2	1	1	1	1	1	1	1	0	0	0	-	-	-	-
3.25	2	1	1	1	1	1	1	0	0	0	-	-	-	-	-
3.50	2	1	1	1	1	1	0	0	0	0	-	-	-	-	-
3.75	2	1	1	1	1	0	0	0	0	0	-	-	-	-	-
4.00	1	1	1	1	0	0	0	0	-	-	-	-	-	-	-
4.25	1	1	1	1	0	0	0	-	-	-	-	-	-	-	-
4.50	1	1	1	0	0	0	-	-	-	-	-	-	-	-	-
4.75	1	1	1	0	0	0	-	-	-	-	-	-	-	-	-
5.00	1	1	1	0	0	-	-	-	-	-	-	-	-	-	-
5.25	1	1	0	0	-	-	-	-	-	-	-	-	-	-	-
5.50	1	1	0	0	-	-	-	-	-	-	-	-	-	-	-
5.75	1	1	0	0	-	-	-	-	-	-	-	-	-	-	-
6.00	1	1	0	-	-	-	-	-	-	-	-	-	-	-	-

When sample size n is fixed the minimum value of expression is obtained. Let $n\mu_1 = \lambda_1$ and$n\mu_2 = \lambda_2$.

$$\text{Minimizing } \alpha + \nu\, \beta = \overline{P_{\lambda_1}}(R) + \overline{P_{\lambda_2}}(A)$$

$$\text{Minimizing} \nu\overline{P_{\lambda_2}}(R) - \overline{P_{\lambda_1}}(A) \quad (9)$$

When λ_1 and $\lambda_2$are given, expression (9) is considered as a function of m and minimized. The resulting value of m is obtained as

$$m = \nu \left(\frac{\lambda_2}{\lambda_1}\right)^2 \left[\frac{s+(m+1)\lambda_1}{s(m+1)\lambda_2}\right]^{s+2} - \left[\frac{s+(m+1+\frac{s+2}{2})\lambda_1}{s(m+1+\frac{s+2}{2})\lambda_2}\right] \quad (10)$$

It is observed that for fixed value of‘s’ the value of ‘ m' increases when ν increases for given $\lambda_1 and\ \lambda_2$. Also for given $\lambda_1 and\ \lambda_2$ and ν the value of m decreases as the value of‘s’ increases and similar to conventional plan for large values of‘s’.

III. REFERENCES

[1]. Calvin, T.W.(1984), How and When to Perform Bayesian Acceptance Sampling, Vol.7, American Society for Quality Control, Milwaukee, Wisconsin, USA

[2]. Dodge. H.F (1955), Chain Sampling Inspection Plan, Industrial Quality Control,11, pp 10-13.

[3]. Golub. A (1953), Designing single sampling inspection plans when the sample is fixed, Journal

of the American Statistical Association, 48, pp. 278-288.

[4]. Govindaraju K and Subramani K(1990), Selection of Multiple Deferred State MDS-1 Sampling Plan for Given Acceptable and Limiting Quality Levels Involving minimum Risks, Journal of Applied Statistics, Vol. 17, No.3, pp. 427-434.

[5]. Govindaraju K and Subramani K(1993), Selection of Multiple Deferred (dependent) State Sampling Plan for Given Acceptable quality level and Limiting Quality Level, Journal of Applied Statistics, Vol. 20, No.3, pp. 423-428.

[6]. Hald A. (1981) Statistical Theory of Sampling Inspection by Attributes (Newyork, Academic Press).

[7]. Latha, M. and Arivagazahan R, S.(2015). Bayesian Chain Sampling Plan Using Binomial Distribution, International Journal of Emerging Trends in Engineering and Development, Vol.3, Issue 5.

[8]. Latha, M. and Jeyabharathi, S.(2012). Selection of Bayesian Chain Sampling Attributes Plans based on Geometric Distribution, International Journal of Scientific and Research Publication, Vol.2, Issue 7.

[9]. Latha, M. and Rajeswari, M.(2013). Asymptotic property for Bayesian Chain Sampling Plan-1, International Journal of Advanced Scientific and Technical Research, Vol 4, issue 3.

[10]. Latha, M. and Subbiah K (2014). Bayesian Multiple Deferred State (BMDS-1) Sampling Plan with weighted Poisson distribution, International Journal of Emerging Trends in Engineering and Development, Vol.3, issue 4, pp. 275- 282.

[11]. Lauer N.G(1978), Acceptance Probabilities for Sampling Plans when the proportion Defective has a Beta Distribution, Journal of Quality Technology, Vol. 10, No. 2, pp. 12-15.

[12]. Schafer,R.,E. (1967), Bayesian Single Sampling Plans by Attributes Based on the Posterior Risks, Navel Research Logistics Quarterly Vol.14, (A) No.1, pp. 81-88.

[13]. Soundararajan, V (1981), Sampling inspection plans when the sample is fixed, Journal of Madras University, Sect. B, 44, pp, 9-10.

[14]. Soundararajan, V and Govindaraju, K (1983), A note on designing single sampling plans, IAPQR Transactions, 18, pp. 37-41.

[15]. Soundararajan, V and Vijayaraghavan, R (1989), On designing multiple deferred state sampling (MDS-1(0,2)) plans involving minimum risks, Journal of Applied Statistics, Vol. 16, No.1, pp. 87-94.

[16]. Soundararajan, V and Vijayaraghavan, R (1990), Construction and Selection of Multiple Dependent (deferred) State Sampling Plan, Journal of Applied Statistics, Vol. 17, No.3, pp. 397-409.

[17]. Suresh, K.K. and Latha, M.(2001). Bayesian Single Sampling Plans for a Gamma Prior, Economic Quality Control, Vol.16,No.1, pp.93-107.

[18]. Suresh, K.K. and Latha, M (2001), Procedures and Tables for selection of Bayesian Single Sampling Plans with weighted risks, Far East Journal of Theoretical Statistics, Vol.5, No.2, pp. 245-252.

[19]. Vaerst, R.(1982): A Procedure to Construct Multiple Deferred State Sampling Plan, Method of Operation Research, Vol.37, pp.477-485.

[20]. Wetherill, G.B and Chiu, W. K (1975). A Review of Acceptance sampling Schemes with emphasis on the economic aspect, International Statistical Review, 43, pp. 191-210.

[21]. Wortham, A.W and Baker, R.C (1976), Multiple Deferred State Sampling Inspection, The International Journal of Production Research, Vol.14, No.6, 719-731.