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Ranking of Heptagonal Fuzzy Number Method to Solve Balanced Zero Assignment Problem

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ABSTRACT

In this paper we consider the balanced zero assignment problem is formulated to the crisp assignment problem in the linear programming form and solved by Hungarian method and using Robust ranking method for the heptagonal fuzzy numbers. The numerical example is shown to find optimal cost and fuzzy optimal cost.

Keywords : Fuzzy number**,** Robust ranking technique, Membership function, Fuzzy assignment problem, Optimal cost.

I. INTRODUCTION

The Fuzzy Assignment problem is a special type of fuzzy linear programming problem and it is a subclass of fuzzy transportation problem. The Fuzzy Assignment problem can be stated as follows: Let n number of jobs is performed by number of persons, where the costs depend on the specific assignments. Each job must be assigned to one and only one worker and each worker has to perform one and only one job. The problem is to find such an assignment so that the total cost is optimized. The fuzzy assignment problem can be applied to nxn fuzzy cost matrix (C_{ij}), where C_{ij} represents the fuzzy cost associated with worker (i=1,2,3….n) who has performed job (j=1,2,3,…n). The fuzzy assignment problem when costs are fuzzy numbers can also be modeled as 0-1 integer programming problem.

II. PRELIMINARIES

2.1 Fuzzy number

A Fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

i) There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$
ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.
iii) $\tilde{A}$ Must be normal and convex.

2.2 Fuzzy set:

A fuzzy set A on a set x is characterized by a mapping m: x → [0, 1] called the membership function. A fuzzy set is denoted as A=(x, m).

2.3 Assignment Problem:

Assignment problem is a special case of the transportation problem in which the number of square and destination are the same and the objective is to assign the given job to most appropriate person so as to optimize the objective function like minimize cost.

2.4 Balanced Assignment Problem:

When the number of rows equals to the number of columns.

Number of rows = Number of columns.

2.5 Row Reduction:

Row reduction subtracts the minimum cost of each row of the cost matrix from all the element of the respective row of the resulting matrix.

2.6 Column Reduction:

Column reduction subtracts the minimum cost of each column of the cost matrix from all the element of the respective column of the resulting matrix.

2.7 Robust ranking technique

Robust ranking technique which satisfy compensation, linearity, and additively Properties and provides results which are consist human intuition. If ã is a fuzzy number then the Robust Ranking is defined by,

R (ã) = $\int_0^1 0.5(\boldsymbol{a_\alpha}^{\boldsymbol{L}}, \boldsymbol{a_\alpha}^{\boldsymbol{U}})$, *where* $(a_h{}^L, {}_h{}^U)$ is a α-level cut of a fuzzy number ã.

Where, (a_α^L, a_α^U) **= {(b-a) α + a, d- (d-c) α} + {(d-c) α +c, e- (e-d) α} +{(f-e) α +e, g- (g-f) α}**

In this paper we use this method for ranking the objective values. The Robust ranking index

R (ã) gives the representative value of the fuzzy number ã.

2.8 α-cut of a fuzzy number:

The α-cut of a fuzzy number A(x) is defined as A(α) = { x ; $\mu(x) \geq \alpha;\ \alpha \in [0,1]$}

2.9 Mathematical formulation of Fuzzy Assignment Problem:

Mathematically, the fuzzy assignment problem is ,

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\ x_{ij}$$

subject to the constraints;

$\sum_{i=1}^{n} x_{ij} = 1$; i= 1,2…….m

$\sum_{j=1}^{m} x_{ij} = 1$; j= 1,2,…….n

$$x_{ij} = \begin{cases} 1, if\ \ i^{th}\ person\ is\ assigned\ j^{th}\ work \\ 0 \qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

where x_{ij} denotes that j^{th} work is to be assigned to the i^{th} person.

2.10 Mathematical formulation of the Assignment Problem:

Consider a problem of assignment of n resources (persons) to n-activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (C_{ij}) is given as follows

Activities						Available
		A_1	A_2	A_3	. . . . . A_n	1
	R_1	C_{11}	C_{12}	C_{13}	. . . . . C_{1n}	1
	R_2	C_{21}	C_{22}	C_{23}	. . . . . C_{2n}	1
Resource	R_3	C_{31}	C_{32}	C_{33}	. . . . . C_{3n}	.
	.	.	.	.	. . . . . .	.
	.	.	.	.	. . . . . .	.
	.	.	.	.	. . . . . .	.
	.	.	.	.	. . . . . .	.
	R_n	C_{n1}	C_{n2}	C_{n3}	. . . . . C_{nn}	1
Required		1	1	1	 1	

This cost matrix is same as that of a transportation problem except that availability at each of the resources and the requirement at each of the destination is unity (due to the fact that assignments are made on a one –to-one basis)

Let x_{ij} denotes the assignment of i^{th} resource to j^{th} activity, such that

$$X_{ij} = \begin{cases} 1, if\ resource\ i^{th}\ is\ assigned\ \ to\ activity\ j^{th} \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

Then the Mathematical formulation of the Assignment Problem is

Minimize $Z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\ x_{ij}$ subject to the constraints;
$\sum_{i=1}^{n} x_{ij} = 1$; i= 1,2…….m

$\sum_{j=1}^{m} x_{ij} = 1$; j= 1,2,…….n

III. HEPTAGONAL FUZZY NUMBERS

The fuzzy number D is a heptagonal fuzzy. A_h is a heptagonal fuzzy number denoted A_h (a, b, c, d, e, f, g;1) and its membership function $\mu.A_h(x)$ is given below:

$$
\mu.A_h(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ \frac{(x-b)}{(c-b)} & b \le x \le c \\ \frac{(x-c)}{(d-c)} & c \le x \le d \\ 1 & x = d \\ \frac{(d-x)}{(d-e)} & d \le x \le e \\ \frac{(e-x)}{(e-f)} & e \le x \le f \\ \frac{(f-x)}{(f-g)} & f \le x \le g \\ 0 & otherwise \end{cases}
$$

IV. NUMERICAL EXAMPLE

Let us consider a fuzzy unbalanced assignment problem with rows representing 4 area A_1, A_2, A_3, A_4 and columns representing the salesman's B_1, B_2, B_3, B_4. The cost matrix ($\widetilde{C_{IJ}}$) is given whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

$$
\widetilde{C_{IJ}} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{pmatrix} (0,2,4,6,8,10,12) & (1,3,5,7,9,11,13) & (-2,0,2,4,6,8,10) & (4,6,8,10,12,14,16) \\ (3,5,7,9,11,13,15) & (2,4,6,8,10,12,14) & (9,11,13,15,17,19,21) & (5,7,9,11,13,15,17) \\ (6,8,10,12,14,16,18) & (7,9,11,13,15,17,19) & (1,3,5,7,9,11,13) & (0,2,4,6,8,10,12) \\ (12,14,16,18,20,22,24) & (-1,1,3,5,7,9,11) & (3,5,7,9,11,13,15) & (4,6,8,10,12,14.16) \end{pmatrix} \end{array}
$$

Solution:

The fuzzy balanced assignment problem can be formulated in the following mathematical programming form.

Min{R $(0,2,4,6,8,10,12)$ x_{11} + R $(1,3,5,7,9,11,13)$ x_{12} + R $(-2,0,2,4,6,8,10)$ x_{13} + R $(4,6,8,10,12,14,16)$ x_{14} + R $(3,5,7,9,11,13,15)$ x_{21} + R $(2,4,6,8,10,12,14)$ x_{22} + R $(9,11,13,15,17,19,21)$ x_{23} + R $(5,7,9,11,13,15,17)$ x_{24} + R $(6,8,10,12,14,16,18)$ x_{31} + R $(7,9,11,13,15,17,19)$ x_{32} + R $(1,3,5,7,9,11,13)$ x_{33} + R $(0,2,4,6,8,10,12)$ x_{34} + R $(12,14,16,18,20,22,24)$ x_{41} + R $(-1,1,3,5,7,9,11)$ x_{42} + R $(3,5,7,9,11,13,15)$ x_{43} + R $(4,6,8,10,12,14.16)$ x_{44} }

Now, we conclude R $(0,2,4,6,8,10,12)$ by applying Robust's Ranking method.

$$
R(\tilde{A}) = \int_0^1 0.5\,(a_\alpha^L, a_\alpha^U)\,d\alpha
$$

Where, (a_α^L, a_α^U) = {(b-a) α + a, d- (d-c) α} + {(d-c) α +c, e- (e-d) α} +{(f-e) α +e, g- (g-f) α}

$\int_0^1 0.5\,(a_\alpha^L, a_\alpha^U)\,d\alpha = \int_0^1 0.5$ {(b-a) α + a, d- (d-c) α} + {(d-c) α +c, e- (e-d) α} +{(f-e) α +e, g-

(g-f) α}. $d\alpha$

R(4,6,8,10,12) = $\int_0^1 0.5$ {(2-0) α + 0, 6- (6-4) α} + {(6-4) α +4, 8- (8-6) α} +{(10-8) α +8, 12-

(12-10) α}. $d\alpha$

$$
= \int_0^1 0.5\,(6+12+20)\,d\alpha
$$

$$
= \int_0^1 0.5\,(38)\,d\alpha
$$

$$
= 19
$$

Similarly,

R $(1,3,5,7,9,11,13)$ =22, R $(-2,0,2,4,6,8,10)$ =13, R $(4,6,8,10,12,14,16)$ =31,

R $(3,5,7,9,11,13,15)$ =28, R $(2,4,6,8,10,12,14)$ =25, R $(9,11,13,15,17,19,21)$ =46

R $(5,7,9,11,13,15,17)$ =34, R $(6,8,10,12,14,16,18)$ =37, R $(7,9,11,13,15,17,19)$ =40

R $(1,3,5,7,9,11,13)$ =22, R $(0,2,4,6,8,10,12)$ =19, R $(12,14,16,18,20,22,24)$ =55

R $(-1,1,3,5,7,9,11)$ =16, R $(3,5,7,9,11,13,15)$ =28, R $(4,6,8,10,12,14.16)$ =31

We replace these values for the corresponding $\widetilde{C_{IJ}}$ which results in assignment problem in the linear programming problem.

We solve it by Hungarian method to get the following optimal solution.

$$
(\widetilde{C_{IJ}}) = \begin{pmatrix} 19 & 22 & 13 & 31 \\ 28 & 25 & 46 & 34 \\ 37 & 40 & 22 & 19 \\ 55 & 16 & 28 & 31 \end{pmatrix}
$$

Row Reduction:

$$
(\widetilde{C_{IJ}}) = \begin{pmatrix} 6 & 9 & 0 & 18 \\ 3 & 0 & 21 & 9 \\ 18 & 21 & 3 & 0 \\ 39 & 0 & 12 & 15 \end{pmatrix}
$$

Column Reduction:

$$(\widetilde{C_{ij}}) = \begin{pmatrix} 3 & 9 & 0 & 18 \\ 0 & 0 & 21 & 9 \\ 15 & 21 & 3 & 0 \\ 36 & 0 & 12 & 15 \end{pmatrix}$$

By using Hungarian Assignment Method:

$$(\widetilde{C_{ij}}) = \begin{pmatrix} 3 & 9 & (0) & 18 \\ (0) & \cancel{0} & 21 & 9 \\ 15 & 21 & 3 & (0) \\ 36 & (0) & 12 & 15 \end{pmatrix}$$

The optimal Assignment

$A_1 \to B_3$, $A_2 \to B_1$, $A_3 \to B_4$, $A_4 \to B_2$

The optimal total minimum cost = Rs

13+28+19+16 = Rs 76

The fuzzy optimal Assignment

$A_1 \to B_3$, $A_2 \to B_1$, $A_3 \to B_4$, $A_4 \to B_2$

The fuzzy optimal total minimum cost

$= \widetilde{C_{13}} + \widetilde{C_{21}} + \widetilde{C_{34}} + \widetilde{C_{42}}$

$= R(-2,0,2,4,6,8,10) + R(3,5,7,9,11,13,15) +$

$R(0,2,4,6,8,10,12) + R(-1,1,3,5,7,9,11)$

$= R(0,8,16,24,32,40,48)$

$= Rs.76$

V. CONCLUSION

In this paper ,the balanced assignment cost are consider as imprecise number described by fuzzy numbers more over the balanced assignment problem has been transformed in to crisp assignment problem using Robust ranking technique, so that numerical example by using method we can have the optimal assignment as well as the crisp and the fuzzy optimal total cost.

VI. REFERENCES

[1]. Kadhirvel. K, Balamurugan.k, Method for solving unbalanced Assignment Problems Using Triangular Fuzzy Numbers Vol. 3, Issue 5, Sep-Oct 2013, pp.359-363.

[2]. Sangeetha.K, HaseenaBegum.H, Pavithra.M, Ranking Of Triangular Fuzzy Number Method To Solve An Unbalanced Assignment Problem Vol2, No. 8, August 2014.

[3]. Er. Prem Kumar Gupta & Dr. D.S. Hira, 2014. Operation Research: Seventh Revised Edition ISBN: 81-219-0281-9.

[4]. Malini.P, Ananthanarayanan.M, Solving Fuzzy Transportation Problem Using Ranking Of Octagonal Fuzzy Numbers Vol 110, No.2 2016, 275-282.

[5]. Dr.IsmailMohideen.S, PrasannaDevi.K, Devi Durga.M, Fuzzy Transportation Problem Of Octagon Fuzzy Numbers With α-Cut And Ranking Technique Vol. 1, Issue. 2, July-2016, Pg. 60-67.

[6]. S. Chandrasekaran, "Ranking of heptagon number using zero suffix method," International Journal: Science and Research, vol. 4, pp. 2319-7064, 2013.

[7]. R. J. Timothy, Fuzzy logic with engineering applications. New Mexico, USA: John Wiley & Sons, 2004.

[8]. S. Poonam, "Fuzzy transportation problem of triangular numbers with alpha cut and ranking technique," IOSR Journal: Engineering, vol. 2, pp. 1162-1164, 2012b.