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$M/\begin{pmatrix} G_1 \\ G_2 \end{pmatrix}/1$ Retrial Queue with Negative Arrivals and Bernoulli Feedback

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ABSTRACT

Single server retrial G-queue with two fluctuating modes of service and feedback is analyzed. If the server is idle upon the arrival of a customer, then the customer receives either of twoservices. Otherwise, he joins the orbit. After completion of service, unsatisfied customer may join the orbit as a feedback customer or depart from the system. Arrival of a negative customer makes the server to failure state and removes the positive customer being in service from the system. Performance analysis are obtained using supplementary variable technique. Stochastic decomposition law is verified. Finally, numerical illustrations are provided.
Keywords: Retrial queue -negative customers – bernoulli feedback.

I. INTRODUCTION

The phenomenon of feedback in the retrial queueing system occurs in many practical situations. Choi and Kulkarni [1] studied an M/G/1 feedback retrial queue. Many authors including Krishnakumar et al. [3,4,5], Mokaddis et al. [7] and Lee and Jang [8] analysed retrial queueing systems with feedback. Ramanath and Lakshmi [9] studied M/G/1 retrial queue with second multi-optional service and feedback.

Queue with negative arrivals called G-queue was first introduced by Gelenbe [2] with a view to modeling neural networks. In recent years, a variety of industrial applications have created interest in the modeling of reliability in G-queues. Liu et al. [6] analysed an M/G/1 G-queue with pre-emptive resume and feedback under N-policy vacation. Wu and Lian [10] discussed an M/G/1 retrial G-queue with priority resume, Bernoulli vacation and server breakdown. In this paper, we analyzed batch arrival retrial G-queue with two fluctuating modes of service and feedback.

II. METHODS AND MATERIAL

A. Model Description

Consider a single server retrial queueing system with two types of independent arrivals, positive and negative. Positive customers arrive according to Poisson process with rate λ^+. The server provides two types of service- type 1 and type 2. Customers choose type 1 service with probability p1 or type 2 service with probability p2 (p1+p2=1). If the positive customer finds the server free, then the customer receives any one of the two services immediately. Otherwise he joins the retrial queue.The retrial time is generally distributed with distribution function A(x), density function a(x), Laplace-Stieltje's transform A*(θ) and conditional completion rate η(x)=a(x)/[1-A(x)].

The service time of type i(i=1,2) follows a general distribution with distribution function Bi(x), density function bi(x), Laplace stieltje's transform Bi*(θ), nth factorial moments μin and conditional completion rate μi(x) = bi(x)/[1-Bi(x)], for (i=1,2). After receiving service, the customer may again join the orbit as a feedback withcondition probability δ or depart the system with its complementary probability 1-δ.

Negative customers arrive according to Poisson process with rate λ^-. The arrival of a negative customer removes the positive customer being in service from the system and makes the server breakdown. When the server fails, it stops providing service and is sent for repair immediately. The repair time also follows a general distribution with distribution function R(x), density function r(x), Laplace stieltje's transform R*(θ), nth factorial moments rn and conditional completion rate β(x) = r(x)/[1-R(x)]. Various stochastic process involved in the system are independent of each other.

B. Analysis of the Steady State Distribution

The state of the system at time t can be described by the Markov process $\{X(t); t \geq 0\} = \{C(t), N(t), \xi(t); t \geq 0\}$, where C(t) denotes the server state 0,1,2 or 3 according as the server being idle, busy in type-1 service, busy in type -2 service or under repair. N(t) corresponds to the number of customers in the orbit . If C(t)=0, then ξ(t) represents the elapsed retrial time. If C(t) =1 or 2, then ξ(t) represents the elapsed service time. If C(t)=3, then ξ(t) represents the elapsed repair time of the failed server.

For the process { C(t) ; t≥0}, define the probabilities

$I_0(t) = P\{C(t)=0, N(t) = 0\}$

$I_n(x,t)dx = P\{C(t)=0, N(t) = n,\ x < \xi(t) \leq x+dx\},\ n \geq 1$

$P_n^{(i)}(x,t)dx = P\{C(t)=i, N(t) = n,\ x < \xi(t) \leq x+dx\},\ n \geq 0, i=1,2$

$R_n(x,t)dx = P\{C(t)=3, N(t)= n,\ x < \xi(t) \leq x+dx\},\ n \geq 0$

The steady state equations governing the model under consideration are

$$\lambda^+ I_0 = \bar{\delta}\left[\int_0^\infty P_0^{(1)}(x)\mu_1(x)dx + \int_0^\infty P_0^{(2)}(x)\mu_2(x)dx\right] + \int_0^\infty R_0(x)\beta(x)dx \tag{1}$$

$$\frac{d}{dx} I_n(x) = -(\lambda^+ + \eta(x)) I_n(x), n \geq 1 \tag{2}$$

$$\frac{d}{dx} P_n^{(i)}(x) = -(\lambda^+ + \lambda^- + \mu_i(x)) P_n^{(i)}(x) + (1-\delta_{0n})\lambda^+ P_{n-1}^{(i)}(x), \quad n \geq 0, i = 1,2 \tag{3}$$

$$\frac{d}{dx} R_n(x) = -(\lambda^+ + \beta(x)) R_n(x) + (1-\delta_{0n})\lambda^+ R_{n-1}(x), n \geq 0 \tag{4}$$

with boundary conditions

$$I_n(0) = \delta\left[\int_0^\infty P_{n-1}^{(1)}(x)\mu_1(x)dx + \int_0^\infty P_{n-1}^{(2)}(x)\mu_2(x)dx\right] + \bar{\delta}\left[\int_0^\infty P_n^{(1)}(x)\mu_1(x)dx + \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx\right], n \geq 1 \tag{5}$$

$$P_0^{(1)}(0) = p_1\left[\lambda^+ I_0 + \int_0^\infty I_1(x)\eta(x)dx\right] \tag{6}$$

$$P_n^{(1)}(0) = p_1\left[\int_0^\infty I_{n+1}(x)\eta(x)dx + \lambda^+ \int_0^\infty I_n(x)dx\right], n \geq 1 \tag{7}$$

$$P_0^{(2)}(0) = p_2\left[\lambda^+ I_0 + \int_0^\infty I_1(x)\eta(x)dx\right] \tag{8}$$

$$P_n^{(2)}(0) = p_2\left[\int_0^\infty I_{n+1}(x)\eta(x)dx + \lambda^+ \int_0^\infty I_n(x)dx\right], n \ge 1 \tag{9}$$

$$R_n(0) = \lambda^-\left[\int_0^\infty P_n^{(1)}(x)dx + \int_0^\infty P_n^{(2)}(x)dx\right], n \ge 0 \tag{10}$$

To solve the above equations, define the probability generating functions

$I(x,z) = \sum_{n=1}^{\infty} I_n(x)z^n$, $P^{(i)}(x,z) = \sum_{n=0}^{\infty} P_n^{(i)}(x)z^n$,i=1,2 and $R(x,z) = \sum_{n=0}^{\infty} R_n(x)z^n$.

Multiplying equations (2) by z^n and summing over all possible values of n, we get

$$\left[\frac{d}{dx} + (\lambda^+ + \eta(x))\right]I(x,z) = 0 \tag{11}$$

Solving the partial differential equation (11) , we obtain

$$I(x,z) = I(0,z)e^{-\lambda^+ x}[1 - A(x)] \tag{12}$$

Solving the differential equations obtained from equation (3) and (4), we get

$$P^{(i)}(x,z) = P^{(i)}(0,z)e^{-(\lambda^+ + \lambda^- - \lambda^+ z)x}[1 - B_i(x)], \text{ for } \quad i = 1,2 \tag{13}$$

$$R(x,z) = R(0,z)e^{-(\lambda^+ - \lambda^+ z)x}[1 - R(x)] \tag{14}$$

Equations (5) to (10) yield

$$I(0,z) = \delta z\left[\int_0^\infty P^1(x,z)\mu_1(x)dx + \int_0^\infty P^2(x,z)\mu_2(x)dx\right] + \bar{\delta}\left[\int_0^\infty P^1(x,z)\mu_1(x)dx + \int_0^\infty P^2(x,z)\mu_2(x)dx\right] + \int_0^\infty R(x,z)\beta(x)dx - \lambda^+ I_0 \tag{15}$$

$$P^{(1)}(0,z) = \frac{p_1}{z}\left[\int_0^\infty I(x,z)\eta(x)dx\right] + P_1\lambda^+\left[I(x,z)dx + I_0\right] \tag{16}$$

$$P^{(2)}(0,z) = \frac{p_2}{z}\left[\int_0^\infty I(x,z)\eta(x)dx\right] + P_2\lambda^+\left[I(x,z)dx + I_0\right] \tag{17}$$

$$R(0,z) = \lambda^-\left[\int_0^\infty P^{(1)}(x,z)dx + \int_0^\infty P^{(2)}(x,z)dx\right] \tag{18}$$

Using equation (12) in equations (16) and (17), we have

$$P^{(i)}(0,z) = \frac{p_{(i)}}{z}\left[\lambda^+ zI_0 + I(o,z)[z + (1-z)A^*(\lambda^+)]\right], \text{ i=1,2} \tag{19}$$

Using equation (13) in (18), we obtain

$$R(0,z)=\frac{\lambda^-}{g(z)}\left[P^{(1)}(0,z)(1-B_1*(g(z))+P^{(2)}(0,z)(1-B_2*(g(z))\right] \quad (20)$$

Using equations (13) , (14), (19) and (20) in equation (15) and simplifying, we have

$$I(0,z)=\lambda^+ I_0 z[p_1T_1(z)+p_2T_2(z)-g(z)]\big/D(z) \quad (21)$$

where $T_1(z)=[\delta(z-1)+1]B_1^*(g(z))g(z)+\lambda^-(1-B_1*(g(z)))R*(h(z))$

$T_2(z)=[\delta(z-1)+1]B_2^*(g(z))g(z)+\lambda^-(1-B_2*(g(z)))R*(h(z))$

$D(z)=zg(z)-[A*(\lambda^+)+z(1-A*(\lambda^+))][p_1T_1(z)+p_2T_2(z)]$

h(z) $=\lambda^+-\lambda^+z$ and $g(z)=\lambda^++\lambda^--\lambda^+z$

Using equation (21), the equations(19) and (20), yield

$$P^{(1)}(0,z)=\lambda^+ I_0 A^*(\lambda^+)p_1 g(z)[z-1]\big/D(z) \quad (22)$$

$$P^{(2)}(0,z)=\lambda^+ I_0 A^*(\lambda^+)p_2 g(z)[z-1]\big/D(z) \quad (23)$$

$$R(0,z)=\lambda^+\lambda^- I_0 A^*(\lambda^+)(z-1)[1-p_1B_1^*(g(z))-p_2B_2^*(g(z))]\big/D(z) \quad (24)$$

Now the expressions of I(x,z), $P^{(i)}$ (x,z) and R(x,z) become

$$I(x,z)=\lambda^+ I_0 z[p_1T_1(z)+p_2T_2(z)-g(z)]e^{-\lambda^+x}[1-A(x)]\big/D(z) \quad (25)$$

$$P^{(i)}(x,z)=\lambda^+ I_0 A^*(\lambda^+)p_i g(z)[z-1]e^{-g(z)x}[1-B_i(x)\big/D(z)\text{, for i=1,2.} \quad (26)$$

$$R(x,z)=\lambda^+\lambda^- I_0 A^*(\lambda^+)(z-1)[1-p_1B_1^*(g(z))-p_2B_2^*(g(z))]e^{-h(z)x}[1-R(x)]\big/D(z) \quad (27)$$

The partial probability generating functions I(z),$P^{(i)}$(z) and R(z) are derived as

$$I(z)=\int_0^\infty I(x,z)dx=I_0 z[1-A^*(\lambda^+)][p_1T_1(z)+p_2T_2(z)-g(z)]\big/D(z) \quad (28)$$

$$P^{(i)}(z)=\int_0^\infty P^{(i)}(x,z)dx=\lambda^+ I_0 A^*(\lambda^+)p_i(z-1)[1-B_i^*(g(z))]\big/D(z)\text{,i=1,2} \quad (29)$$

$$R(z)=\int_0^\infty R(x,z)dx=-\lambda^- I_0 A^*(\lambda^+)[1-p_1B_1^*(g(z))-p_2B_2^*(g(z))][1-R*(h(z))]\big/D(z) \quad (30)$$

The unknown constant I_0 can be obtained by using the normalizing condition

$I_0+\lim_{z\to1}[I(z)+P^{(1)}(z)+P^{(2)}(z)+R(z)]=1$ as

$$I_0 = \frac{\lambda^-[A^*(\lambda^+) - \delta[p_1B_1^*(\lambda^-) + p_1B_1^*(\lambda^-)]] - \lambda^+(1+\lambda^- r_1)(1 - p_1B_1^*(\lambda^-) - p_1B_1^*(\lambda^-))}{\lambda^- A^*(\lambda^+)[1-\delta(p_1B_1^*(\lambda^-) + p_1B_1^*(\lambda^-))]} \quad (31)$$

Probability that the server is idle in the non-empty system is given by

$$I = \lim_{z\to 1} I(z) = \frac{(1-A^*(\lambda^+))\{\lambda^-\delta[p_1B_1^*(\lambda^-) + p_1B_1^*(\lambda^-)] + \lambda^+(1+\lambda^- r_1)(1 - p_1B_1^*(\lambda^-) - p_1B_1^*(\lambda^-)\}}{\lambda^- A^*(\lambda^+)[1-\delta(p_1B_1^*(\lambda^-) + p_1B_1^*(\lambda^-))]} \quad (32)$$

Probability that the server is busy in type i service is given by

$$P^{(i)} = \lim_{z\to 1} P^{(i)}(z) = \frac{\lambda^+ p_i(1-B_i^*(\lambda^-))}{\lambda^-(1-\delta(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-)))}, \text{ for i = 1,2.} \quad (33)$$

Probability that the server is under repair is given by

$$R = \lim_{z\to 1} R(z) = \frac{\lambda^+\lambda^- r_1(1 - p_1B_1^*(\lambda^-) - p_2B_2^*(\lambda^-))}{[1-\delta(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))]} \quad (34)$$

The probability generating function of the orbit size is

$$P_q(z) = I_0 A^*(\lambda^+) g(z)[z-1][1-\delta(B_1^*(g(z)) + B_2^*(g(z))] \big/ D(z) \quad (35)$$

The probability generating function of the system size is

$$P_s(z) = I_0 A^*(\lambda^+)[z-1][\lambda^- + (h(z) - \delta g(z))(p_1B_1^*(g(z)) + p_2B_2^*(g(z)))] \big/ D(z) \quad (36)$$

C. Performance Measures

Mean number of customers in the orbit is derived as

$$L_q = P_q'(1) = \frac{N''(1)D'(1) - N'(1)D''(1)}{2D'^2(1)} \quad (37)$$

where N(z) and D(z)are the numerator and denominator of $P_q(z)$

$$N'(1) = I_0\lambda^-[1-\delta(p_1B_1^*(\lambda^-) + p_1B_1^*(\lambda^-))]$$

$$N''(1) = -2I_0\lambda^+[1+\lambda^-\delta(p_1\mu_{11} + p_2\mu_{21}) - \delta(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))]$$

$$D'(1) = \lambda^-[A^*(\lambda^+)) - \delta(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))] - \lambda^+(1+\lambda^- r_1)(1 - p_1B_1^*(\lambda^-) - p_2B_2^*(\lambda^-))$$

$$D''(1) = -2\lambda^+ - \lambda^-\lambda^+(\lambda^+ r_2 - r_1) + 2(\lambda^{+2} - \partial\lambda^+\lambda^- - \lambda^{+2}\lambda^- r_1)(p_1\mu_{11} + p_2\mu_{21})$$
$$-\{[2(1-A^*(\lambda^+))(\lambda^+\lambda^- r_1 + \partial\lambda^- - \lambda^+)] + \lambda^{+2}\lambda^- r_2 - \lambda^+ - 2\partial\lambda^+\}(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))$$

Mean number of customers in the system is

$$L_s = P_s'(1) = \frac{N_1''(1)D'(1) - N_1'(1)D''(1)}{2D'^2} \quad (38)$$

where $N_1(z)$ denotes the numerator of $P_s(z)$

$$N_1^{'}(1) = \lambda^{-}[1-\delta(p_1B_1^*(\lambda^{-}) + p_1B_1^*(\lambda^{-}))]$$

$$N''(1) = 2\lambda^{+}[(\delta-1)(p_1B_1^*(\lambda^{-}) + p_2B_2^*(\lambda^{-})) - \lambda^{-}\delta(p_1\mu_{11} + p_2\mu_{21})]$$

D. Reliability Indices

The system availability A(t) at time t is the probability that the server is either working for a customer or in an idle period. Then under steady state condition availability of the server is shown to be

$$A = 1\text{-R} = 1 - \frac{\lambda^{+}r_1(1-p_1B_1^*(\lambda^{-}) - p_2B_2^*(\lambda^{-}))}{[1-\delta(p_1B_1^*(\lambda^{-}) + p_2B_2^*(\lambda^{-}))]} \quad (39)$$

Steady state failure frequency of the server is

$$F = \lambda^{-}[P^{(1)} + P^{(2)}] = \frac{\lambda^{+}(1-p_1B_1^*(\lambda^{-}) - p_2B_2^*(\lambda^{-}))}{[1-\delta(p_1B_1^*(\lambda^{-}) + p_2B_2^*(\lambda^{-}))]} \quad (40)$$

E. Stochastic Decomposition

Theorem The decomposition property states that the number of customers in the system in steady state at a random point of time (L_s) is distributed as the sum of two independent random variables, one of which is the number of customers in the corresponding standard queueing system in steady state at a random point of time L and the other random variable is the number of customers in the system when the server is idle.

III. RESULTS AND DISCUSSION

F. Proof

The probability generating function π(z) of the system size in the classical single arrivalqueue with negative arrivals and feedback is

$$\pi(z) = [z-1]L_1.L_2/S_1.\,S_2 \quad (41)$$

where, $L_1 = \lambda^{-} - \delta g(z)(p_1B_1^*(g(z)) + p_1B_1^*(g(z))) + h(z)(p_1B_1^*(g(z)) + p_1B_1^*(g(z)))$

$$L_2 = \lambda^{-}[1-\delta(p_1B_1^*(\lambda^{-}) + p_2B_2^*(\lambda^{-}))] - \lambda^{+}(1+\lambda^{-}r_1)(1-p_1B_1^*(\lambda^{-}) - p_2B_2^*(\lambda^{-}))$$

$$S_1 = zg(z) - (p_1T_1(z) + p_1T_1(z))$$

$$S_2 = \lambda^{-}[1-\delta(p_1B_1^*(\lambda^{-}) + p_2B_2^*(\lambda^{-}))]$$

The probability generating function $\chi(z)$ of the number of customers in the orbit when the system is idle is given by

$$\chi(z) = \frac{I_0 + I(z)}{I_0 + I}$$

$$= \frac{[zg(z) - p_1T_1(z) - p_2T_2(z)]x\{\lambda^-[1-(1-A^*(\lambda^+)) - \delta(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))] - \lambda^+(1+\lambda^- r_1)(1-(p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))\}}{[\lambda^-[1-\delta((p_1B_1^*(\lambda^-) + p_2B_2^*(\lambda^-))] - \lambda^+(1+\lambda^- r_1)(1-(p_1B_1^*(\lambda^-) - p_2B_2^*(\lambda^-)))]D(z)} \quad (42)$$

From equations (36),(41) and (42), it is observed that probability generating function of the number of customers in the system $P_s(z)$ is decomposed as $P_s(z) = \pi(z)\,\chi(z)$.

Hence, $L_s = L + L_I$.

G. Numerical Results

Numerical examples are presented to study the effect of the parameters on the system characteristics. It is assumed that the retrial time, service time and repair time are exponentially distributed with respective parameters η ,μ_i and r where i=1,2.

The following arbitrary values are selected for the parameters in such a way that stability condition holds, λ^+=0.7, λ^-=11, η=0.9, μ_1 =3.6, μ_2=0.1, r_1=2, r_2=2, p_1=0.8, p_2=0.2, δ=0.9 and 1-δ=0.1.

Effect of the Parameters on the performance measures I_0 – the probability that the server is idle in empty system, I- the probability that the server is idle in non-empty system, $P^{(i)}$ – the probabilitythat the server is busy in type 1 or type 2 service, R- the probability that the server is under repair are displayed in figure 1 to 4.

From the figures it is observed that L_q decreases for increasing values of λ^-, μ_1 , η, r_1 and r_2 and increasing for λ^+ , μ_2 and δ.

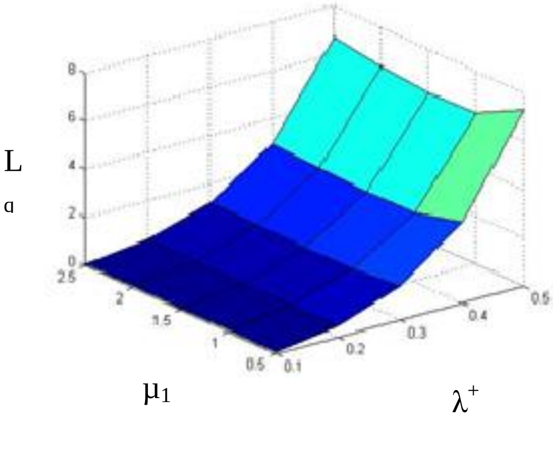

Figure 1. Effect of (μ_1,λ^+) on L_q

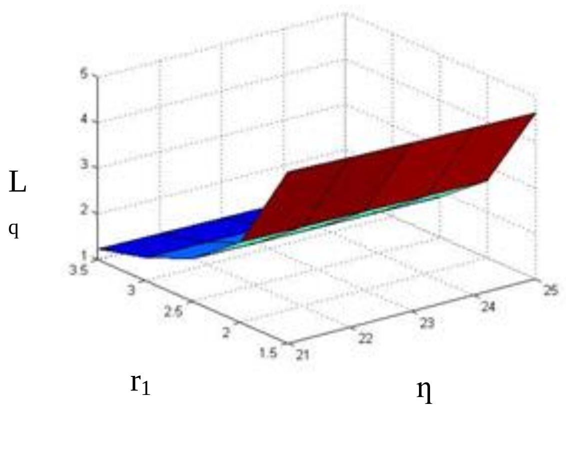

Figure 2. Effect of (r_1,η) on L_q

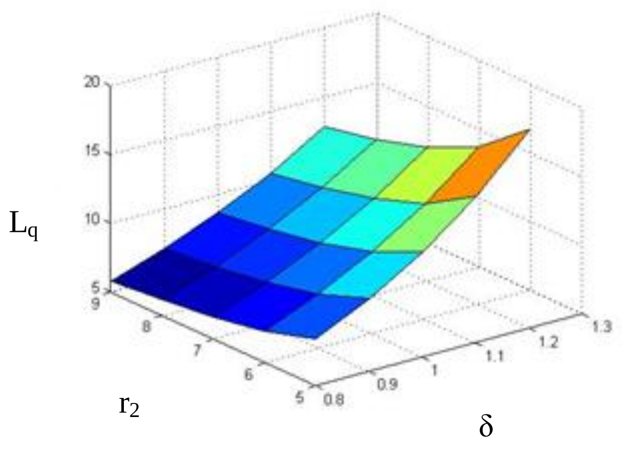

Figure 3. Effect of (r_2,δ) on L_q

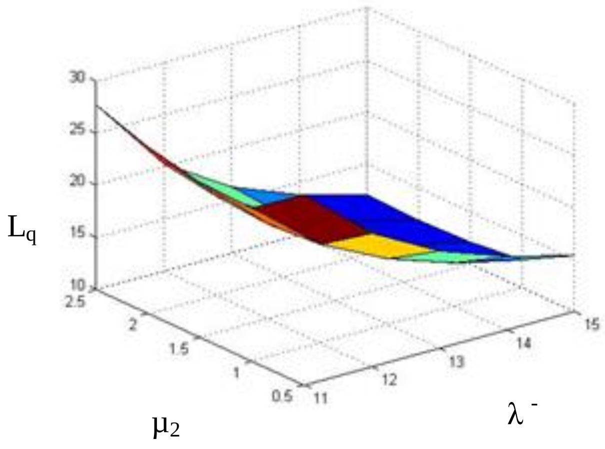

Figure 4. Effect of (μ_1,λ^+) on L_q

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