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A New generalization of the associated k-Fibonacci Numbers related to Left k-Fibonacci Numbers and their Identity

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ABSTRACT

In this paper we define the associated left $k-$Fibonacci numbers and we give basic identities of it.

Keywords: Associated $k-$Fibonacci numbers, associated left $k-$Fibonacci numbers, Pell numbers, recurrence relation.

I. INTRODUCTION

The subject matter of the paper is connected with the study of some aspects of generalization ofFibonacci numbers. The aim of the paper is to study various properties of the family of generalized Fibonacci numbers.

The Fibonacci sequence $\{F_n\}_{n=0}^{\infty}$, starting with the integer pair 0 and 1 and each Fibonacci number is obtained by the sum of the two preceding it. We define as $F_n = F_{n-1} + F_{n-2}$; $n \geq 2$ [5,8]. The first few terms of the Fibonacci numbers are 0,1,1,2,3,5,8,13,21,34, 55, 89, ….. The Fibonacci sequence has been generalized in many ways, some by preserving the initial conditions, and others by preserving the recurrence relation [5,6,7,9].

In [1] Alvaro H. Salas defined the sequence $\{A_{k,n}\}_{n=0}^{\infty}$ associated to $\{F_{k,n}\}_{n=0}^{\infty}$ as $A_{k,n} = F_{k,n} + F_{k,n-1}$ where $A_{k,0} = 1$ for n=1,2,3,…. He then defines associated k-Fibonacci numbers by recurrence relation

$$A_{k,n} = kA_{k,n-1} + A_{k,n-2};\ n \geq 2 \text{ where } A_{k,0} = 1.$$

In this paper we define *associated left k- Fibonacci sequence* $\{A_{k,n}^L\}$ and prove some interesting properties of *associated left k- Fibonacci numbers* [2,3]. Despite its simple appearance, this sequence contains a wealth of subtle and fascinating properties. In this paper we explore several of the fundamental identities related with $A_{k,n}^L$.

II. The associated left k-Fibonacci numbers:

Definition: The sequence of *associated left k-Fibonacci numbers* $\{A_{k,n}^L\}$ associate to left k-Fibonacci sequence $\{F_{k,n}^L\}$ [4] is defined as

$$A_{k,0}^L = 1 \text{ and } A_{k,n}^L = F_{k,n}^L + F_{k,n-1}^L, n = 1,2,3\ldots.$$

We observe that the expression $A_{k,n}^L$ is the sum of the two consecutive *left k-Fibonacci numbers* $F_{k,n}^L$ and its predecessor $F_{k,n-1}^L$. The members of the sequence $\{A_{k,n}^L\}$ will be called *associated left k-Fibonacci numbers*. An equivalent definition for the sequence $\{A_{k,n}^L\}$ is

$$A_{k,n}^L = \begin{cases} 1 & ,if\ n=0 \\ 1 & ,if\ n=1 \\ (k+1)F^L{}_{k,n-1} + F^L{}_{k,n-2} & ,if\ n \geq 2 \end{cases}$$

Observe that

$$A_{k,n}^L = F_{k,n}^L + F_{k,n-1}^L = kF_{k,n-1}^L + F_{k,n-2}^L + kF_{k,n-2}^L + F_{k,n-3}^L$$

$$= k(F_{k,n-1}^L + F_{k,n-2}^L) + F_{k,n-2}^L + F_{k,n-3}^L$$

$$= kA^L_{k,n-1} + A^L_{k,n-2}$$

This allows defining recursively the sequence of associated left *k*-Fibonacci numbers as follows:

$$A^L_{k,n} = \begin{cases} 1, if \ \ n = 0 \\ 1, if \ \ n = 1 \\ kA^L_{k,n-1} + A^L_{k,n-2}, if \ \ n \geq 2 \end{cases}$$

Members of *associated left k- Fibonacci sequence* $\left\{A^L_{k,n}\right\}$ will be called *associated left k- Fibonacci numbers*. Some of them are

n	$A^L_{k,n}$
0	1
1	1
2	$k+1$
3	k^2+k+1
4	k^3+k^2+2k+1
5	$k^4+k^3+3k^2+2k+1$
6	$k^5+k^4+4k^3+3k^2+3k+1$
7	$k^6+k^5+5k^4+4k^3+6k^2+3k+1$
8	$k^7+k^6+6k^5+5k^4+10k^3+6k^2+4k+1$
9	$k^8+k^7+7k^6+6k^5+15k^4+10k^3+10k^2+4k+1$
10	$k^9+k^8+8k^7+7k^6+21k^5+15k^4+20k^3+10k^2+5k+1$
11	$k^{10}+k^9+9k^8+8k^7+28k^6+21k^5+35k^4+20k^3+15k^2+5k+1$

III. Basic identities of associated left k-Fibonacci numbers

One of the purposes of this paper is to develop many identities and results. We use the technique of induction as a useful tool in proving many of these identities and theorems involving Fibonacci numbers.

Identity 3.1 $\gcd(A^L_{k,n}, A^L_{k,n+1}) = 1, \ \forall \ n = 0, 1, 2, 3, ...$

Proof: Suppose that $A^L_{k,n}$ and $A^L_{k,n+1}$ are both divisible by a positive integer d. Then their difference

$$A^L_{k,n+1} - A^L_{k,n} = kA^L_{k,n} + A^L_{k,n-1} - A^L_{k,n}$$
$$= (k-1)\,A^L_{k,n} + A^L_{k,n-1}$$

will also be divisible by d. Then right hand side of this equation is divisible by d. Thus $d \mid A^L_{k,n-1}$. Continuing we see that $d \mid A^L_{k,n-2}$, $d \mid A^L_{k,n-3}$ and so on…

Eventually, we must have $d \mid A^L_{k,1}$. But $A^L_{k,1} = 1$ then $d = 1$. Since the only positive integer which divides successive terms of the *associated left k - Fibonacci sequence* is 1. This proves the required result.

Next we derive the formula for sum of the first n *associated left k - Fibonacci numbers*.

Identity 3.2

$$A^L_{k,1} + A^L_{k,2} + A^L_{k,3} + \cdots \quad = \frac{1}{k}(A^L_{k,n+1} + A^L_{k,n} - 2).$$

Proof: We have $A^L_{k,n} = kA^L_{k,n-1} + A^L_{k,n-2}, \ \ n \geq 2$.

Replacing n by 2, 3, 4… we have

$$A^L_{k,2} = kA^L_{k,1} + A^L_{k,0},$$
$$A^L_{k,3} = kA^L_{k,2} + A^L_{k,1},$$
$$A^L_{k,4} = kA^L_{k,3} + A^L_{k,2}$$
$$\vdots$$
$$A^L_{k,n-1} = kA^L_{k,n-2} + A^L_{k,n-3}$$
$$A^L_{k,n} = kA^L_{k,n-1} + A^L_{k,n-2}$$

Adding all these equations term by term, we get

$$A^L_{k,1} + A^L_{k,2} + A^L_{k,3} + \cdots$$
$$= A^L_{k,1} + (k+1)(A^L_{k,1} + A^L_{k,2} + \cdots \quad) + kA^L_{k,n-1} + A^L_{k,0}$$
$$= A^L_{k,0} + A^L_{k,1} + (k+1)(A^L_{k,1} + A^L_{k,2} + \cdots \quad - A^L_{k,n-1} - (k+1)A^L_{k,n}$$
$$\therefore \ (1-k-1)(A^L_{k,1} + A^L_{k,2} + \cdots \quad = A^L_{k,0} + A^L_{k,1} - (A^L_{k,n-1} + kA^L_{k,n} + A^L_{k,n})$$
$$\therefore \ -k(A^L_{k,1} + A^L_{k,2} + \cdots \quad = 2 - (kA^L_{k,n+1} + A^L_{k,n})$$
$$\therefore \ A^L_{k,1} + A^L_{k,2} + A^L_{k,3} + \cdots \quad = \frac{1}{k}(A^L_{k,n+1} + A^L_{k,n} - 2).$$

An alternate method of proving Identity 3.2 is to apply the principle of mathematical induction. Using the same process or by induction we can derive formulae for the sum of the first n *associated left k-Fibonacci numbers* with various subscripts.

We next derive the sum of first n *associated left k −Fibonacci numbers* with only odd or even subscripts.

Identity 3.3

$$A^L_{k,1} + A^L_{k,3} + A^L_{k,5} + \cdots \quad = \frac{1}{k}(A^L_{k,2n} - 1).$$

Proof: We have $A^L_{k,n} = kA^L_{k,n-1} + A^L_{k,n-2}, \ \ n \geq 2$.

Replacing n by 2, 4, 6… we have

$$A^L_{k,2} = kA^L_{k,1} + A^L_{k,0}$$

$$A_{k,4}^{L} = kA_{k,3}^{L} + A_{k,2}^{L}$$
$$A_{k,6}^{L} = kA_{k,5}^{L} + A_{k,4}^{L}$$
$$\vdots$$
$$A_{k,2n-2}^{L} = kA_{k,2n-3}^{L} + A_{k,2n-4}^{L}$$
$$A_{k,2n}^{L} = kA_{k,2n-1}^{L} + A_{k,2n-2}^{L}$$

Adding all these equations term by term,

$$A_{k,2}^{L} + A_{k,4}^{L} + A_{k,6}^{L} + \cdots$$
$$= k(A_{k,1}^{L} + A_{k,3}^{L} + \cdots \quad \cdots$$
$$= A_{k,0}^{L} + k(A_{k,1}^{L} + A_{k,3}^{L} + \cdots \quad \cdots$$
$$\therefore \quad 0 = A_{k,0}^{L} + k(A_{k,1}^{L} + A_{k,3}^{L} + \cdots \quad {}_{1}) - A_{k,2n}^{L}$$
$$\therefore \quad A_{k,1}^{L} + A_{k,3}^{L} + \cdots \quad {}_{1} = \frac{1}{k}(A_{k,2n}^{L} - 1).$$

Identity 3.4

$$A_{k,2}^{L} + A_{k,4}^{L} + A_{k,6}^{L} + \cdots \quad = \frac{1}{k}(A_{k,2n+1}^{L} - 1).$$

Proof: We know that $A_{k,n}^{L} = kA_{k,n-1}^{L} + A_{k,n-2}^{L}, \quad n \geq 2$

Replacing *n* by 1, 3, 5 … we have

$$A_{k,1}^{L} = 1$$
$$A_{k,3}^{L} = kA_{k,2}^{L} + A_{k,1}^{L}$$
$$A_{k,5}^{L} = kA_{k,4}^{L} + A_{k,3}^{L}$$
$$\vdots \quad \vdots \quad \vdots$$
$$A_{k,2n-1}^{L} = kA_{k,2n-2}^{L} + A_{k,2n-3}^{L}$$

Adding all these equations term by term,

$$A_{k,1}^{L} + A_{k,3}^{L} + A_{k,5}^{L} + \cdots \quad {}_{1}$$
$$= A_{k,1}^{L} + k(A_{k,2}^{L} + A_{k,4}^{L} + \cdots \quad \cdots$$
$$= A_{k,1}^{L} + k(A_{k,2}^{L} + A_{k,4}^{L} + \cdots \quad) - kA_{k,2n}^{L} +$$
$$(A_{k,1}^{L} + A_{k,3}^{L} + \cdots \quad {}_{1}) - A_{k,2n-1}^{L}$$
$$\therefore \quad 0 = 1 + k(A_{k,2}^{L} + A_{k,4}^{L} + \cdots \quad) - (kA_{k,2n}^{L} + A_{k,2n-1}^{L}).$$
$$\therefore \quad A_{k,2}^{L} + A_{k,4}^{L} + A_{k,6}^{L} + \cdots \quad = \frac{1}{k}(A_{k,2n+1}^{L} - 1).$$

The following results follow immediately from above two lemmas.

Corollary 3.5 $A_{k,2n}^{L} \equiv 1 (\bmod k)$ and $A_{k,2n+1}^{L} \equiv 1 (\bmod \mathrm{k})$.

Proof: We use Mathematical Induction to prove the first result.

For $n = 1$, we have $A_{k,2}^{L} = 1 + k \equiv 1(mod\, k)$

Suppose it is true for $n = r$, Thus $F_{k,2\mathrm{r}}^{R} \equiv 1(mod\, k)$ holds.

Now, $A_{k,2\mathrm{r}+2}^{L} = kA_{k,2\mathrm{r}+1}^{L} + A_{k,2\mathrm{r}}^{L} \equiv 1(mod\, k)$.

So the result is true for $n = r + 1$ also. This proves the result for all integers *n*.

Similarly, we can prove the second result.

IV. CONCLUSION

New generalized k- Fibonacci and associated k-Fibonacci sequences has been introduced and deducted their identities and results.

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