© 2016 IJSRSET | Volume 2 | Issue 2 | Print ISSN : 2395-1990 | Online ISSN : 2394-4099
Themed Section: Engineering and Technology

Markov Model for Internet Traffic Sharing Analysis with Unique Conception of Rest State in Multi-Market Environment

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ABSTRACT

In today's scenario internet used as an effective resource for advertising and marketing by the users in daily life. Generally, these kinds of services are managed by service providers. Owing to demands of same kind of services by the users, internet traffic load and traffic share of operators oscillated. Naldi(2002) made a pioneering contribution to the model based internet traffic share problem and developed a relationship between traffic share and network blocking probability with the supposition of call-by-call basis, Further this suggestion improved in two-call-basis with a special reference of rest state based analysis by Shukla, Tiwari, Thakur and Deshmukh(2010). In this paper market based internet traffic share expressions has been derived in case of multi operator situation by using markov chain model. However, relatively few studies have taken into consideration on effect of rest state on traffic share of operators. In addition, simulation studies have also been focused to analyze the supporting results. Herein, we found that marketing plan like rest state inclusion has the potential to enhance the traffic proportion of operators in multi-market environment.

Keywords: Markov Model, Internet Traffic Share, Network Blocking, User Behaviour, Rest State

I. INTRODUCTION

The excess of internet traffic can cause even the fastest connections to slow down. Besides the broadband services, a large group of internet users still using dialup connection for connectivity. However, it is cheaper in term of cost as compared to broadband connection and provides slow speed of connectivity. The impact of location is also an important issue for analyzing the traffic distribution of any service provider. In this paper two prime markets have been considered having two operators in each market. User's behavior categorised as faithful user, partial impatient users and completely impatient user with the interrelationship of some network parameters like initial share, blocking probability and abandon probability in two -call based setup. The effect of rest state on traffic distribution of service provider need special consideration which is a package of facility to attract the customer and is a part of marketing strategies. Moreover location based traffic share have been computed and analyzed through simulation study, which is based on model parameters.

II. LITERATURE REVIEW

Koetter and Medard (2003) discussed an algebraic approach in the field of networking, in which network coding were used and some new result derived. Naldi (2002) presented internet traffic share problem in a multi-operator environment case and traffic management between two service providers were studied through markov chain. Newby and Dagg (2008) focused on optical inspection and maintenance for stochastically deteriorating systems in average cost criteria point of view and some facts related to maintenance were discussed. Altman, Avrachenkov and Brakat (2002) examined TCP network calculus and a case of large delay-bandwidth product was suggested where as Hambali and Ramani (2002) initiated for a performance based study of at multicast switch with different traffics modes. Agarwal and Kaur (2008) advocate on reliability analysis of fault-tolerant in case of multistage interconnection networks and limit of fault was analyzed. Gangele and Dongre (2014) attempted for two-call index based analysis for internet traffic share case of cyber crime environment in computer network field. Shukla and Gadewar (2007) utilize markov chain model for cell movement in a knockout switch in computer network where as Shukla, Gadewar and Pathak (2007) also used this model for space-division switches and derived some result on it. Shukla and Thakur (2009) have a view point approach on state probability analysis for internet traffic sharing in two operator environment case. Similar contribution was given by Shukla *et al.*(2010) for two call based analysis of internet traffic sharing and some new result were derived. Shukla, Verma and Gangele (2012) have another interesting contribution on curve fitting approximation on internet traffic distribution in two market environment case. Shukla, Gangele, Singhai and Verma (2011) performed elasticity analysis of web-browsing behaviour of users in case when two browsers install in a computer system. Gangele and Shukla (2014) have given a methodology on area computation of internet traffic share problem with special reference to cyber crime users. Park and Willinger (2000) explored Self-Similar network traffic and a performance evaluation were measured with some new constraints. Shukla and Singhai (2011) explored a model based study for the analysis of user's web browsing behaviour and derived browser share expression for each browser. Shukla *et al.* (2012) analyzed curve fitting approximation for internet traffic distribution in computer networking field when there is two market environments. Similar contribution was given by Shukla, Verma and Gangele (2015) in which bounded area was estimated with the help of Simpson 3/8 rule in traffic share scenario in a computer network. Naldi (1999) have put on a new look on measurement based modelling of internet dial-up access connections and some entrusting result arrived. Perzen (1992) has given detail description on stochastic processes and various fundamental concept related to randomness was discussed. Gangele and Patil (2015) have innovatively presented internet traffic distribution analysis in case of multi-operator as well as multi-market environment and derived some new results for each kind of operator. Shukla *et al.*(2015) have conductive a extension analysis on an approximating the probability of traffic sharing through numerical analysis techniques between two operators in a computer network environment.

III. USER BEHAVIOR AS A SYSTEM

(As proposed Shukla *et al.*(2010))

Under the hypothesis of user's behaviour can be modeled by a eleven-state discrete time markov chain $\{D^{(n)}, n \geq 0\}$ such that $D^{(n)}$ stands for the state of random variable D at n^{th} attempt made by user over the state space $\{M_1, M_2, O_1, O_2, O_3, O_4, Z_1, Z_2, R_1, R_2$ & $A\}$ where:

State M_1: Market-I
State M_2: Market-II
State O_1: First operator in market-I
State O_2: Second operator in market-I
State O_3: Third operator in market-II

State O_4: Fourth operator in market-II

State Z_1: Success (in connectivity) in market-I

State Z_2: Success (in connectivity) in market-II

State R_1: Temporary short time rest in market-I

State R_2: Temporary short time rest in market-II

State A: abandon to call attempt process

Some underlying assumptions of the model are:

(a) User first selects the Market-I with probability q and Market-II with probability (1-q) as per ease.

(b) After that User, in a shop, chooses the first operator O_i *(i=1,3)*with probability p or to next O_j *(j=2,4)* with (1-p).

(c) The blocking probability experienced by O_i *(i=1,3)* is L_1 and by O_j *(j=2,4)* is L_2.

(d) Connectivity attempts of user between operators are on two-call basis, which means when first attempt of connectivity is failed, the users attempts one more to the same operator, and thereafter, attempts for the effort of connectivity.

(e) Whenever call connects through either O_i *(i=1,3)* or O_j *(j=2,4)* we say system reaches to the state of success (Z_1, Z_2).

(f) The user can terminate call attempt process, marked as system to abandon state A with probability P_A (either from O_i *(i=1,3)* or from O_j *(j=2,4)*)

(g) After each failed call attempt, the user has three choices: he can abandon with probability pA, switch over to other operator for a new attempt or moves for a little rest (on R_k).

(h) If user reaches to rest state R_k *(k=1,2)* from O_i *(i=1,3)* or O_j then in next attempt he may either with a call on O_i *(i=1,3)* or O_j *(j=2,4) (j=2,4)*with probability r_k and (1-r_k) respectively but cannot abandon.

(i) Switching among O_i, O_j and R_k is on two call-basis depending just on the latest attempt.

(j) From state R_k*(k=1,2)*user cannot move to states Z_k and A.

(k) State Z_k*(k=1,2)* and A are absorbing State.

IV. USER'S ATTITUDE

A user may be dedicating for an operator or opportunist which reveals attitude and categorized as:

(a) Faithful User (FU):

Who is faithful to an operator O_i (i = 1, 2) only, otherwise, prefers to take rest on R or abandon, but does not attempt for other competitive operator O_j (i ≠ j).

(b) Partially Impatient User (PIU):

Who attempt only between O_1 an O_2, all time until call completes or abandon but never goes to RS (rest state).

(c) Completely Impatient User (CIU):

Who is among O_1, O_2, R in different call attempts (or physical movements) or prefers to abandon.

V. MARKOV CHAIN MODEL

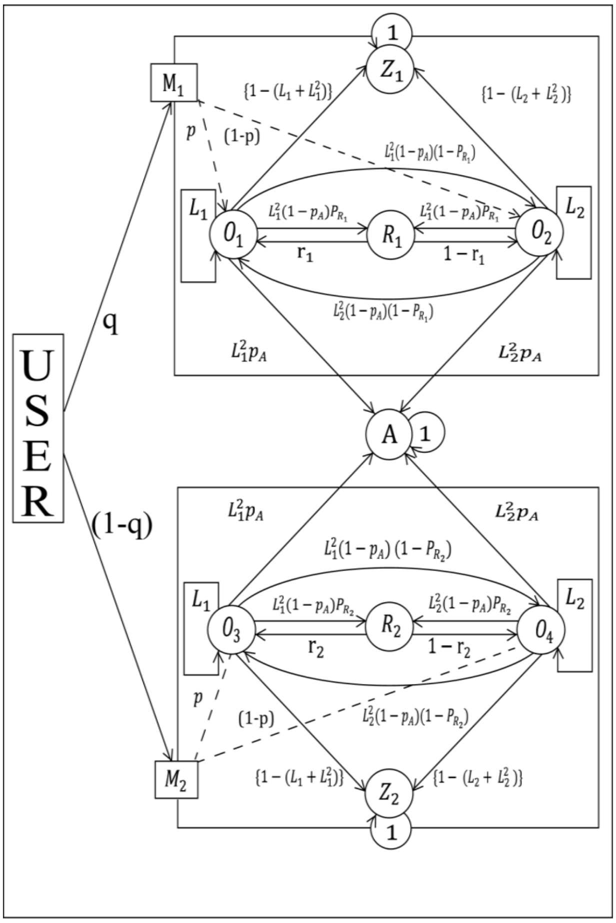
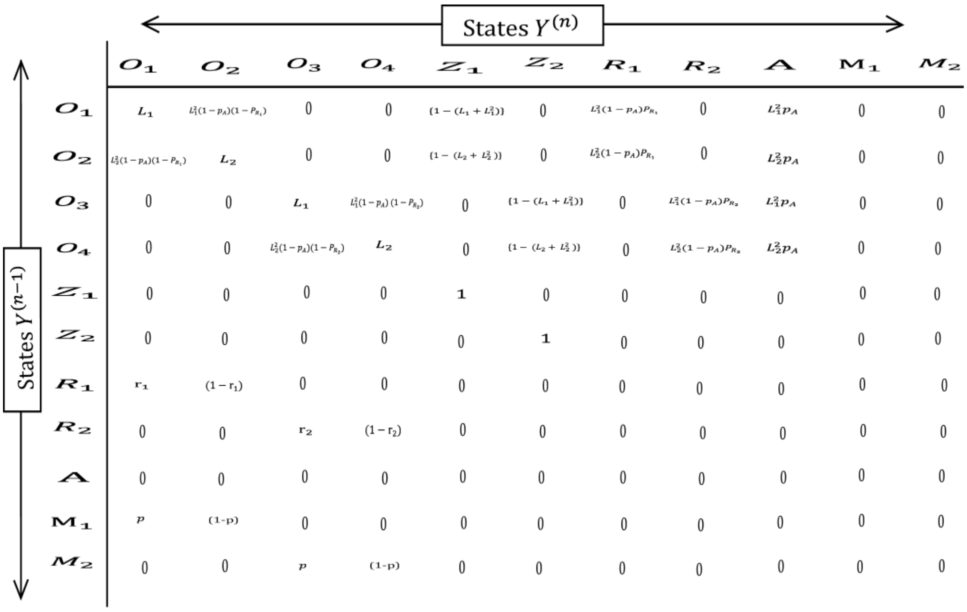


Fig.1-Markov Chain Model for the User's Behaviour in Multi Operators Case



States $Y^{(n)}$

States $Y^{(n-1)}$

	O_1	O_2	O_3	O_4	Z_1	Z_2	R_1	R_2	A	M_1	M_2
O_1	L_1	$L_1^2(1-p_A)(1-P_{R_1})$	0	0	$\{1-(L_1+L_1^2)\}$	0	$L_1^2(1-p_A)P_{R_1}$	0	$L_1^2p_A$	0	0
O_2	$L_2^2(1-p_A)(1-P_{R_1})$	L_2	0	0	$\{1-(L_2+L_2^2)\}$	0	$L_2^2(1-p_A)P_{R_1}$	0	$L_2^2p_A$	0	0
O_3	0	0	L_1	$L_1^2(1-p_A)(1-P_{R_2})$	0	$\{1-(L_1+L_1^2)\}$	0	$L_1^2(1-p_A)P_{R_2}$	$L_1^2p_A$	0	0
O_4	0	0	$L_2^2(1-p_A)(1-P_{R_2})$	L_2	0	$\{1-(L_2+L_2^2)\}$	0	$L_2^2(1-p_A)P_{R_2}$	$L_2^2p_A$	0	0
Z_1	0	0	0	0	1	0	0	0	0	0	0
Z_2	0	0	0	0	0	1	0	0	0	0	0
R_1	r_1	$(1-r_1)$	0	0	0	0	0	0	0	0	0
R_2	0	0	r_2	$(1-r_2)$	0	0	0	0	0	0	0
A	0	0	0	0	0	0	0	0	0	0	0
M_1	p	(1-p)	0	0	0	0	0	0	0	0	0
M_2	0	0	p	(1-p)	0	0	0	0	0	0	0

Transition Probability Matrix

Fig.2- T.P.M

VI. LOGICAL RULES FOR TRANSITION PROBABILITIES IN MODEL

(As discussed Shuka *et al.* (2010))

There are some certain rules for transition mechanism in model and probabilities

Rule 1:

The starting conditions (state distribution before the first call attempt) are:

$P[D^{(0)}=O_1]=0$ and $P[D^{(0)}=O_2]=0$,
$P[D^{(0)}=R_1]=0$, and $P[D^{(0)}=R_2]=0$,
$P[D^{(0)}=Z_1]=0$, and $P[D^{(0)}=Z_2]=0$, $P[D^{(0)}=A]=0$
$P[D^{(0)}=M_1]=q$, and $P[D^{(0)}=M_2]=(1-q)$

Rule 2:

User attempts to operator O_i *(i=1, 3)* or Oj *(j=2,4)* with initial probabilities p (based on quality of service).

Rule 3:

If users (customers) fail in connectivity in first attempt then reattempt to operator O_1.

Rule 4:

User may succeed to O_1 in one attempt or in the next. Since the blocking probabilities for O_1 in the attempt is L_1, therefore blocking probabilities for O_1 in the next attempt is

=*P [O_1 Blocked in an attempt] P [O_1 blocked in next attempt / previous attempt to O_1 was blocked]*

$= (L_1L_1) = L_1^2$

The total blocking probabilities is ($L_1+L_1^2$) is inclusive of both attempts.

Hence success probability for O_1 is [1- ($L_1+L_1^2$)]. Similarly could be derived for operator O_2 in form [1-($L_2+L_2^2$)].

Rule 5:

User shift to O_2 if call blocked in both attempts to O_1 and does not abandon the attempting process.

The transition probability is = *P [O_1 blocked in an attempt]. P [O_1 blocked in next attempt/previous attempt to O_1 was blocked].P[does not abandon attempting process]*

$= L_1^2\ (1- p_A)$

Rule 6:

User earliest abandons the system only after two attempts to an operator which is a compulsive assumption with this model. This leads to probabilities that user abandons process after two attempts over O_1 is

= P [O_1 blocked in attempt]. P [O_1 blocked in next attempt / Previous attempt to O_1 was blocked].P [abandon the attempting process]

$=L_1^2\ p_A$

Rule 7:

At O_1, when call blocked in (n-1)th attempt, user doesn't want to abandon, but wants a little rest then,

$P[D^{(n)} = R / D^{(n-1)}=O_1]$ = P[blocked at O_1].P[abandon the process].[a little rest]$=L^2{}_1.(1-P_A).p_R$

Rule 8:

At O_1, if call is blocked in (n-1)th attempt ,user doesn't want both abandon and rest, then he shifts to O_2 .

$P[D^{(n)} = O_2 / D^{(n-1)}=O_1]$ = P[blocked at O_1].P[not abandon].P[not rest] $=L^2{}_1.(1-P_A).(1-p_R)$

Rule 9:

Also assume for, 0<r<1

$P[D^{(n)} = O_1 / D^{(n-1)}=R\,] = r, P[D^{(n)} = O_2 / D^{(n-1)}=R\,] = (1-r)$

VII. SOME RESULT FOR n^{th} CONNECTIVITY ATTEMPT

Theorem 7.1: If user restricts to only O_1 and R_1 then n^{th} attempt state probability for market –I is:

Attempts are classified into four different categories.

Type A: When t=4n+1, n>0

$$\mathrm{P}[D^{(4n+1)} = O_1]_{FU}^{M_1} = [\mathrm{qp}{L_1}^{4n+1}(1-p_A)^{2n} P_{R_1}^{2n} {r_1}^{2n}$$

Type B: When t=4n-1, n>0

$$\mathrm{P}[D^{(4n-1)} = O_1]_{FU}^{M_1} = [\mathrm{qp}{L_1}^{4n-1}(1-p_A)^{2n-1} P_{R_1}^{2n-1} {r_1}^{2n-1}]$$

Type C: When t=4n, n>0

$$\mathrm{P}[D^{(4n)} = O_1]_{FU}^{M_1} = [\mathrm{qp}{L_1}^{4n}(1-p_A)^{2n} P_{R_1}^{2n} {r_1}^{2n}]$$

Type D: When t=4n-2, n>0

$$\mathrm{P}[D^{(4n-2)} = O_1]_{FU}^{M_1} = [\mathrm{qp}{L_1}^{4n-2}(1-p_A)^{2n-1} P_{R_1}^{2n-1} {r_1}^{2n-1}]$$

Theorem 7.2: If user restricts to only O_2 and R_1 then n^{th} attempts state probabilities for market-I is:

Type A: When t=4n+1, n>0

$$\mathrm{P}[D^{(4n+1)} = O_2]_{FU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_2}^{4n-1}(1-p_A)^{2n} P_{R_1}^{2n}(1-r_1)^{2n}]$$

Type B: When t=4n-1, n>0

$$\mathrm{P}[D^{(4n-1)} = O_2]_{FU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_2}^{4n-1}(1-p_A)^{2n-1} P_{R_1}^{2n-1}(1-r_1)^{2n-1}]$$

Type C: When t=4n, n>0

$$\mathrm{P}[D^{(4n)} = O_2]_{FU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_2}^{4n}(1-p_A)^{2n} P_{R_1}^{2n}(1-r_1)^{2n}]$$

Type D: When t=4n-2, n>0

$$\mathrm{P}[D^{(4n-2)} = O_2]_{FU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_2}^{4n-2}(1-p_A)^{2n-1} P_{R_1}^{2n-1}(1-r_1)^{2n-1}]$$

Theorem 7.3: If user restricts to only between O_1 and O_2 not interested for R_1 for partially impatient user (PIU) for market-I we have

Type A: When t=4n+1, n>0

$$\mathrm{P}[D^{(4n+1)} = O_1]_{PIU}^{M_1} = [(\mathrm{q}L_1)\mathrm{p}{L_1}^{3n}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}}]$$

$$\mathrm{P}[D^{(4n+1)} = O_2]_{PIU}^{M_1} = [(\mathrm{q}L_2)(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}}]$$

Type B: When t=4n-1, n>0

$$\mathrm{P}[D^{(4n-1)} = O_1]_{PIU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_1}^{3n-2}{L_2}^{3n}(1-p_A)^{2n-1}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}-1}]$$

$$\mathrm{P}[D^{(4n-1)} = O_2]_{PIU}^{M_1} = [\mathrm{qp}{L_1}^{3n}{L_2}^{3n-2}(1-p_A)^{2n-1}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}-1}]$$

Type C: When t=4n, n>0

$$\mathrm{P}[D^{(4n)} = O_1]_{PIU}^{M_1} = [\mathrm{qp}{L_1}^{3n}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}}]$$

$$\mathrm{P}[D^{(4n)} = O_2]_{PIU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}}]$$

Type D: When t=4n-2, n>0

$$\mathrm{P}[D^{(4n-2)} = O_1]_{PIU}^{M_1} = [\mathrm{q}(1-\mathrm{p}){L_1}^{3n-3}{L_2}^{3n}(1-p_A)^{2n-1}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}-1}]$$

$$\mathrm{P}[D^{(4n-2)} = O_2]_{PIU}^{M_1} = [\mathrm{qp}{L_1}^{3n}{L_2}^{3n-3}(1-p_A)^{2n-1}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}-1}]$$

Theorem 7.4: If call attempt is among O_1, O_2 and R_1 only then for n^{th} state probability the approximate expression (CIU) for market-I we have

Type A: When t=4n+1, n>0

$$\mathrm{P}[D^{(4n+1)} = O_1]_{CIU}^{M_1} = \mathrm{q}[\{\mathrm{p}{L_1}^{3n-1}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}}\} + \{2\mathrm{n}(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}-1}\mathrm{P}_{\mathrm{R}_1}\mathrm{r}_1\}]$$

$$\mathrm{P}[D^{(4n+1)} = O_2]_{CIU}^{M_1} = \mathrm{q}[\{(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n+1}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}}\} + \{2\mathrm{np}{L_1}^{3n}{L_2}^{3n}(1-p_A)^{2n}(1-\mathrm{P}_{\mathrm{R}_1})^{2\mathrm{n}-1}\mathrm{P}_{\mathrm{R}_1}(1-\mathrm{r}_1)\}]$$

Type B: When t=4n-1, n>0

$$P[D^{(4n-1)} = O_1]^{M_1}_{CIU} = q[\{(1-p)L_1{}^{3n-2}L_2{}^{3n}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-1}\} + \{2npL_1{}^{3n}L_2{}^{3n-2}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-2}P_{R_1}r_1\}]$$

$$P[D^{(4n-1)} = O_2]^{M_1}_{CIU} = q[\{pL_1{}^{3n}L_2{}^{3n-2}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-1}\} + \{2n(1-p)L_1{}^{3n-2}L_2{}^{3n}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-2}P_{R_1}(1-r_1)\}]$$

Type C: When t=4n, n>0

$$P[D^{(4n)} = O_1]^{M_1}_{CIU} = q[\{pL_1{}^{3n}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_1})^{2n}\} + \{(2n-1)(1-p)L_1{}^{3n-1}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_1})^{2n-1}P_{R_1}r_1\}]$$

$$P[D^{(4n)} = O_2]^{M_1}_{CIU} = q[\{(1-p)L_1{}^{3n}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_1})^{2n}\} + \{(2n-1)pL_1{}^{3n}L_2{}^{3n-1}(1-p_A)^{2n}(1-P_{R_1})^{2n-1}P_{R_1}(1-r_1)\}]$$

Type D: When t=4n-2, n>0

$$P[D^{(4n-2)} = O_1]^{M_1}_{CIU} = q[\{(1-p)L_1{}^{3n-3}L_2{}^{3n}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-1}\} + \{(2n-1)pL_1{}^{3n-1}L_2{}^{3n-3}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-2}P_{R_1}r_1\}]$$

$$P[D^{(4n-2)} = O_2]^{M_1}_{CIU} = q[\{pL_1{}^{3n}L_2{}^{3n-3}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-1}\} + \{(2n-1)(1-p)L_1{}^{3n-3}L_2{}^{3n-1}(1-p_A)^{2n-1}(1-P_{R_1})^{2n-2}P_{R_1}(1-r_1)\}]$$

Theorem 7.5: If user restricts to only O_3 and R_2 then n^{th} attempt state probabilities for market-II are:

Type A: When t=4n+1, n>0

$$P[D^{(4n+1)} = O_3]^{M_2}_{FU} = [(1-q)pL_1{}^{4n+1}(1-p_A)^{2n}P_{R_2}^{2n}r_2{}^{2n}]$$

Type B: When t=4n-1, n>0

$$P[D^{(4n-1)} = O_3]^{M_2}_{FU} = [(1-q)pL_1{}^{4n-1}(1-p_A)^{2n-1}P_{R_2}^{2n-1}r_2{}^{2n-1}]$$

Type C: When t=4n, n>0

$$P[D^{(4n)} = O_3]^{M_2}_{FU} = [(1-q)pL_1{}^{4n}(1-p_A)^{2n}P_{R_2}^{2n}r_2{}^{2n}]$$

Type D: When t=4n-2, n>0

$$P[D^{(4n-2)} = O_3]^{M_2}_{FU} = [(1-q)pL_1{}^{4n-2}(1-p_A)^{2n-1}P_{R_2}^{2n-1}r_2{}^{2n-1}]$$

Theorem 7.6: If user restricts to only O_4 and R_2 then n^{th} attempts state probabilities for market-II are:

Type A: When t=4n+1, n>0

$$P[D^{(4n+1)} = O_4]^{M_2}_{FU} = [(1-q)(1-p)L_2{}^{4n-1}(1-p_A)^{2n}P_{R_2}^{2n}(1-r_2)^{2n}]$$

Type B: When t=4n-1, n>0

$$P[D^{(4n-1)} = O_4]^{M_2}_{FU} = [(1-q)(1-p)L_2{}^{4n-1}(1-p_A)^{2n-1}P_{R_2}^{2n-1}(1-r_2)^{2n-1}]$$

Type C: When t=4n, n>0

$$P[D^{(4n)} = O_4]^{M_2}_{FU} = [(1-q)(1-p)L_2{}^{4n}(1-p_A)^{2n}P_{R_2}^{2n}(1-r_2)^{2n}]$$

Type D: When t=4n-2, n>0

$$P[D^{(4n-2)} = O_4]^{M_2}_{FU} = [(1-q)(1-p)L_2{}^{4n-2}(1-p_A)^{2n-1}P_{R_2}^{2n-1}(1-r_2)^{2n-1}]$$

Theorem 7.7: If user restricts to only between O_3 and O_4 not interested for R_2 for partially impatient users (PIU) for market-II we have

Type A: When t=4n+1, n>0

$$P[D^{(4n+1)} = O_3]^{M_2}_{PIU} = [\{(1-q)L_1\}pL_1{}^{3n}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_2})^{2n}]$$

$$P[D^{(4n+1)} = O_4]^{M_2}_{PIU} = [\{(1-q)L_2\}(1-p)L_1{}^{3n}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_2})^{2n}]$$

Type B: When t=4n-1, n>0

$$P[D^{(4n-1)} = O_3]^{M_2}_{PIU} = [(1-q)(1-p)L_1{}^{3n-2}L_2{}^{3n}(1-p_A)^{2n-1}(1-P_{R_2})^{2n-1}]$$

$$P[D^{(4n-1)} = O_4]^{M_2}_{PIU} = [(1-q)pL_1{}^{3n}L_2{}^{3n-2}(1-p_A)^{2n-1}(1-P_2)^{2n-1}]$$

Type C: When t=4n, n>0

$$P[D^{(4n)} = O_3]^{M_2}_{PIU} = [(1-q)pL_1{}^{3n}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_2})^{2n}]$$

$$P[D^{(4n)} = O_4]^{M_2}_{PIU} = [(1-q)(1-p)L_1{}^{3n}L_2{}^{3n}(1-p_A)^{2n}(1-P_{R_2})^{2n}]$$

Type D: When t=4n-2, n>0

$$\mathrm{P}[D^{(4n-2)} = O_3]_{PIU}^{M_2} = [(1-\mathrm{q})(1-\mathrm{p}){L_1}^{3n-3}{L_2}^{3n}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}]$$

$$\mathrm{P}[D^{(4n-2)} = O_4]_{PIU}^{M_2} = [(1-\mathrm{q})\mathrm{p}{L_1}^{3n}{L_2}^{3n-3}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}]$$

Theorem 7.8: If call attempt is among O_3, O_4 and R_2 only then for n^{th} state probability the approximate expression (CIU) for market-II we have

Type A: When t=4n+1, n>0

$$\mathrm{P}[D^{(4n+1)} = O_3]_{CIU}^{M_2} = (1-\mathrm{q})[\{\mathrm{p}{L_1}^{3n-1}{L_2}^{3n}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}}\} + \{2\mathrm{n}(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\mathrm{P}_{\mathrm{R}_2}\mathrm{r}_2\}]$$

$$\mathrm{P}[D^{(4n+1)} = O_4]_{CIU}^{M_2} = (1-\mathrm{q})[\{(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n+1}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}}\} + \{2\mathrm{np}{L_1}^{3n}{L_2}^{3n}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\mathrm{P}_{\mathrm{R}_2}(1-\mathrm{r}_2)\}]$$

Type B: When t=4n-1, n>0

$$\mathrm{P}[D^{(4n-1)} = O_3]_{CIU}^{M_2} = (1-\mathrm{q})[\{(1-\mathrm{p}){L_1}^{3n-2}{L_2}^{3n}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\} + \{2\mathrm{np}{L_1}^{3n}{L_2}^{3n-2}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-2}\mathrm{P}_{\mathrm{R}_2}\mathrm{r}_2\}]$$

$$\mathrm{P}[D^{(4n-1)} = O_4]_{CIU}^{M_2} = (1-\mathrm{q})[\{\mathrm{p}{L_1}^{3n}{L_2}^{3n-2}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\} + \{2\mathrm{n}(1-\mathrm{p}){L_1}^{3n-2}{L_2}^{3n}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-2}\mathrm{P}_{\mathrm{R}_2}(1-\mathrm{r}_2)\}]$$

Type C: When t=4n, n>0

$$\mathrm{P}[D^{(4n)} = O_3]_{CIU}^{M_2} = (1-\mathrm{q})[\{\mathrm{p}{L_1}^{3n}{L_2}^{3n}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}}\} + \{(2\mathrm{n}-1)(1-\mathrm{p}){L_1}^{3n-1}{L_2}^{3n}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\mathrm{P}_{\mathrm{R}_2}\mathrm{r}_2\}]$$

$$\mathrm{P}[D^{(4n)} = O_4]_{CIU}^{M_2} = (1-\mathrm{q})[\{(1-\mathrm{p}){L_1}^{3n}{L_2}^{3n}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}}\} + \{(2\mathrm{n}-1)\mathrm{p}{L_1}^{3n}{L_2}^{3n-1}\,(1-p_A)^{2n}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\mathrm{P}_{\mathrm{R}_2}(1-\mathrm{r}_2)\}]$$

Type D: When t=4n-2, n>0

$$\mathrm{P}[D^{(4n-2)} = O_3]_{CIU}^{M_2} = (1-\mathrm{q})[\{(1-\mathrm{p}){L_1}^{3n-3}{L_2}^{3n}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-1}\} + \{(2\mathrm{n}-1)\mathrm{p}{L_1}^{3n-1}{L_2}^{3n-3}\,(1-p_A)^{2n-1}\,(1-\mathrm{P}_{\mathrm{R}_2})^{2\mathrm{n}-2}\mathrm{P}_{\mathrm{R}_2}\mathrm{r}_2\}$$

$$\mathrm{P}[D^{(4n-2)} = O_4]_{CIU}^{M_2} = (1-\mathrm{q})[\{\mathrm{p}{L_1}^{3n}{L_2}^{3n-3}\,(1-p_A)^{2n-1}\left(1-\mathrm{P}_{\mathrm{R}_2}\right)^{2\mathrm{n}}\} + \{(2\mathrm{n}-1)(1-\mathrm{p}){L_1}^{3n-3}{L_2}^{3n-1}\,(1-p_A)^{2n-1}\left(1-\mathrm{P}_{\mathrm{R}_2}\right)^{2\mathrm{n}-2}\mathrm{P}_{\mathrm{R}_2}(1-\mathrm{r}_2)\}$$

VIII. TRAFFIC SHARING AND CALL CONNECTION

The traffic is shared between O_i $(i = 1{,}3)$ and O_j $(j = 2{,}4)$ operators. The aim is to calculate the probability of completion of a call with the assumption that it is achieve at n^{th} attempt with operator O_i (i=1,3) in market-I (M_1)

$\overline{\mathrm{P}_1}^{\,n}$ = P (call completes in n^th^ attempt with operator O~1~).= P[At $(n-1)^{th}$ attempt user is on O~1~].P[user is at z in n^th^ attempt when he was at O~1~ in (n-1)^th^ attempt]

$$\overline{\mathrm{P}_1}^{\,n} = \mathrm{P}\left[D^{(n-1)} = O_1\right].\mathrm{P}[D^{(n)} = Z_1\,/D^{(n-1)} = O_1]$$

$$= \{1-(L_1+L_1^2)\}\,.\textstyle\sum_{i=1}^{n} P[D^{(i-1)} = O_1]$$

$$\overline{\mathrm{P}_1}^{\,n} = \{1-(L_1+L_1^2)\}[\sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ A}}}^{n-1} P[D^{(i)} = O_1] + \sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ B}}}^{n-1} P[D^{(i)} = O_1] + \sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ C}}}^{n-1} P[D^{(i)} = O_1] + \sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ D}}}^{n-1} P[D^{(i)} = O_1]$$

Similarly for operator O~2~

$$\overline{\mathrm{P}_1}^{\,n} = \mathrm{P}\left[D^{(n-1)} = O_2\right].\mathrm{P}[D^{(n)} = Z_2\,/D^{(n-1)} = O_2]$$

$$= \{1-(L_2+L_2^2)\}\,.\textstyle\sum_{i=1}^{n} P[D^{(i-1)} = O_2]$$

$$\overline{\mathrm{P}_1}^{\,n} = \{1-(L_2+L_2^2)\}[\sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ A}}}^{n-1} P[D^{(i)} = O_2] + \sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ B}}}^{n-1} P[D^{(i)} = O_2] + \sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ C}}}^{n-1} P[D^{(i)} = O_2] + \sum_{\substack{\mathrm{i}=1\\ \mathrm{t=Type\ D}}}^{n-1} P[D^{(i)} = O_2] \qquad \ldots(8.1)$$

IX. TRAFFIC SHARE FOR FAITHFUL USERS

The FU are those who are hard core to an operator and never think about others to take services. We write for market-I (M_1)

For operator O₁

$$[\overline{P}_1^{(n)}]_{FU}^{M_1} = q\{1-(L_1+L_1^2)\}[p(1+L_1)+\{\frac{1-\{L_1^4(1-p_A)^2P_{R_1}^2{r_1}^2\}^{n-1}+1}{1-L_1^4(1-p_A)^2P_{R_1}^2{r_1}^2}\}.\{pL_1^2(1-p_A)P_{R_1}r_1\}(1+L_1)\{1+L_1^2(1-p_A)P_{R_1}r_1\}] \quad \ldots(9.1)$$

For operator O₂

$$[\overline{P}_2^{(n)}]_{FU}^{M_1} = q\{1-(L_2+L_2^2)\}[(1-p)(1+L_2)+\{\frac{1-\{L_2^4(1-p_A)^2P_{R_1}^2(1-r_1)^2\}^{n-1}+1}{1-L_2^4(1-p_A)^2P_{R_1}^2(1-r_1)^2}\}\{(1\text{-}p)L_2^2(1-p_A)P_{R_1}(1-r_1)\}(1+L_2)\{1+L_2^2(1-p_A)P_{R_1}(1-r_1)\}] \quad \ldots(9.2)$$

For Market-II for operator O₂

$$[\overline{P}_3^{(n)}]_{FU}^{M_2} = (1-q)\{1-(L_1+L_1^2)\}[p(1+L_1)+\{\frac{1-\{L_1^4(1-p_A)^2P_{R_2}^2{r_2}^2\}^{n-1}}{1-L_1^4(1-p_A)^2P_{R_2}^2{r_2}^2}\}\{pL_1^2(1-p_A)P_{R_2}r_2\}(1+L_1)\{1+L_1^2(1-p_A)P_{R_2}{r_2}^2\}] \quad \ldots(9.3)$$

$$[\overline{P}_4^{(n)}]_{FU}^{M_2} = (1-q)\{1-(L_1+L_1^2)\}[(1-p)(1+L_2)+\{\frac{1-\{L_2^4(1-p_A)^2P_{R_2}^2(1-r_2)^2\}^{n-1}}{1-L_2^4(1-p_A)^2P_{R_2}^2(1-r_2)^2}\}\{(1\text{-}p)L_2^2(1-p_A)P_{R_2}(1-r_2)\}(1+L_2)\{1+L_2^2(1-p_A)P_R(1-r_2)\} \quad \ldots(9.4)$$

X. BEHAIOUR OVER LARGE NO. OF ATTEMPTS

Suppose n is very large then

$$\overline{P}_k = \left[\underset{n\to\infty}{\text{limit}}\ \overline{P}_k^{(n)}\right], k = 1,2,3\ldots$$

And we get final traffic shares for market I

$$[\overline{P}_1]_{FU}^{M_1} = q\{1-(L_1+L_1^2)\}[p(1+L_1)\{\frac{\{1-L_1^4(1-p_A)^2P_{R_1}^2{r_1}^2\}+1}{1-L_1^4(1-p_A)^2P_{R_1}^2{r_1}^2}\}\{pL_1^2(1-p_A)P_{R_1}r_1\}(1+L_1)\{1+L_1^2(1-p_A)P_{R_1}r_1\}] \quad \ldots(10.1)$$

$$[\overline{P}_2]_{FU}^{M_1} = q\{1-(L_2+L_2^2)\}[(1-p)(1+L_2)\{\frac{\{1-L_2^4(1-p_A)^2P_{R_1}^2(1-r_1)^2\}+1}{1-L_2^4(1-p_A)^2P_{R_1}^2(1-r_1)^2}\}\{(1\text{-}p)L_2^2(1-p_A)P_{R_1}(1-r_1)\}(1+L_2)\{1+L_2^2(1-p_A)P_{R_1}(1-r_1)\}] \quad \ldots(10.2)$$

For market II we have

$$[\overline{P}_3]_{FU}^{M_2} = (1-q)\{1-(L_1+L_1^2)\}[p(1+L_1)\{\frac{\{1-L_1^4(1-p_A)^2P_{R_2}^2{r_2}^2\}+1}{1-L_1^4(1-p_A)^2P_{R_2}^2{r_2}^2}\}\{pL_1^2(1-p_A)P_{R_2}r_2\}(1+L_1)\{1+L_1^2(1-p_A)P_{R_2}r_2\}] \quad \ldots(10.3)$$

$$[\overline{P}_4]_{FU}^{M_2} = (1-q)\{1-(L_2+L_2^2)\}[(1-p)(1+L_2)\{\frac{\{1-L_2^4\left(1-p_A\right)^2P_{R_2}^2(1-r_2)^2\}+1}{1-L_2^4(1-p_A)^2P_{R_2}^2(1-r_2)^2}\}\{(1\text{-}p)L_2^2(1-p_A)P_{R_2}(1-r_2)\}(1+L_2)\{1+L_2^2(1-p_A)P_{R_2}(1-r_2)\}] \quad \ldots(10.4)$$

XI. SIMULATION STUDY

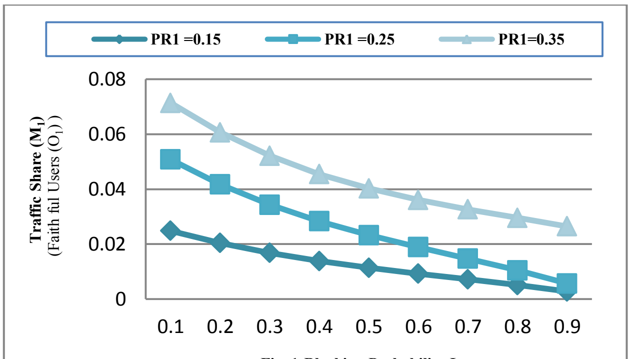


Fig. 1-Blocking Probability L_1
(When q=0.2, r_1=0.15, L_2=0.3, p=0.25, p_A=0.15)

In view of fig. 1, it is found that for faithful user in market-I traffic share pattern is in downward trend with the condition when rest state probability increases by 10% with some fixed network parameter q=20%, r_1=15%, L_2=30%, p=25% and p_A=15%.

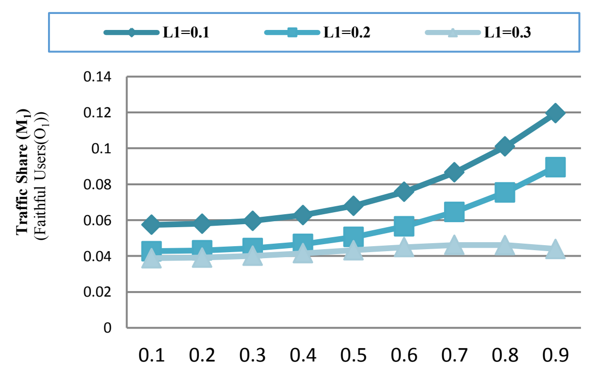


Fig. 2-Blocking Probability L_2
(When PR$_1$=0.2, r$_1$=0.25, q=0.2, p=0.35,p$_A$=0.25)

From fig. 2 it is seems that when L_1 is high the traffic pattern is nearly12% and the traffic pattern is upward for the variation of L_2 with rest probability P_{R_1}=20% and some constant network parameter.

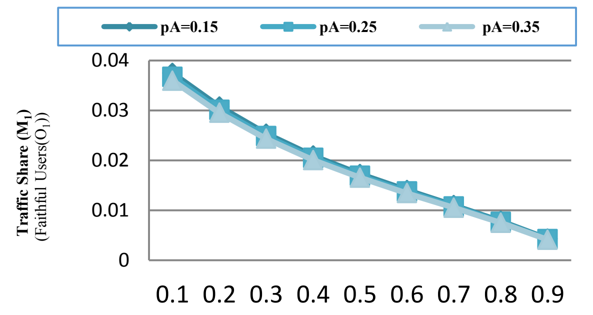


Fig. 3-Blocking Probability L_1
(When P$_{R1}$=0.25, r$_1$=0.25, q=0.25, p=0.35, L$_2$=0.15)

Fig. 3 indicate that in market-Ist at 25% probability of P_{R_1} and with little growth in abandon probability p$_A$ for faithful users traffic pattern is over lapped in case when r_1=25% with some constant parameter.

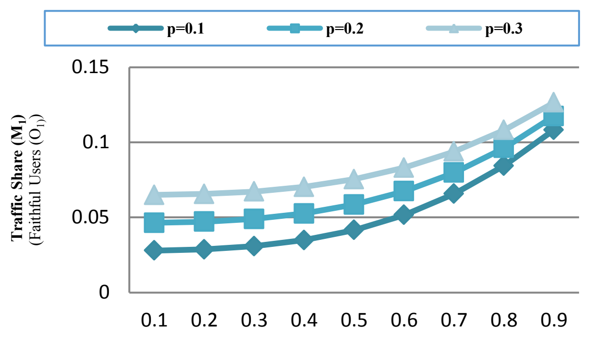


Fig. 4-Blocking Probability L_2
(When P$_{R1}$=0.3, r$_1$=0.15, q=0.25,p$_A$=0.15,L$_1$=0.2)

With reference to fig. 4 it is seen that traffic pattern is in upward trend for the constant increment in initial share p by 10% and opponent blocking probability L$_1$=20%.

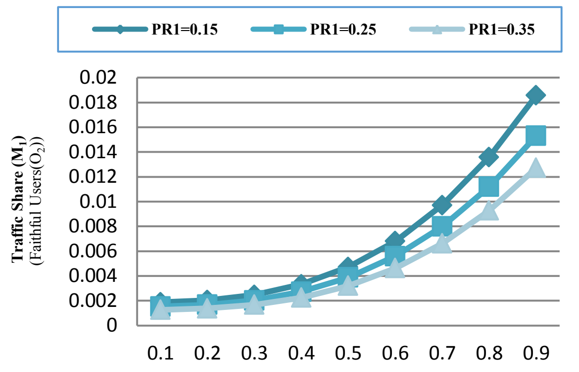


Fig. 5-Blocking Probability L_2
(When r$_1$=0.05, L$_1$=0.35, q=0.25, p=0.3, p$_A$=0.15)

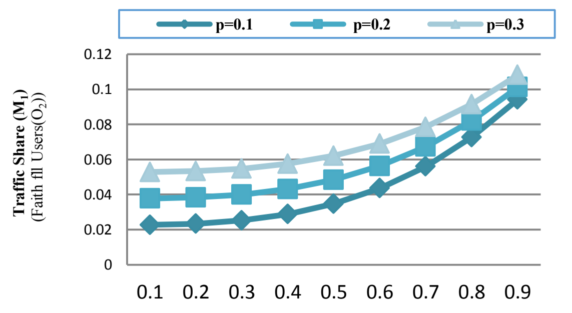


Fig. 6-Blocking Probability L_2
(When P$_{R1}$=0.15,r$_1$=0.25,q=0.35,L$_1$=0.25,p$_A$=0.35)

According to fig. 5 shows that if rest state probability increases then traffic share pattern also increases in case when opponent blocking probability L_2 also increase for some constant network parameter r_1=15%, L_1=35%, p_A=15%, q=25% and p=30% in faithful users prospect. Similar pattern exist in view of fig. 6 like fig. 4 but the network parameter are differ.

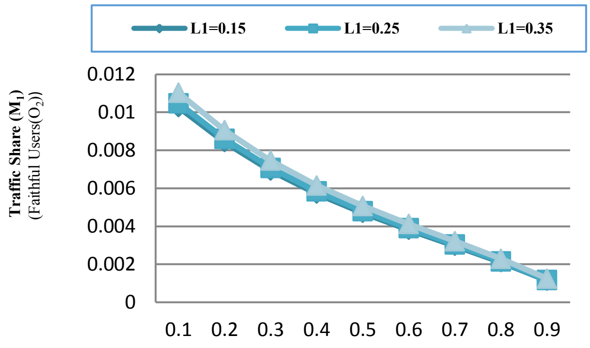


Fig. 7-Blocking Probability L_2
(When P_{R1}=0.2, r_1=0.3, q=0.1, p=0.2, p_A=0.15)

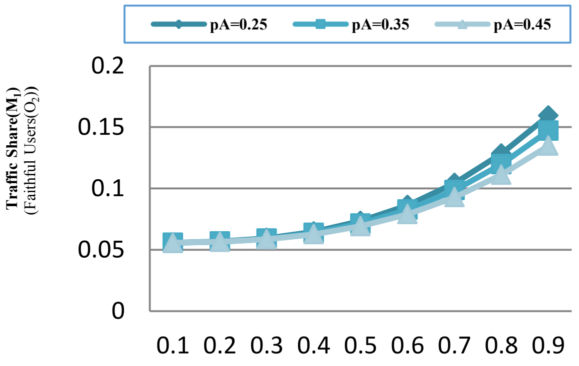


Fig. 8 -Blocking Probability L_2
(When P_{R1}=0.35,r_1=0.25, q=0.3,p=0.25, L_1=0.15)

While looking over figure 8 it is analyzed that overall increasing traffic share pattern exist with little increment in abandon probability p_A.. Fig.7 reflects that when P_{R_1} =20% and r=30% traffic pattern decreases rapidly for constant increment in blocking probabilityL_1.

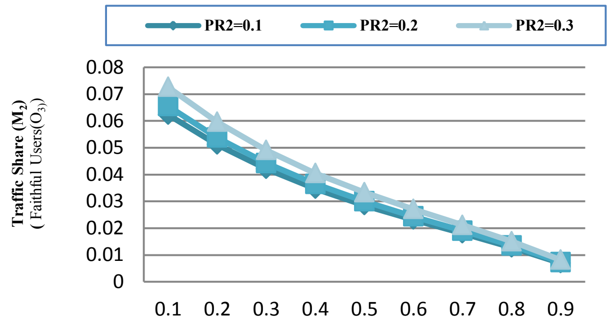


Fig. 9-Blocking Probability L_1
(When L_2=0.3, r_2=0.15 , q=0.25, p=0.25, p_A=0.25)

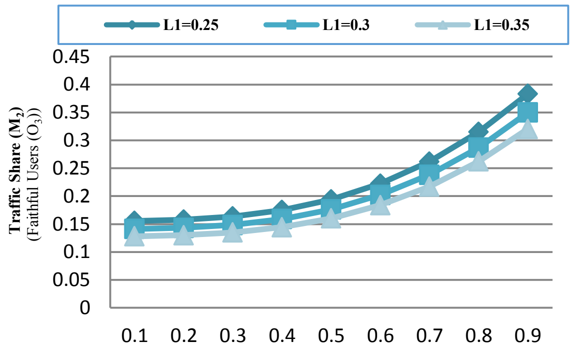


Fig. 10-Blocking Probability L_2
(When PR_2=0.25, r_2=0.15 , q=0.25, p=0.15, p_A=0.35)

In fig. 9 it is observe that in market-II traffic share pattern decrease for operator O_3 . But in fig. 10 opposite pattern was found and it is observe that 40% traffic covered when rest state probability P_{R_2}=25%.

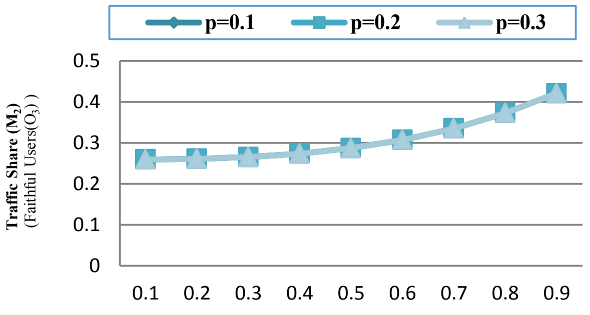


Fig. 11-Blocking Probability L_2
(When P_{R2}=0.35, r_2=0.25, q=0.1, p=0.35, p_A=0.25)

With reference to fig. 11 when the rest state probability P_{R_2}=35% and r_2=25% traffic pattern was overlapped in a particular case with some fixed increment in initial share p by 10%.

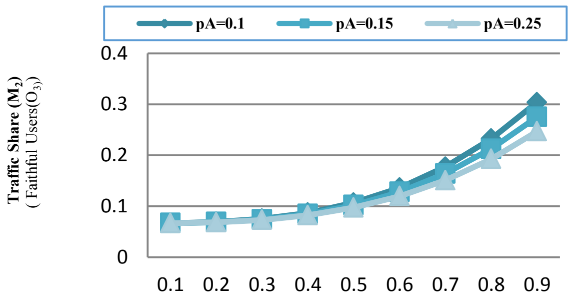


Fig. 12-Blocking Probability L_2
(When P_{R2}=0.2,r_2=0.25, q=0.2,p=0.15,L_1=0.3)

In view of fig.12 similar patterns exist like fig. 10 with some fixed network parameters.

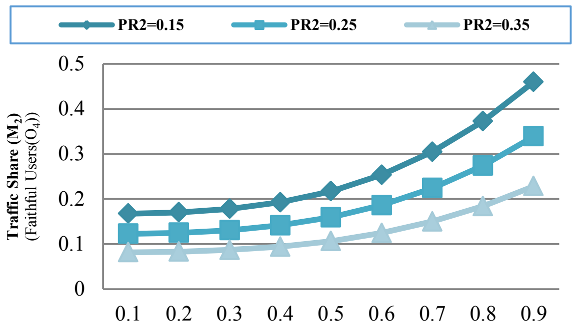


Fig. 13-Blocking Probability L_1
(When L_2=0.3, r_1=0.25, q=0.35, p=0.15, p_A=0.25)

Fig. 13 we depicts that in market-II for operator O_4 as the faithful users traffic share increased ,an upward pattern was observed at rest state probability P_{R_2} when L_2=35%, r_1=25%, q=35% and abandon probability p_A=25%.

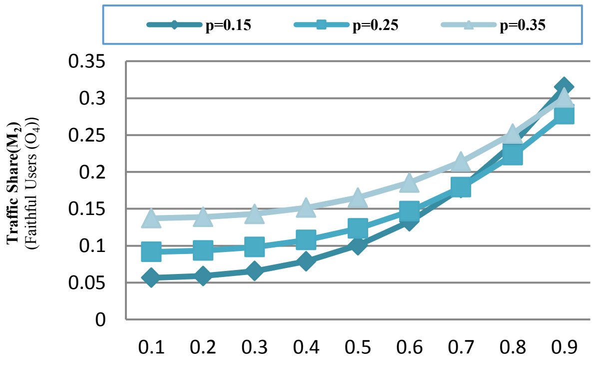


Fig. 14-Blocking Probability L_1
(When r_2=0.25, P_{R2}=0.05 , q=0.2, p=0.1, p_A=0.3)

Looking into fig. 14 it is observed that the similar pattern like fig. 4 and fig. 6 and for P_{R_2}=5% the traffic share pattern goes to upward with some varying blocking probability L_1 and prefixed network parameter q=20%, p=10% and abandon probability p_A=30%.

XII. CONCLUSION

In our findings it is found that rest state based markov chain model plays an imperative role for analyzing the users behavior. Users categories like faithful users, impatient users and partially impatient users have been discussed and network traffic share between operators have also been studied. Herein, we observed that an exponential traffic pattern exist in market-I for second kind of operator in case when r_1=5%, L_1=35%, q=25%, p=30% and p_A=15%. Similar traffic pattern also found in market-II in second kind of operator when L_2=30%, r_1=25%, q=35%, p=15% and p_A=25%. This work further reveals that in case of faithful users traffic pattern overlapped when P_{R_1}=25% and P_{R_2}=35% with some constant network parameters for first and second kind of market respectively. Moreover one can conclude that p_R and r if both have increases, then faithful user proportion for operator uplifts and they have hard core stuff for increasing internet traffic of operators. If stuff is high in proportions, then the operators gets benefits but operators have to control their network blocking probability in a particular case of faithful user in two market environment.

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