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Optimized Multiplicative Derivations in Close to Rings

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ABSTRACT

In the present paper, we investigate the commutatively of 3-prime near-rings satisfying certain conditions and identities involving left generalized multiplicative derivations. Moreover, examples have been provided to justify the necessity of 3-primeness condition in the hypotheses of various results.
Keywords: 3-Primeness Condition, Hypotheses, Derivation, Nonzero Multiplicative Derivation

I. INTRODUCTION

Throughout the paper, N will denote a left near-ring. N is called a 3-prime near-ring if xN y = {0} implies $x = 0$ or $y = 0$. N is called a semiprime near-ring if xN x = {0} implies $x = 0$. A nonempty subset A of N is called a semigroup left ideal (resp. semigroup right ideal) if N A ⊆ A (resp. AN ⊆ A) and if A is both a semigroup left ideal as well as a semigroup right ideal, it will be called a semigroup ideal of N . The symbol Z will denote the multiplicative center of N , that is, $Z = \{x \in N \mid xy = yx$ for all $y \in N\}$. For any $x, y \in N$ the symbol $[x, y] = xy - yx$ stands for the multiplicative commutator of x and y, while the symbol xoy stands for $xy + yx$. An additive mapping $d : N \to N$ is called a derivation of N if $d(xy) = xd(y) + d(x)y$ holds for all $x, y \in N$. The concept of derivation has been generalized in different directions by various authors (for reference see [1, 3, 9]). A map $d : N \to N$ is called a multiplicative derivation of N if $d(xy) = xd(y) + d(x)y$ holds for all $x, y \in N$. We, together with M. Ashraf and A. Boua have generalized the notion of multiplicative derivation by introducing the notion of generalized multiplicative derivations in [1] as follows: A map $f : N \longrightarrow N$ is called a left generalized multiplicative derivation of N if there exists a multiplicative derivation d of N such that $f(xy) = xf(y) + d(x)y$ for all $x, y \in N$. The map f will be called a left generalized multiplicative derivation of N with associated multiplicative derivation d of N . Similarly a map $f : N \longrightarrow N$ is called a right generalized multiplicative derivation of N if there exists a multiplicative derivation d of N such that $f(xy) = xd(y)+f(x)y$ for all $x, y \in N$. The map f will be called a right generalized multiplicative derivation of N with associated multiplicative derivation d of N . Finally, a map $f : N \longrightarrow N$ will be called a generalized multiplicative derivation of N if it is both a right as well as a left generalized multiplicative derivation of N with associated multiplicative derivation d of N . Note that if in the above definition both d and f are assumed to be additive mappings, then f is said to be a generalized derivation with associated derivation d of N. The following example shows that there exists a left generalized multiplicative derivation which is not a right generalized multiplicative derivation. For more properties of generalized multiplicative derivations one can refer to [1].

Example 1.1. Let S be a zero-symmetric left near-ring. Suppose that

$$N = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix} \;\middle|\; x, y, z, 0 \in S \right\}$$

It can be easily shown that N is a zero symmetric left near-ring with regard to matrix addition and matrix multiplication. Define $d, f :$ N $\longrightarrow$ N such that

$$d\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x^2 & 0 & 0 \end{pmatrix}$$

$$f\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & z^2 & 0 \end{pmatrix}$$

It can be easily proved that d is a multiplicative derivation of N and f is a left generalized multiplicative derivation of N with an associated multiplicative derivation d of N. But f is not a right generalized multiplicative derivation of N associated with multiplicative derivation d. It can be also verified that the maps d, f defined here are non-additive.

The study of commutativity of 3-prime near-rings was initiated by using derivations by H.E. Bell and G. Mason [6] in 1987. Subsequently a number of authors have investigated the commutativity of 3-prime near-rings admitting different types of derivations, generalized derivations, generalized multiplicative derivations(for reference see [1, 3, 4, 5, 6, 7, 8, 9], where further references can be found). In the present paper, we have obtained the commutativity of 3-prime near-rings, equipped with left generalized multiplicative derivations and satisfying some differential identities or conditions.

II. Preliminary Results

In this section we give some well-known results and we add some new lemmas which will be used throughout the next section of the paper. The proofs of the Lemmas 2.1 − 2.4 can be found in [6, 4], while those of Lemmas 2.5 − 2.7, can be found in [6, Lemma 1],[11,

Lemma 2.1] and [14, Lemma 2] respectively.

Lemma 2.1. Let N be a 3-prime near-ring. If $Z \setminus \{0\}$ contains an element z for which $z + z \in Z$, then $(N, +)$ is abelian.

Lemma 2.2. Let N be a 3-prime near-ring. If $z \in Z \setminus \{0\}$ and x is an element of N such that $xz \in Z$ or $zx \in Z$ then $x \in Z$.

Lemma 2.3. Let N be a 3-prime near-ring and A be nonzero semigroup ideal of N. Let d be a nonzero derivation on N. If $x \in$ N and $xd(\text{A}) = \{0\}$, then $x = 0$.

Lemma 2.4. Let N be a 3-prime near-ring. If N admits a nonzero derivation d for which $d(\text{A}) \subseteq Z$, then N is a commutative ring.

Lemma 2.5. Let N be a near-ring and d be a derivation on N. Then $(xd(y) + d(x)y)z = xd(y)z + d(x)yz$ for all $x, y, z \in$ N.

Lemma 2.6. A near-ring N admits a multiplicative derivation if and only if it is zero-symmetric.

Lemma 2.7 Let N be a near-ring with center Z and let d be derivation on N. Then $d(Z) \subseteq Z$.

Lemma 2.8. Let N be 3-prime near-ring. If N admits a left generalized multiplicative derivation f with associated multiplicative derivation d such that $f(u)v = uf(v)$ for all $u, v \in$ N, then $d = 0$.

Proof. We are given that $f(u)v = uf(v)$ for all $u, v \in$ N . Now replacing v by vw, where $w \in$ N , in the previous relation, we obtain that $f(u)vw = uf(vw)$ i.e.; $f(u)vw = u(vf(w) + d(v)w)$. By using hypothesis we arrive at $ud(v)w = 0$ i.e.; uN $d(v)w = \{0\}$. Now using the facts that N $f= \{0\}$ and N is a 3-prime near-ring, weobtain that $d(v)w = 0$, for all $v, w \in$ N. This shows that $d(v)w = 0$ i.e.; d(N)N $w = \{0\}$. Again 3-primeness of N and N $f= \{0\}$ force us to conclude that d(N) $= \{0\}$. We get $d = 0$

III. Main Results

We facilitate our discussion with the following theorem.

Theorem 3.1**.** Let f be a nonzero left generalized multiplicative derivation with associated nonzero multiplicative derivation d of a 3-prime near-ring N such that $f([x, y]) = 0$ for all $x, y \in$ N. Then N is a commutative ring.

Proof. Assume that $f([x, y]) = 0$ for all $x, y \in$ N . Putting xy in place of y, we obtain that $f([x, xy]) = f(x[x, y]) = xf([x, y]) + d(x)[x, y] = 0$. Using hypothesis, it is clear that $d(x)xy = d(x)yx$ for all $x, y \in$ N. (3.1) Replacing y by yr where $r \in$ N in (3.1) and using this relation again, we get $d(x)$N $[x, r] = \{0\}$ for all $x, r \in$ N . Hence by 3-primeness of N , for each $x \in$ N either $d(x) = 0$ or $x \in Z$. Let $u \in$ N. It is clear that either $d(u) = 0$ or $u \in Z$. We claim that if $d(u) = 0$, then also $u \in Z$. Suppose on contrary i.e.; $u\ f\in Z$. Now in the present situation, we prove that $d(uv)$ 0, for all $v \in$ N . For otherwise, we have $d(uv) = 0$ for all $v \in$ N, which gives us $ud(v) + d(u)v = 0$. This implies that $ud(v) = 0$ for all $v \in$ N . Replacing v by vr, where $r \in$ N , in the previous relation and using the same again, we arrive at uN $d(r) = \{0\}$. Using the facts that N is 3-prime and d 0, we obtain that $u = 0 \in Z$, which leads to a contradiction. Thus, we have seen that if $d(u) = 0$ and $u\ f\in Z$, then there exists $v \in$ N , such that $d(uv)\ f= 0$ and obviously v 0. Since $u, v \in$ N , we have $uv \in$ N.

We obtain that either $d(uv) = 0$ or $uv \in Z$. But as $d(uv)\ f= 0$, we infer that $uv \in Z$. Next we claim that $v\ f\in Z$, for otherwise we have $uvr = ruv$ i.e.; $v[u, r] = 0$ for all $r \in$ N . This shows that vN $[u, r] = \{0\}$. Now by 3-primeness of N , we conclude that $u \in Z$, as $v\ f= 0$, leading to a contradiction. Including all the above arguments, we conclude that if $d(u) = 0$ and $u\ f\in Z$, then there exists $v \in$ N , such that $d(uv)$0 and $v\ f\in Z$. As $v\ f\in Z$, shows that $d(v) = 0$. Finally, we get $d(uv) = ud(v) + d(u)v = u0 + 0v = 0$, leading to a contradiction again. We have proved that if $d(u) = 0$, then also $u \in Z$ i.e.; N $\subseteq Z$. Thus we obtain that $N = Z$ i.e; N is a commutative near-ring. If $N = \{0\}$ then N is trivially a commutative ring. If $N\ f= \{0\}$ then there exists 0 $x \in N$ and hence $x+x \in N = Z$. Now by Lemma 2.1; we conclude that N is a commutative ring.

Theorem 3.2**.** Let f be a nonzero left generalized multiplicative derivation with associated nonzero multiplicative derivation d of N such that $f[x, y] = x^k[x, y]x^l$, k, l; being some given fixed positive integers, for all $x, y \in$ N. Then N is a commutative ring.

Proof. It is given that $f[x, y] = x^k[x, y]x^l$, for all $x, y \in$ N. Replacing y by xy in the previous relation, we obtain that $f[x, xy] = x^k[x, xy]x^l$, i.e.; $f(x[x, y]) = x^k(x[x, y])x^l$. This implies that $xf[x, y] + d(x)[x, y] = x^kx[x, y]x^l = xx^k[x, y]x^l = xf[x, y]$. Now we obtain that $d(x)[x, y] = 0$ for all $x, y \in$ N, which is same as the relation (3.1) of Theorem.

3.1. Now arguing in the same way as in the Theorem 3.1., we conclude that N is a commutative ring.

The following example shows that the restriction of 3-primeness imposed on the hypothe- ses of Theorems 3.1 and 3.2 is not superfluous.

Example 3.2. Consider the near-ring N, taken as in Example 1.1. N is not 3-prime and (*i*) $f([x, y]) = 0$,

(*ii*) $f[x, y] = x^k[x, y]x^l$, k, l; being some given fixed positive integers, for all $x, y \in$ N.

However N is not a commutative ring.

Theorem 3.3. Let N be a 3-prime near-ring. If N admits a nonzero left generalized multiplicative derivation f with associated nonzero multiplicative derivation d such that either (*i*) $f([x, y]) = [f(x), y]$ for all $x, y \in$ N, or (*ii*) $f([x, y]) = [x, f(y)]$, for all $x, y \in$ N, then N is a commutative ring.

Proof. (*i*) Given that $f([x, y]) = [f(x), y]$, for all $x, y \in$ N. Replacing y by xy in the previous relation, we get $f([x, xy]) = [f(x), xy]$ i.e.; $f(x[x, y]) = [f(x), xy]$. This shows that $xf([x, y]) + d(x)[x, y] = f(x)xy - xyf(x)$. Using the given condition and the fact that $[f(x), x] = 0$, the previous relation reduces to $x(f(x)y - yf(x)) + d(x)[x, y] = xf(x)y - xyf(x)$. This gives us $d(x)[x, y] = 0$, for all $x, y \in$ N, which is the same as the relation (3.1) of Theorem 3.1. Now arguing in the similar way as in the Theorem 3.1., we conclude that N is a commutative ring.

(*ii*) We have $f([x, y]) = [x, f(y)]$, for all $x, y \in$ N. Replacing x by yx in the given condition, we obtain that $f([yx, y]) = [yx, f(y)]$ i.e.; $f(y[x, y]) = yxf(y) - f(y)yx$. This gives us $yf([x, y]) + d(y)[x, y] = yxf(y) - f(y)yx$. With the help of the given condition and using the fact that $[f(y), y] = 0$, previous relation reduces to $yxf(y) - yf(y)x + d(y)[x, y] = yxf(y) - yf(y)x$. As a result, we obtain that $d(y)[x, y] = 0$ i.e.; $d(y)[y, x] = 0$. This implies that $d(x)[x, y] = 0$, which is identical with the relation (3.1) of Theorem 3.1. Now arguing in the similar way as in the Theorem 3.1., we conclude that N is a commutative ring.

Theorem 3.4. Let N be a 3-prime near-ring. If N admits a nonzero left generalized multiplicative derivation f with associated nonzero multiplicative derivation d such that either (*i*) $f([x, y]) = [d(x), y]$ for all $x, y \in$ N, or (*ii*) $d([x, y]) = [f(x), y]$, for all $x, y \in$ N, then N is a commutative ring.

Proof. (*i*) We are given that $f([x, y]) = [d(x), y]$. Replacing y by xy in the previous relation we get $f([x, xy]) = [d(x), xy]$. This relation gives $f(x[x, y]) = [d(x), xy]$ i.e.; $xf([x, y]) + d(x)[x, y] = d(x)xy - xyd(x)$. Using the given condition and the fact that $[d(x), x] = 0$, we obtain that $xd(x)y - xyd(x) + d(x)[x, y] = xd(x)y - xyd(x)$. Finally we get $d(x)[x, y] = 0$, for all $x, y \in$ N, which is the same as the relation (3.1) of Theorem.

3.1. Now arguing in the similar way as in the Theorem 3.1., we conclude that N is a

commutative ring.

(ii) We have $d([x, y]) = [f(x), y]$. Putting xy in the place of y in the previous relation we get $d([x, xy]) = [f(x), xy]$. This implies that $d(x[x, y]) = f(x)xy - xyf(x)$ i.e.; $xd[x, y] + d(x)[x, y] = f(x)xy - xyf(x)$. Using the fact that $[f(x), x] = 0$, we obtain that $xd[x, y] + d(x)[x, y] = xf(x)y - xyf(x)$ i.e.; $xd[x, y] + d(x)[x, y] = x[f(x), y]$. Now using the hypothesis, we get $xd[x, y] + d(x)[x, y] = xd([x, y])$. Finally we have $d(x)[x, y] = 0$, for all $x, y \in N$, which is identical with the relation (3.1) of Theorem 3.1. Now ar- guing in the similar way as in the Theorem 3.1., we conclude that N is a commutative ring.

Theorem 3.5. Let N be a 3-prime near-ring. If N admits a nonzero left generalized multiplicative derivation f with associated nonzero multiplicative derivation d such that either (i) $f([x, y]) = \pm[x, y]$ for all $x, y \in N$, or (ii) $f([x, y]) = \pm(xoy)$ for all $x, y \in N$, then under the condition (i) N is a commutative ring and under the condition (ii) N is a commutative ring of characteristic 2.

Proof. Assume that condition (i) holds i.e.; $f([x, y]) = \pm[x, y]$ for all $x, y \in N$. Putting xy in place of y, we obtain, $f([x, xy]) = f(x[x, y]) = xf([x, y]) + d(x)[x, y] = \pm x[x, y]$. Using our hypothesis we get $d(x)xy = d(x)yx$ for all $x, y \in N$, which is identical with the relation (3.1) of Theorem 3.1. Now arguing in the similar way as in the Theorem 3.1., we conclude that N is a commutative ring. Under the condition (ii), using similar arguments, it is easy to show that N a commutative ring. But now under this situation, condition (ii) reduces to $xoy = 0$ for all $x, y \in N$ i.e.; $2xy = 0$. Suppose on contrary i.e.;

characteristic N = 2. As N is a prime ring, N will be a 2-torsion free ring. Now we get $xy = 0$, for all $x, y \in N$ i.e.; $xNy = \{0\}$. Finally, we have $N = \{0\}$, leading to a contradiction.

Theorem 3.6. Let N be a 3-prime near-ring. If N admits a nonzero left general- ized multiplicative derivation f with associated multiplicative derivation d such that $f(xy) = \pm(xy)$ for all $x, y \in N$, then $d = 0$.

Proof. Let $f(xy) = xy$ for all $x, y \in N$. Putting yz, where $z \in N$ for y in the previous relation, we obtain that $f(x(yz)) = x(yz)$ i.e.; $xf(yz) + d(x)yz = xyz$.

Using the hypothesis we get $d(x)yz = 0$ i.e.; $d(x)N z = \{0\}$. Since $N = \{0\}$, by 3-primeness of N, we get $d = 0$. Similar arguments hold if $f(xy) = -(xy)$ for all $x, y \in N$.

Very recently, Boua and Kamal [7, Theorem 1] proved that if N is a 3-prime near-ring, which admits nonzero derivations d_1 and d_2 such that $d_1(x)d_2(y) \in Z$, for all $x, y \in A$, where A is a nonzero semigroup ideal of N, then N is a commutative ring. Motivated by this result, we have obtained the following:

Theorem 3.7. Let N be a 3-prime near-ring and f be a nonzero left generalized multiplicative derivation with associated nonzero multiplicative derivation d of N such that either (i) $f(x)d(y) \in Z$, for all $x, y \in N$, or (ii) $d(x)f(y) \in Z$, for all $x, y \in N$ and d is a nonzero derivation of N. Then N is a commutative ring.

Proof. (i) We are given that $f(x)d(y) \in Z$, for all $x, y \in N$. Replacing y by yz, where $z \in N$ in the previous relation, we get $f(x)d(yz) \in Z$. This implies that f

$(x)(yd(z) + d(y)z) \in Z$ i.e.; $f(x)yd(z) + f(x)d(y)z \in Z$. This gives us $(f(x)yd(z) + f(x)d(y)z)z = z(f(x)yd(z) + f(x)d(y)z)$. Now using Lemma 2.5, we obtain that $f(x)yd(z)z + f(x)d(y)zz = zf(x)yd(z) + zf(x)d(y)z$ for all $x, y, z \in$ N . Using the hypothesis we infer that $f(x)yd(z)z + f(x)d(y)z^2 = zf(x)yd(z) + f(x)d(y)z^2$ i.e.; $f(x)yd(z)z = zf(x)yd(z)$. Putting $d(t)y$ for y, where $t \in$ N in the relation $f(x)yd(z)z = zf(x)yd(z)$, we get $f(x)d(t)yd(z)z = zf(x)d(t)yd(z)$ and now us- ing the hypothesis again, we arrive at $f(x)d(t)(yd(z)z - zyd(z))$. This shows that $f(x)d(t)$N $(yd(z)z - zyd(z)) = \{0\}$. Hence 3-primeness of N shows that either $f(x)d(t) = 0$ or $yd(z)z - zyd(z) = 0$. We claim that $f(x)d(t)$ 0 for all $x, t \in$ N . For otherwise if $f(x)d(t) = 0$ for all $x, t \in$ N , we have $f(x)d(\text{N}) = \{0\}$. Using Lemma 2.3, we find that $f(x) = 0$ for all $x \in$ N , leading to a contradiction. Thus there exist $x_0, t_0 \in$ N such that $f(x_0)d(t_0)$ 0. Hence, we arrive at $yd(z)z - zyd(z) = 0$ for all $y, z \in$ N . Now replacing y by $yf(x)$, where $x \in$ N in the previous relation and using the hypothesis again, we get $f(x)d(z)(yz - zy) = 0$ i.e.; $f(x)d(z)\text{N}(yz - zy) = \{0\}$. By hypothesis we have $f(\text{N}) \neq \{0\}$, hence there exists $u_0 \in$ N such that $f(u_0) \neq 0$. By Lemma 2.3, there exists $z_0 \in$ N such that $f(u_0)d(z_0) \neq 0$ and hence obviously $d(z_0) \neq 0$Again 3-primeness of N and the relation $f(x)d(z)\text{N}(yz - zy) = \{0\}$, ultimately give us $yz_0 = z_0y$ for all $y \in$ N . Now Lemma 2.5 insures that $z_0 \in Z$ and using Lemma 2.7, we obtain that $d(z_0) \in Z$. Since $f(u_0)d(z_0) \in Z$ and $0 \neq d(z_0) \in Z$, Lemma 2.2, implies that $f(u_0) \in Z$. Using the given hypothesis again we have $f(u_0)d(y) \in Z$. But 0 $f(u_0) \in Z$, thus Lemma 2.2, shows that $d(\text{N}) \subseteq Z$. Finally the Lemma 2.4, gives the required result.

(ii) Using the similar arguments as used in (i) with necessary variations, it can be easily shown that under the condition $d(x)f(y) \in Z$, for all $x, y \in$ N , N is a commutative ring.

Theorem 3.8**.** Let N be a 3-prime near-ring and f be a left generalized multiplica- tive derivation of N such that either (i) $d(y)f(x) = [x, y]$, for all $x, y \in$ N , or (ii) $d(y)f(x) = -[x, y]$, for all $x, y \in$ N and d is a nonzero derivation of N . Then N is a commutative ring.

Proof. (i) We are given that $d(y)f(x) = [x, y]$, for all $x, y \in$ N . (3.2) Case I: Let $f = 0$. Under this condition the equation (3.2) reduces to $[x, y] = 0$ for all $x, y \in$ N . This implies that $xy = yx$, for all $x, y \in$ N . Replacing x by xr, where $r \in$ N in the previous relation and using the same relation again we arrive at $\text{N}[r, y] = \{0\}$ i.e.; $[r, y]\text{N}[r, y] = \{0\}$. Now using 3-primeness of N, we conclude that $r \in Z$. This implies that N $\subseteq Z$. If $N = \{0\}$, then N is trivially a commutative ring. If N $\{0\}$ then there exists $0 \neq x \in N$ and hence $x + x \in N = Z$. Now by Lemma 2.1; we conclude that N is a commutative ring.

Case II: Let f 0. Replacing y by xy in the relation (3.2), we obtain that

$d(xy)f(x) = x[x, y]$ i.e.; $(xd(y) + d(x)y)f(x) = x[x, y]$. Using Lemma 2.5 and the relation (3.2), we arrive at $xd(y)f(x) + d(x)yf(x) = xd(y)f(x)$. This shows that $d(x)yf(x) = 0$ i.e.; $d(x)\text{N}f(x) = \{0\}$. Using 3-primeness of N , we conclude that for any given $x \in$ N , either $d(x) = 0$ or $f(x) = 0$. If for any given $x \in$ N , $f(x) = 0$, then relation (3.2) reduces to $[x, y] = 0$ for all $y \in$ N i.e.; $x \in Z$. By Lemma 2.7, this shows that $d(x) \in Z$. Finally using both possibilities, we deduce that $d(\text{A}) \subseteq Z$. By Lemma 2.4, we get our required result.

Using the similar arguments as used in (i), it can be easily proved that under the condition $d(y)f(x) = -[x, y]$, for all $x, y \in$ N , N is a commutative ring.

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