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Some Examples of Application of Differential Equation

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ABSTRACT

A differential equation is a mathematical equation that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two. Because such relations are extremely common, differential equations play an important role in many disciplines including engineering, physics, economics, and biology. Many fundamental laws of physics and chemistry can be formulated as differential equations. In biology and economics, differential equations are used to model the manners of complex systems. In pure mathematics, differential equations are studied from several different perspectives, mostly concerned with their solutions—the set of functions that satisfy the equation. Only the simplest differential equations are solvable by explicit formulas; however, some properties of solutions of a given differential equation may be determined without finding their exact form. The aim of study is not limited to mathematics and physics only, but variety of problems, decomposition of radioactive substances like in chemistry, the growth rate of bacteria and virus in biology, the rate of spreading infectious diseases in medical science. The most difficult problem of social science the rate of increase of population or the relation between demand and supply in commerce and calculation of interest on principle amount can be expressed in the form of a differential equation

Keywords : A Differential Equation and Its Application.

I. INTRODUCTION

The study of differential equations is a wide field in pure and applied mathematics, physics, and engineering. All of these disciplines are concerned with the properties of differential equations of various types. Many fundamental laws of physics and chemistry can be formulated as differential equations. In biology and economics, differential equations are used to model the behavior of complex systems. The mathematical theory of differential equations first developed together with the sciences where the equations had originated and where the results found application. Differential equations can be divided into several types, (i) An ordinary differential equation (ODE): An ordinary differential equation is an equation containing a function of one independent variable and its derivatives.(ii) Linear differential equation : A differential equation is linear if the unknown function and its derivatives have degree 1.

Newton's law of cooling states that the rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings provided the temperature difference is small and the nature of radiating surface remains same. in a simple differential equation for temperature-difference as a function of time. This equation has a solution that specifies a simple negative exponential rate of temperature-difference decrease, over time. This characteristic time function for temperature-difference behavior is also associated with Newton's law of cooling. Radioactive decay or radioactivity is the process by which an unstable atomic nucleus loses energy (in terms of mass in its rest frame) by emitting radiation, such as an alpha particle, beta particle with neutrino or only a neutrino in the case of electron capture, gamma ray, or electron in the case of internal conversion. The mathematics of radioactive decay depend on a key assumption that a nucleus of a radionuclide has no "memory" or way of translating its history into its present behavior.

Suppose that P(t) describes the quantity of a population at time t. For example, P(t)could be the number of milligrams of bacteria in a particular beaker for a biology experiment, or P(t) could be the number of people in a particular country at a time t. A model of population growth tells plausible rules for how such a population changes over time. The simplest model of population growth is the exponential model, which assumes that there is a constant parameter k, called the growth parameter, such that P'(t) = k P(t) holds for all time t. This differential equation itself might be called the exponential differential equation, The fundamental difficulty is that the exponential differential equation ignores the fact that there are limits to resources needed for the population to grow. It ignores the needs for food, oxygen, and space; and it ignores the accumulation of waste products that inevitably arise.

Exmples:

(i)The temperature of the dead body of a person was 90°F at 10:30 A.M. After one hour the dead body temperature is reduced to 88°F.If room temperature is 70° F,at what time did person die?(normal body temperature of living person is 98.6°F)

By Newton's law, the rate of cooling of a body is proportional to the difference of the temperature of the atmosphere and the temperature of the body. If the temperature of the body is T°F at time t, then by Newton's law, $\frac{dT}{dt} \propto (70 - T) \Rightarrow \frac{dT}{dt} = -k(70 - T) \Rightarrow \frac{dT}{dt} = k(T - 70)$

$\Rightarrow \frac{dT}{T-70} = k\,dt \Rightarrow \int \frac{dT}{T-70} = k \int dt \Rightarrow \log(T - 70) = kt + c$

Now if t = 0 (10:30A.M.) , $T = 90^\circ$F $\therefore \log(90-70) = k(0) + c \Rightarrow c = \log(20)$

If t = 1(at 11:30A.M) then $T = 88^\circ$F $\therefore \log(88-70) = k(1) + \log(20)$

$\therefore \log(18) = k + \log(20) \Rightarrow k = \log(18) - \log(20) = \log\frac{18}{20} = \log\frac{9}{10}$

$\therefore \log(T - 70) = t\log\frac{9}{10} + \log(20) \Rightarrow t = \frac{\log(\frac{T-70}{20})}{\log(\frac{9}{10})}$

Now let us find the temperature of the body when the person died. i.e. at the time the body temperature was 98.6°F.

$\therefore t = \frac{\log(\frac{98.6-70}{20})}{\log(\frac{9}{10})} = \frac{0.1533}{-0.0457} = -3.35 \therefore t = -3.35$ hours (approximately)

∴ The person died at 6:55 A.M.

(ii) Rate of decay of a radio-active body is proportional to its mass at that time. After a decay of two hours the mass of the body is 100 grams and after four hours it is 80 grams. Find initial mass of the body.

Let initial mass of the body is m_0 grams. If the mass of the body is m grams after time t, then from the rate of decay we have, $\frac{dm}{dt} \propto m \Rightarrow \frac{dm}{dt} = -kt \Rightarrow \frac{dm}{m} = -kdt \Rightarrow \int \frac{dm}{m} = -k \int dt$

$\Rightarrow \log(m) = -kt + c$

Now if t = 0, $m = m_0$ $\therefore \log(m_0) = k(0) + c \Rightarrow c = \log(m_0)$

$\Rightarrow \log(m) = -kt + \log(m_0) \Rightarrow \log(m) - \log(m_0) = -kt \Rightarrow \log(\frac{m}{m_0}) = -kt \Rightarrow \frac{m}{m_0} = e^{-kt}$

$m = m_0 e^{-kt}$

If t = 2 then m = 100 $\therefore 100 = m_0 e^{-2k}$ and if t = 4 then m = 80

$\therefore 80 = m_0 e^{-4k}$

$\therefore \frac{100}{80} = \frac{m_0 e^{-2k}}{m_0 e^{-4k}} = e^{2k} \Rightarrow e^{-2k} = \frac{80}{100} = \frac{4}{5}$

$\therefore 100 = m_0 e^{-2k} \Rightarrow 100 = \frac{4}{5} m_0 \Rightarrow m_0 = 125$grams

∴initially the mass of the body will be 125 grams.

(iii)In an experiment of culture of bacteria in a laboratory, rate of increase of bacteria is proportional to the number of bacteria present at that time.If in two hour the number of bacteria gets doubled,then (a) What number of bacteria will be there at the end of 6 hours? (b) If the number of bacteria is 48000 at the end of 4 hour, find the number of bacteria in the beginning.

Let initial number of bacteria is x_0 . If number of bacteria is x at time t, then from the rate of increase of bacteria we have, $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx \Rightarrow \frac{dx}{x} = kdt \Rightarrow \int \frac{dx}{x} = k \int dt$

$\Rightarrow \log(x) = kt + c$

Now if t = 0, $x = x_0$ $\therefore \log(x_0) = k(0) + c \Rightarrow c = \log(x_0)$

$\Rightarrow \log(x) = kt + \log(x_0) \Rightarrow \log(x) - \log(x_0) = kt \Rightarrow \log(\frac{x}{x_0}) = kt \Rightarrow \frac{x}{x_0} = e^{kt}$

$x = x_0 e^{kt}$……..(I)

If t = 2 then $x = 2x_0$ $\therefore 2x_0 = x_0 e^{2k} \Rightarrow 2 = e^{2k}$

(a) Now if t = 6 then $x = x_0 e^{6k} \Rightarrow x = x_0 (e^{2k})^3 \Rightarrow x = x_0 (2)^3$ ($\because 2 = e^{2k}$)

$\therefore x = 8x_0$

∴ At the end of 6 hour number of bacteria is eight times of the beginning.

(b) Now If t = 4 then x = 48000 , ∴ By (I) we have $4800 = x_0 e^{4k}$

$\therefore 4800 = x_0 (e^{2k})^2 \Rightarrow 4800 = x_0 (2)^2$ ($\because 2 = e^{2k}$) $\Rightarrow x_0 = \frac{48000}{4} = 12000$

∴ The number of bacteria in the beginning is 12000

(iv) The population of a city is growing at a rate that is proportional to the population of the city.The population in 2000 was 20,00000 and in 2010 the population was 2200000.Estimate the population 2020.

Suppose population of city at time t is P.As per given condition $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = kP \Rightarrow \frac{dP}{P} = kdt \Rightarrow \int \frac{dP}{P} = k \int dt$

$\Rightarrow \log(P) = kt + c$

Now if t = 0, P = 20,00000 $\therefore \log(20{,}00000) = k(0) + c \Rightarrow c = \log(20{,}00000)$

$\Rightarrow \log(P) = kt + \log(20{,}00000) \Rightarrow \log(x) - \log(20{,}00000) = kt \Rightarrow \log(\frac{P}{20{,}00000}) = kt$

$\Rightarrow \frac{P}{20{,}00000} = e^{kt}$

$P = 20{,}00000 e^{kt}$……..(I)

Now If t = 10 then P = 2200000 ∴ By (I) we have $2200000 = 20{,}00000 e^{10k}$

$\Rightarrow e^{10k} = \frac{22}{20} = \frac{11}{10}$

Now If t = 20 then by (I) we have $P = 20{,}00000 e^{20k}$

$\therefore P = 20{,}00000 (e^{10k})^2 \Rightarrow P = 20{,}00000 (\frac{11}{10})^2$ ($\because e^{10k} = \frac{11}{10}$)

$\Rightarrow P = 20{,}00000(\frac{121}{100}) = 20000 \times 121 = 2420000$

∴ In 2020 the population of the city will be 2420000.

II. RESULTS AND DISCUSSION

Above examples of application of differential equation is useful in study of physics,microbiology and in population growth model.

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