

Vertex Coloring of A Complement Fuzzy Graph

R. Vinitha¹, H. Haseena Begum²

¹M.Phil Mathematics Department of Mathematics, Dr.SNS Rajalakshmi College of Arts and Science,
(Autonomous) Coimbatore

²Assistant professor Department of Mathematics, Dr.SNS Rajalakshmi College of Arts and Science,
(Autonomous) Coimbatore

ABSTRACT

$G = (V, \gamma, \delta)$ $G(V, \gamma, \delta)$ be a simple connected undirected graph where V is a set of vertices and each vertices has a membership value γ . Colors are assigned to the vertices so that no two adjacent vertices share the same colors. Such an assignment is called vertex coloring. Chromatic number of that graph is the minimum number of color needed to color the graph. In this paper we introduced coloring function of complement of a fuzzy graph.

Keywords : Complement Fuzzy Graph, Vertex Coloring, α Cut of Fuzzy Graph

I. INTRODUCTION

A graph represents information involving relationship between objects. The vertices represents object and edges represents relation. We get partial information about that problem in many real world problems. When the descriptions of the objects or its relationship or in both are not clear, we need to design fuzzy graph model. One of the most important problems of fuzzy graph theory is Fuzzy graph coloring. Vertex coloring and edge coloring are the two types of coloring usually associated with any graph. Coloring of the vertices with minimal number of colors such that no two adjacent vertices should have the same color is called the proper coloring of a graph. The graph is called properly colored graph if minimum numbers of colors are given to the graph.

COLORING OF COMPLEMENT FUZZY GRAPH

To calculate all the different membership value of vertices and edges in the complement of a fuzzy graph. The membership value will work as α Cut of this complement fuzzy graph. Depending upon the values of α Cut different types of fuzzy graphs can be

calculated for the same complement fuzzy graph. Then we color all the vertices of the complement fuzzy graph so that no incident vertices will not get the same color and find the fewest number of colors will need to color the complement fuzzy graph is known as chromatic number.

For solving the problem we done the calculation into three steps

- In the first step, take a fuzzy graph which has 6 vertices and 6 edges.
- In the second step, find the complement of this fuzzy graph.
- In third step, define the vertex coloring function to color the complement fuzzy graph.

STEP 1.

Consider the fuzzy graph with 6 vertices $v_1, v_2, v_3, v_4, v_5, v_6$ and the membership values are 0.9, 0.7, 0.8, 0.7, 0.9, 1. The graph consists of 6 edges $e_1, e_2, e_3, e_4, e_5, e_6$ and the membership values are 0.5, 0.4, 0.7, 0.8, 0.6, 0.8. The fuzzy graph is shown in figure 3.

$$\delta_1 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.5 & 0.0 & 0.4 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.0 & 0.7 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.0 & 0.8 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.6 \\ 0.8 & 0.0 & 0.0 & 0.0 & 0.6 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 1

$$E_1 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & e_1 & 0.0 & 0.0 & 0.0 & e_6 \\ e_1 & 0.0 & e_2 & 0.0 & 0.0 & 0.0 \\ 0.0 & e_2 & 0.0 & e_3 & 0.0 & 0.0 \\ 0.0 & 0.0 & e_3 & 0.0 & e_4 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_4 & 0.0 & e_5 \\ e_6 & 0.0 & 0.0 & 0.0 & e_5 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 2

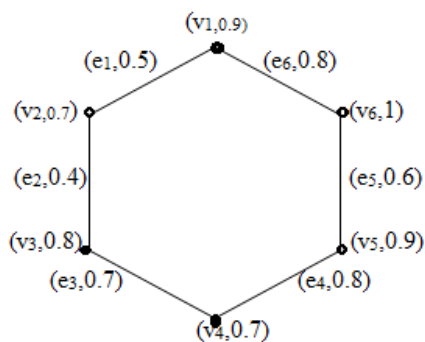


Figure 3

Matrix 1 represent the membership value of edges and matrix 2 represent the name of edges between the vertices.

STEP 2

Calculate the complement of a fuzzy graph

$$\delta_2 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.5 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.4 \\ 0.5 & 0.0 & 0.0 & 0.0 & 0.7 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.6 & 0.0 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.0 & 0.8 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 3

$$E_2 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & e_1 & 0.0 & e_5 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_4 & 0.0 & e_2 \\ e_1 & 0.0 & 0.0 & 0.0 & e_3 & 0.0 \\ 0.0 & e_4 & 0.0 & 0.0 & 0.0 & e_6 \\ e_5 & 0.0 & e_3 & 0.0 & 0.0 & 0.0 \\ 0.0 & e_2 & 0.0 & e_6 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 4

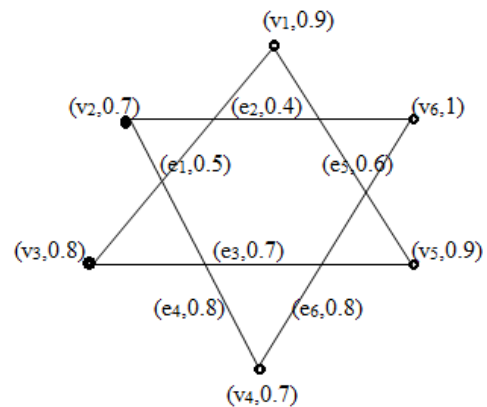


Figure 4

The matrix 3 represent the membership value of edges and the matrix 4 represent the name of edges between the vertices.

STEP 3

A fuzzy graph $G = (V_F, E_F)$, the fuzzy numbers $\chi(G) = \{(C_\alpha, \alpha)\}$ where C_α is edge chromatic number of G_α and α values are different membership value of vertex and edge of graph G.

This graph there is six α Cuts. There are $\{0.5, 0.4, 0.7, 0.6, 0.8, 1\}$

When $\alpha = 0.5$

$$\delta_3 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.5 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.4 \\ 0.5 & 0.0 & 0.0 & 0.0 & 0.7 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.6 & 0.0 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.0 & 0.8 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 5

$$E_3 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & e_1 & 0.0 & e_5 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_4 & 0.0 & e_2 \\ e_1 & 0.0 & 0.0 & 0.0 & e_3 & 0.0 \\ 0.0 & e_4 & 0.0 & 0.0 & 0.0 & e_6 \\ e_5 & 0.0 & e_3 & 0.0 & 0.0 & 0.0 \\ 0.0 & e_2 & 0.0 & e_6 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 6

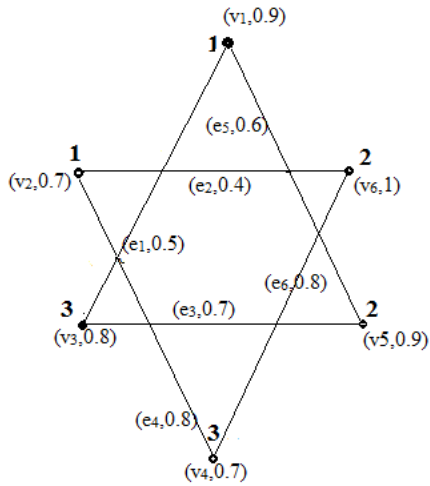


Figure 5 $\chi(0.5) = 3$

Chromatic number of this graph is 3.

When $\alpha = 0.4$

$$\delta_4 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.4 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.7 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.6 & 0.0 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.0 & 0.8 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 7

$$E_4 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & e_5 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_4 & 0.0 & e_2 \\ 0.0 & 0.0 & 0.0 & 0.0 & e_3 & 0.0 \\ 0.0 & e_4 & 0.0 & 0.0 & 0.0 & e_6 \\ e_5 & 0.0 & e_3 & 0.0 & 0.0 & 0.0 \\ 0.0 & e_2 & 0.0 & e_6 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 8

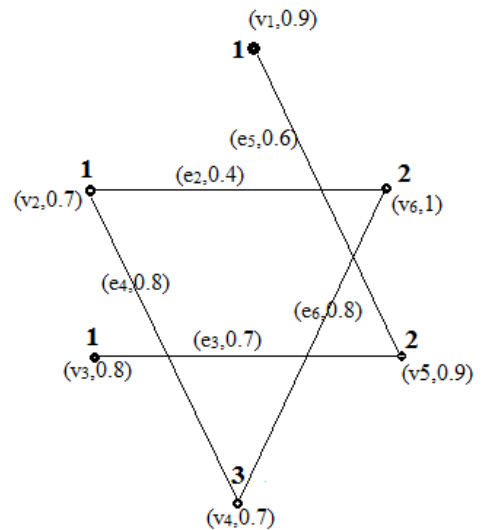


Figure 6 $\chi(0.4) = 3$

Chromatic number of this graph is 3.

When $\alpha = 0.7$

$$\delta_5 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.7 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.6 & 0.0 & 0.7 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 9

$$E_5 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & e_5 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & e_3 & 0.0 \\ 0.0 & e_4 & 0.0 & 0.0 & 0.0 & e_6 \\ e_5 & 0.0 & e_3 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_6 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 10

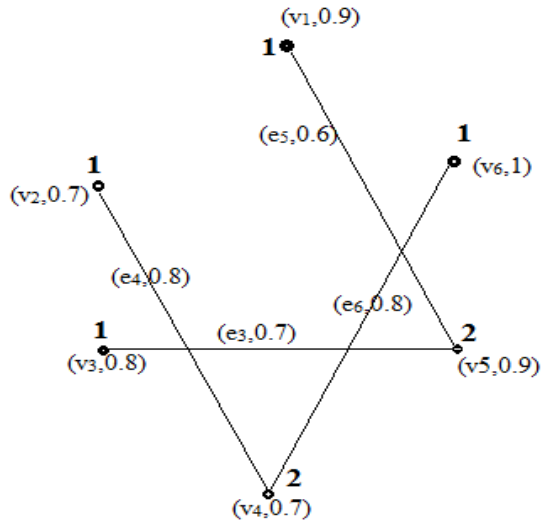


Figure 7 $\chi(0.7) = 2$

Chromatic number of this graph is 2.

When $\alpha = 0.6$

$$\delta_6 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.0 & 0.8 \\ 0.6 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 11

$$E_6 = \begin{matrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & e_5 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & e_4 & 0.0 & 0.0 & 0.0 & e_6 \\ e_5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & e_6 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 12

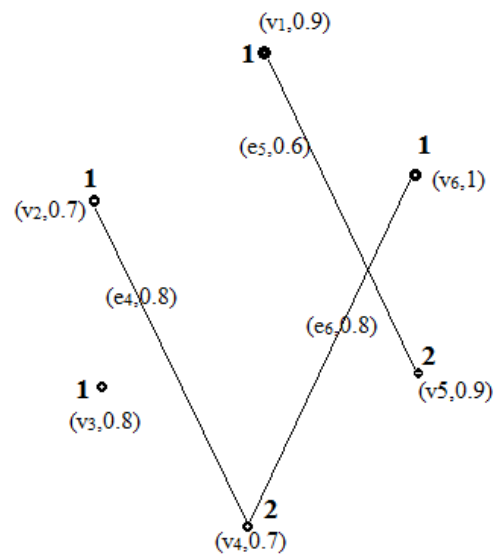


Figure 8 $\chi(0.6) = 2$

Chromatic number of this graph is 2.

When $\alpha = 0.8$

$$\delta_7 = \begin{matrix} & v_1 & v_2 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.8 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 13

$$E_7 = \begin{matrix} & v_1 & v_2 & v_4 & v_5 & v_6 \\ \begin{matrix} v_1 \\ v_2 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & e_4 & 0.0 & 0.0 \\ 0.0 & e_4 & 0.0 & 0.0 & e_6 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & e_6 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 14

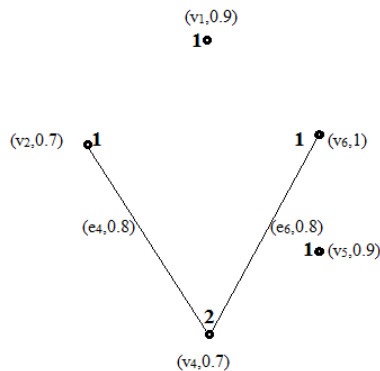


Figure 9 $\chi(0.8) = 2$

Chromatic number of this graph is 2.

When $\alpha = 1$

$$\delta_8 = \begin{matrix} & v_2 & v_4 & v_6 \\ \begin{matrix} v_2 \\ v_4 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 15

$$E_8 = \begin{matrix} & v_2 & v_4 & v_6 \\ \begin{matrix} v_2 \\ v_4 \\ v_6 \end{matrix} & \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \end{matrix}$$

Matrix 16

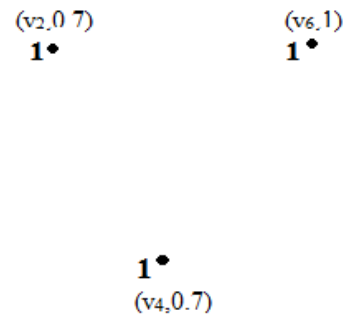


Figure 10 $\chi(1) = 1$

Chromatic number of this graph is 1.

The chromatic number of a fuzzy graph G is

$$\chi(G) = \{(3,0.5), (3,0.4), (2,0.7), (2,0.6), (2,0.8), (1,1)\}$$

II. CONCLUSION

In this paper find the complement of a given fuzzy graph and color all the vertices of that complement fuzzy graph. Here chromatic number of fuzzy graph will decrease when the value of α Cut of the fuzzy graph increase.

III. REFERENCES

- [1]. Arindam Dey, Anita pal, "Vertex coloring of a fuzzy graph using alpa cut ".
- [2]. Daniel Mark, "Graph coloring problems and their applications in Scheduling ".
- [3]. Arindam Dey, Dhrubajyoti Ghosh, Anita pal, "Edge coloring of a complement fuzzy graph ".
- [4]. Mordeson J.N, " Operations on Fuzzy graphs".
- [5]. Mordeson J.N, "Fuzzy line graph ".