

Thermo Diffusion Effect on Three Dimensional Mhd Free Convective Flow Through A Porous Plate With Periodic Permeability

R. Panneerselvi*1, M. Suchithra²

*1Department of Mathematics, Bharathiar University / PSGR Krishnammal College for Women, Coimbatore, Tamilnadu, India 2Department of Mathematics, Bharathiar University / PSGR Krishnammal College for Women, Coimbatore, Tamilnadu, India

ABSTRACT

The analysis is made to study the effect of Thermal Diffusion on a three dimensional free convective flow with mass transfer of an incompressible viscous electrically conducting fluid past a porous vertical plate with transverse sinusoidal suction velocity. A uniform magnetic field is assumed to be applied transversely to the direction normal to the plate. The magnetic Reynolds number is considered to be small so that the induced magnetic field can be neglected. The momentum, temperature and concentration equations are non-dimensionalised and solved using perturbation technique. The graphs are plotted for various governing parameters viz, Grashof number (G), Hartmann number (M), Reynolds number (Re), Prandtl number (Pr), Schmidt number (Sc) and Soret number (Sr).

Keywords: Thermal Diffusion, Mixed Convection, MHD Flow, Soret Number, Electrically Conducting Fluid

I. INTRODUCTION

The MHD laminar flow through a porous medium accomplish a critical position in agricultural engineering resources of underground water, leakage of water in river beds; in chemical engineering for filtration and purification system; in petroleum generation look away the motion of natural gasoline, oil and water via the oil channels/reservoirs. The magneto-hydrodynamic drift many packages in designing, cooling structures, petroleum industry, purification of crude oil, and in industry/polymer. The unfastened convection flow through a porous medium has much practicable benefit to geophysical software.

The flux of mass is brought on because of temperature gradient is referred to as the Soret effect or the thermal diffusion impact. The Soret impact is applied for isotope separation and in combos between gases with very mild molecular weight and medium molecular weight in which the diffusion – thermo impact is observed to be of significance such that it cannot be neglected.

Acharya, Dash and Singh, (2000), investigated the free convection and mass transfer flow of a viscous

incompressible and electrically conducting fluid through a porous medium bounded by vertical infinite surface with constant suction velocity and constant heat flux under the action of uniform magnetic field applied normal to the direction of the flow. The Soret effect and the magnetic field effect on the flow with heat and mass transfer characteristics is studied by Ahmed, Sarmah, and Kalita, (2011).

Reddy (2016), analysed the Dufour and Soret effects on an MHD free convection flow past a vertical porous plate placed in a porous medium in the presence of chemical reaction, thermal radiation and heat source. The effects of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous, incompressible and electrically conducting fluid on a moving inclined heated porous plate is discussed by Raju and Veeresh, (2015). Mohammed Ibrahim and Suneetha, (2015), investigated the effect of Soret and chemical reaction effect on steady MHD mixed convective heat and mass transfer flow embedded in a porous medium in the presence of heat source, viscous and Joules dissipation. Sushila Chand Pradhan, Panda and Satapathy (2015), discussed the effect of mass transfer on three dimensional free convective flow of viscous incompressible fluid through a highly porous medium in the presence of uniform transverse magnetic field. Das, Mitra and Mishra, (2011), studied the effect of magnetic field and the permeability of the medium on the three dimensional flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by an infinite vertical porous plate in the presence of a transverse magnetic field and periodic suction.

In view of the importance of the effect of the magnetic field and thermal diffusion, it is proposed to study a problem of three dimensional MHD free convective flows with mass transfer past a porous vertical infinite plate taking into account the effect of thermal diffusion. Although the flow over a flat plate is the simplest case of boundary layer development in external flow, yet its significance cannot be undervalued because of its relevance to numerous engineering applications. Several configurations such as flow over airfoils, turbine blades, ship hulls etc. can initially be estimated as flow past flat plates. Of course we have chosen a simple model of a three-dimensional flow caused by transverse sinusoidal suction velocity. Here, our main objectives are to study the Soret effect and the magnetic field effect on the flow and heat and mass transfer characteristics.

II. MATHEMATICAL ANALYSIS

Consider three dimensional flows of a viscous, incompressible electrically conducting fluid with simultaneous heat and mass transfer through a highly porous medium which is bounded by an infinite vertical porous plate with constant suction. The plate is lying vertically on the \tilde{x} - \tilde{z} plane with \tilde{x} axis taken in the upward direction. The \tilde{y} - axis is taken normal to flow laminarly with a uniform free stream velocity \tilde{u} . The permeability of the porous medium is assumed to be of the form

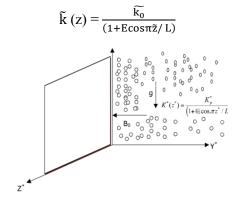


Figure 1. Schematic diagram of the flow

where $\widetilde{k_0}$ is the mean permeability of the medium, L is the wave length of the permeability distribution and E (<<1) is the amplitude of the permeability variation. All the fluid properties are assumed to be constant except that the influence of the density variation with temperature is considered only in the body force term. Denoting velocity components \widetilde{u} , \widetilde{v} , \widetilde{w} in the direction \widetilde{x} , \widetilde{y} , \widetilde{z} and temperature by \widetilde{T} , the flow through a highly porous medium is governed by the following equations:-

$$\frac{\partial \tilde{\mathbf{v}}}{\partial \tilde{\mathbf{v}}} + \frac{\partial \tilde{\mathbf{w}}}{\partial \tilde{\mathbf{z}}} = 0 \tag{1}$$

$$\widetilde{v} \frac{\partial \widetilde{u}}{\partial \widetilde{v}} + \widetilde{w} \frac{\partial \widetilde{u}}{\partial \widetilde{z}} = g \beta \left(\widetilde{T} - \widetilde{T}_{\infty} \right) + g \widetilde{\beta} \left(\widetilde{C} - \widetilde{C}_{\infty} \right) + \upsilon \left(\frac{\partial^2 \widetilde{u}}{\partial \widetilde{v}^2} + \frac{\partial^2 \widetilde{u}}{\partial \widetilde{v}^2} + \frac{\partial^2 \widetilde{u}}{\partial \widetilde{v}^2} + \frac{\partial^2 \widetilde{u}}{\partial \widetilde{v}^2} \right) + \varepsilon \left(\frac{\partial \widetilde{u}}{\partial \widetilde{v}^2} + \frac{\partial^2 \widetilde{u}}{\partial \widetilde{v}^2} + \frac{\partial^$$

$$\frac{\partial^2 \widetilde{\mathbf{u}}}{\partial \widetilde{\mathbf{z}}^2} - \frac{\sigma B_0^2 (\widetilde{\mathbf{u}} - \mathbf{u}_0)}{\rho} - \frac{\upsilon}{\widetilde{\mathbf{k}}} (\widetilde{\mathbf{u}} - \mathbf{u}_0)$$
 (2)

$$\widetilde{v}\frac{\partial\widetilde{v}}{\partial\widetilde{y}} + \widetilde{w}\frac{\partial\widetilde{v}}{\partial\widetilde{z}} = -\frac{1}{\rho}\frac{\partial\widetilde{p}}{\partial\widetilde{y}} + v\left(\frac{\partial^{2}\widetilde{v}}{\partial\widetilde{v}^{2}} + \frac{\partial^{2}\widetilde{v}}{\partial\widetilde{z}^{2}}\right)$$
(3)

$$\widetilde{v} \frac{\partial \widetilde{w}}{\partial \widetilde{y}} + \widetilde{w} \frac{\partial \widetilde{w}}{\partial \widetilde{z}} = -\frac{1}{\rho} \frac{\partial \widetilde{p}}{\partial \widetilde{z}} + \upsilon \left(\frac{\partial^2 \widetilde{w}}{\partial \widetilde{y}^2} + \frac{\partial^2 \widetilde{w}}{\partial \widetilde{z}^2} \right) - \frac{\sigma B_0^2}{\rho} \widetilde{w}$$
(4)

$$\widetilde{\mathbf{v}} \frac{\partial \widetilde{\mathbf{T}}}{\partial \widetilde{\mathbf{v}}} + \widetilde{\mathbf{w}} \frac{\partial \widetilde{\mathbf{T}}}{\partial \widetilde{\mathbf{z}}} = \frac{k}{\rho C_{n}} \left(\frac{\partial^{2} \widetilde{\mathbf{T}}}{\partial \widetilde{\mathbf{v}}^{2}} + \frac{\partial^{2} \widetilde{\mathbf{T}}}{\partial \widetilde{\mathbf{z}}^{2}} \right)$$
 (5)

$$\widetilde{v} \frac{\partial \widetilde{\phi}}{\partial \widetilde{y}} + \widetilde{w} \frac{\partial \widetilde{\phi}}{\partial \widetilde{z}} = D_{M} \left(\frac{\partial^{2} \widetilde{\phi}}{\partial \widetilde{y}^{2}} + \frac{\partial^{2} \widetilde{\phi}}{\partial \widetilde{z}^{2}} \right) + D_{T} \left(\frac{\partial^{2} \widetilde{T}}{\partial \widetilde{y}^{2}} + \frac{\partial^{2} \widetilde{T}}{\partial \widetilde{z}^{2}} \right)$$

$$(6)$$

where g, \tilde{p} , β , $\tilde{\beta}$, μ , C_p , k denote respectively, the acceleration due to gravity, fluid pressure, coefficient of thermal expansion, coefficient of concentration expansion, viscosity, specific heat at constant pressure and thermal conductivity.

The corresponding boundary conditions become:

$$\begin{split} \widetilde{y} &= 0 \colon \widetilde{u} = 0, \, \widetilde{v} = -v \left(1 + E \cos \pi \frac{\widetilde{z}}{L} \right), \, \widetilde{w} = 0, \, \widetilde{T} = \widetilde{T}_{\infty}, \, \widetilde{\phi} = \widetilde{\phi}_{w} \\ (7) \\ \widetilde{y} &\to \infty \colon \widetilde{u} = 0, \, \widetilde{P} = \widetilde{P}_{\infty}, \, \widetilde{w} = 0, \, \widetilde{T} = \widetilde{T}_{\infty}, \, \widetilde{\phi} = \widetilde{\phi}_{\infty} \end{split} \tag{8}$$

where \widetilde{T}_w , \widetilde{T} are the temperatures of the plate and the temperature of the fluid far away from the plate, \widetilde{p}_{∞} is a constant pressure in the free stream and v>0 is a constant and the negative sign indicates that suction is towards the plate.

Introducing the following non dimensional quantities

$$\begin{split} y &= \frac{\widetilde{y}}{L} \;,\;\; z \;= \frac{\widetilde{z}}{L} \;,\;\; u \;= \frac{\widetilde{u}}{U} \;,\;\; v \;= \frac{\widetilde{v}}{U}, \;\; w \;= \frac{\widetilde{w}}{U}, \;\; p \;= \frac{\widetilde{P}}{\rho U^2} \;,\\ \theta &= \frac{\widetilde{T} - \widetilde{T}_{\infty}}{\widetilde{T}_{W} - \widetilde{T}_{\infty}} \;,\; \phi = \frac{\widetilde{C} - \widetilde{C}_{\infty}}{\widetilde{C}_{W} - \widetilde{C}_{\infty}} \;, \end{split} \tag{9}$$

Equations (1) – (6) reduce to the following form

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{10}$$

$$\begin{array}{lll} v \, \frac{\partial u}{\partial y} + \, w \, \frac{\partial u}{\partial z} = \, \operatorname{Gr} \, \operatorname{Re} \, \theta + \operatorname{Gm} \, \operatorname{Re} \, C + \frac{1}{\operatorname{Re}} \, \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - & \frac{\partial^2 \theta_0}{\partial y^2} - \operatorname{Pr} \operatorname{Re} \, v_0 \, \frac{\partial \theta_0}{\partial y} = 0 \\ \frac{(u-1)(1+ \in \cos \pi z)}{\operatorname{Re} \, K_0} - \operatorname{M}(u-1) & (11) & \frac{\partial^2 \phi_0}{\partial y^2} - \operatorname{ScRe} \, v_0 \, \frac{\partial \phi_0}{\partial y} + \frac{S}{\operatorname{Re}} \\ v \, \frac{\partial v}{\partial y} + w \, \frac{\partial v}{\partial z} = - \frac{\partial P}{\partial y} + \frac{1}{\operatorname{Re}} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) & (12) \\ v \, \frac{\partial w}{\partial y} + w \, \frac{\partial w}{\partial z} = - \frac{\partial P}{\partial z} + \frac{1}{\operatorname{Re}} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \operatorname{Mw} & (13) & y = 0 \colon u_0 = 0, \, v_0 = -1, \, w_0 = 0 \\ v \, \frac{\partial \theta}{\partial y} + w \, \frac{\partial \theta}{\partial z} = \frac{1}{\operatorname{RePr}} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{\operatorname{So}}{\operatorname{Re}} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) & (14) & y \to \infty \colon u_0 = 1, v_0 = -1, w_0 = 0 \\ v \, \frac{\partial \phi}{\partial y} + w \, \frac{\partial \phi}{\partial z} = \frac{1}{\operatorname{ReSc}} \left(\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) + \frac{\operatorname{So}}{\operatorname{Re}} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) & \text{The solution for the about } \\ w \, \frac{\partial \phi}{\partial z} = \frac{\operatorname{Vg} \beta(\tilde{T}_W - \tilde{T}_\infty)}{\operatorname{U}^3} & \text{Grashof number for heat} & \theta_0 = e^{-\operatorname{Re} Pr} \, y \\ \theta_0 = (1-B_0) \, e^{-\operatorname{Re} \operatorname{Sc} \, y} + B_0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \, w_0 = 0 \\ w \, \text{in } v_0 = -1, \, p_0 = p_\infty, \,$$

$$Fr = \frac{\frac{UL}{U}}{\frac{U^3}{U}} \quad \text{(Grashof number for mass transfer}$$

$$Re = \frac{\frac{UL}{U}}{\frac{U}{U}} \quad \text{Reynolds number}$$

$$Pr = \frac{\frac{\mu C_p}{k}}{k} \quad \text{Prandtl number}$$

$$Sc = \frac{\upsilon}{D} \quad \text{Schmidt number}$$

$$M = \frac{\sigma B_0^2 L}{\rho U} \quad \text{Magnetic parameter}$$

$$K_0 = \frac{\widetilde{K_0}}{L^2} \quad \text{Permeability parameter}$$

$$So = \frac{D_T(\widetilde{T}_W - \widetilde{T}_\infty)}{\upsilon(\widetilde{C}_W - \widetilde{C}_\infty)} \quad \text{Soret number}$$

The non-dimensionalized boundary conditions are y = 0: u = 0, $v = -(1 + E\cos\pi z)$, w = 0, $\theta = 1$, $\phi = 1$ $y \rightarrow \infty$: u = 1, v = -1, w = 0, $p = p_{\infty}$, $\theta = 0$, $\phi = 0$ (17)

Method Of Solution

Assuming the solutions to be of the following form

$$\begin{array}{l} u \ (y,z) = u_0(y) + E \ u_1(y,z) + E^2 \ u_2(y,z) + \\ v \ (y,z) = v_0(y) + E \ v_1(y,z) + E^2 \ v_2(y,z) + \\ w \ (y,z) = w_0(y) + E \ w_1(y,z) + E^2 \ w_2(y,z) + \\ p \ (y,z) = p_0(y) + E \ p_1(y,z) + E^2 \ p_2(y,z) + \\ \theta \ (y,z) = \theta \ _0(y) + E \ \theta \ _1(y,z) + E^2 \ \theta \ _2(y,z) + \\ \phi \ (y,z) = \phi_0(y) + E \ \phi_1(y,z) + E^2 \ \phi_2(y,z) + \\ \end{array}$$

when E = 0 the problem reduces to two-dimensional free convective flow past a porous vertical plate with constant suction and transversely applied uniform magnetic field. When we take terms of order 1, the equations are

$$\frac{\partial v_0}{\partial y} = 0$$

$$\frac{\partial^2 u_0}{\partial y^2} + \text{Re } v_0 \frac{\partial u_0}{\partial y} - \left(\frac{1}{K_0} + \text{ReM}\right) u_0 = -\text{GRe}^2(\theta_0 + \phi_0)$$

$$-\left(\frac{1}{K_0} + \text{MRe}\right)$$
(20)

$$\frac{\partial^2 \theta_0}{\partial y^2} - \text{PrRe } v_0 \frac{\partial \theta_0}{\partial y} = 0$$
 (21)

$$\frac{\partial^{2} \varphi_{0}}{\partial v^{2}} - \operatorname{ScRe} v_{0} \frac{\partial \varphi_{0}}{\partial v} + \frac{\operatorname{So}}{\operatorname{Re}} \left(\frac{\partial^{2} \theta_{0}}{\partial v^{2}} \right) = 0$$
 (22)

The boundary conditions are

$$y = 0$$
: $u_0 = 0$, $v_0 = -1$, $w_0 = 0$, $\theta_0 = 1$, $\varphi_0 = 1$ (23)

$$y \rightarrow \infty$$
: $u_0 = 1, v_0 = -1, w_0 = 0, p_0 = p_{\infty}, \theta_0 = 0, \phi_0 = 0$ (24)

The solution for the above equation is

$$u_0\!\!=\!\!1\text{-}\;e^{-R_0y}+B_1(e^{-R_0y}\text{-}\;e^{\text{-}RePry})+B_2(e^{-R_0y}\text{-}\;e^{\text{-}ReScy})\;(25)$$

$$\theta_0 = e^{-RePr y} \tag{26}$$

$$\varphi_0 = (1-B_0) e^{-ReSc y} + B_0 e^{-Re Pr y}$$
 (27)

with
$$v_0 = -1$$
, $p_0 = p_{\infty}$, $w_0 = 0$ (28)

When we take coefficient of order E, the equations are

$$\frac{\partial \mathbf{v}_1}{\partial \mathbf{y}} + \frac{\partial \mathbf{w}_1}{\partial \mathbf{z}} = 0 \tag{29}$$

$$v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial z} = GRe (\theta_1 + \phi_1) + \frac{1}{Re} (\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}) -$$

$$\frac{(u_0 - 1)(\cos \pi z + u_1)}{\text{Re } K_0} - Mu_1$$
 (30)

$$-\frac{\partial \mathbf{v}_1}{\partial y} = -\frac{\partial \mathbf{p}_1}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 \mathbf{v}_1}{\partial y^2} + \frac{\partial^2 \mathbf{v}_1}{\partial z^2} \right)$$
(31)

$$-\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{Re} \left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) - Mw_1$$
 (32)

$$v_1 \frac{\partial \theta_0}{\partial y} + \frac{\partial \theta_1}{\partial z} = \frac{1}{\text{RePr}} \left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right)$$
(33)

$$v_{1} \frac{\partial \phi_{0}}{\partial y} + \frac{\partial \phi_{1}}{\partial z} = \frac{1}{\text{ReSc}} \left(\frac{\partial^{2} \phi_{1}}{\partial y^{2}} + \frac{\partial^{2} \phi_{1}}{\partial z^{2}} \right) + \frac{\text{So}}{\text{Re}} \left(\frac{\partial^{2} \theta_{1}}{\partial y^{2}} + \frac{\partial^{2} \theta_{1}}{\partial z^{2}} \right)$$
(34)

The boundary conditions are

$$y = 0$$
: $u_1 = 0$, $v_1 = -\cos\pi z$, $w_1 = 0$, $\theta_1 = 0$, $\varphi_1 = 0$ (35)

$$y \rightarrow \infty$$
: $u_1 = 1, v_1 = -1, w_1 = 0, p_1 = 0, \theta_1 = 0, \phi_1 = 0$
(36)

Cross Flow Solution

Consider the equation (29), (31) and (32) for $v_1(y, z)$, $w_1(y, z)$ and $p_1(y, z)$ which are independent of main flow component u_1 , temperature field θ_1 and concentration field φ_1 . The suction velocity consists of a basic uniform distribution with superimposed weak sinusoidal distribution.

Hence the velocity components v, w and p are also separated into mean and small sinusoidal components v₁, w_1 and p_1 . By assuming v_1 , w_1 and p_1 to be of the following form.

$$v_1(y, z) = v_{11}(y) \cos \pi z$$
 (37)

$$w_1(y, z) = -\frac{1}{\pi}v'_{11}(y)\sin\pi z$$
 (38)

$$p_1(y, z) = p_{11}(y) \cos \pi z$$
 (39)

On substitution of (37) - (39), the equation (29) is satisfied and the equation (31) and (32) reduce to the

following differential equations:-

$$v_{11}^{"}$$
 + Re $v_{11}^{'}$ - $\pi^2 v_{11}$ = Re $p_{11}^{'}$ (40)

$$v_{11}^{"}$$
 + Re $v_{11}^{"}$ - $(\pi^2 + MRe) v_{11}^{'} = \pi^2 Re p_{11}$ (41)

The corresponding boundary conditions become

$$y = 0$$
: $v_{11} = -1$, $v'_{11} = 0$ (42)

$$y \to \infty$$
: $v_{11} = 0$, $v'_{11} = 0$, $p_{11} = 0$ (43)

On solving equations (40) and (41) under the boundary condition (42) and (43), we get

$$v_1 = \frac{1}{R_2 - R_4} [R_4 e^{-R_2 y} - R_2 e^{-R_4 y}] \cos \pi z$$
 (44)

$$w_1 = \frac{R_2 R_4}{\pi (R_2 - R_4)} \left[e^{-R_2 y} - e^{-R_4 y} \right] \sin \pi z \tag{45}$$

Solution For Flow, Temperature And Concentration Field

Consider the equations (30), (33) and (34). The solutions for the velocity component u, temperature field θ and concentration field ϕ are also separated into mean and sinusoidal components u_1 , θ_1 , ϕ_1 . To reduce the partial differential equations (30), (33) and (34) into ordinary differential equations, we consider for the following forms for u_1 , θ_1 and ϕ_1 .

$$u_1(y, z) = u_{11}(y) \cos \pi z$$
 (46)

$$\theta_1(y, z) = \theta_{11}(y) \cos \pi z \tag{47}$$

$$\varphi_1(y, z) = \varphi_{11}(y) \cos \pi z$$
 (48)

Using the expressions for u_1 , θ_1 , ϕ_1 , v_1 in (30), (33) and (34), we get the following ordinary differential equations:

$$u_{11}'' + \text{Re } u_{11}' - (\pi^2 + \frac{1}{K_0} + \text{MRe}) u_{11} = \text{Re } v_{11}u_0'$$

GrRe²
$$\phi_{11}$$
 - GmRe² ϕ_{11} + $\frac{u_0 - 1}{K_0}$ (49)

$$\theta_{11}^{"} + \text{RePr}\theta_{11}^{"} - \pi^2 \theta_{11} = \text{RePr} v_{11}\theta_0^{"}$$
 (50)

$$\varphi_{11}^{"}$$
 + ReSc $\varphi_{11}^{'}$ - π^2 v_{11} = ReSc $p_{11}^{'}$ - ScSr $\theta_{11}^{"}$ - π^2 ScSr θ_{11} (51)

With the boundary condition

$$y = 0; u_{11} = 0, \theta_{11} = 0, \varphi_1 = 0$$
 (52)

$$y \to \infty$$
; $u_{11} = 0$, $\theta_{11} = 0$, $\varphi_1 = 0$ (53)

Solving these equations and using (52) and (53) we get

$$\begin{split} u_1 &= \left(B_0 e^{-h_3 y} - \frac{\pi^2 ScSrRe^2Sc^2}{R_2 - R_4} \left(B_{24} e^{-h_1 y} - \right. \right. \\ &\quad B_{25} e^{-(R_2 + RePr)y} + B_{26} e^{-(R_4 + RePr)y} \right) - \\ &\quad \frac{ScSrRe^2Sc^2}{R_2 - R_4} \left(B_{21} e^{-h_1 y} - B_{22} e^{-(R_2 + RePr)y} - \right. \\ &\quad B_{23} e^{-(R_4 + RePr)y} \right) + \frac{Re}{R_2 - R_4} \left((B_{15} - B_{16})e^{-h_1 y} + B_{17} e^{-(R_2 + RePr)y} - \right. \\ &\quad B_{19})e^{-(R_4 + RePr)y} \right) \cos \pi z \end{aligned} \tag{54}$$

$$\begin{split} \theta_{1} &= \frac{Re^{2}Pr^{2}}{R_{2}-R_{4}} \Big[B_{4} \Big(e^{-h_{1}y} - e^{-(R_{2} + RePr)y} \Big) - \\ & B_{5} \Big(e^{-h_{1}y} - e^{-(R_{4} + RePr)y} \Big) \Big] \cos \pi z \end{split} \tag{55} \\ \phi_{1} &= \frac{1}{R_{2}-R_{4}} \Big[-B_{5} \Big(e^{-h_{2}y} - e^{-(R_{4} + ReSc)y} \Big) + \\ & B_{6} \Big(e^{-h_{2}y} - e^{-(R_{2} + ReSc)y} \Big) - \Big(B_{7} - B_{10} + \\ & B_{5} \Big) \Big(e^{-h_{2}y} - e^{-(R_{4} + RePr)y} \Big) + \Big(B_{8} - B_{11} + \\ & B_{13} \Big) \Big(e^{-h_{2}y} - e^{-(R_{2} + RePr)y} \Big) + \Big(B_{8} - \\ & B_{12} \Big) \Big(e^{-h_{2}y} - e^{-(R_{4} + ReSc)y} \Big) \Big] \cos \pi z \end{aligned} \tag{56}$$

Skin Friction

The non dimensional skin friction is given by

$$\tau = \frac{\tau'}{\rho' u_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + Eu_1'(0)$$

Nusselt Number

The rate of heat transfer in terms of Nusselt number (Nu) is given by

$$Nu = \frac{q'v}{u_0 k} (T'_w - T'_{\infty}) = -\left(\frac{\partial \theta}{\partial y}\right)_{v=0} = -(\theta'_0(0) + E\theta'_1(0))$$

Sherwood Number

The rate of mass transfer in terms of Sherwood number (Sh) is given by

$$\operatorname{Sh} = \frac{-q'}{\rho \operatorname{Uc}_{p}(C'_{\infty} - C'_{w})} = \frac{k}{\rho \operatorname{Uc}_{p} \operatorname{l}} \left(\frac{\partial \varphi}{\partial y}\right)_{y=0} = \varphi'_{0}(0) + E\varphi'_{1}(0)$$

III. RESULT AND DISCUSSION

In order to get physical insight into the problem, we have carried out the calculations for non-dimensional velocity field, temperature field and concentration field by assuming some specific values to the parameters entering into the problem and the effects of these values on the above field are demonstrated graphically. The following are the values that we are used throughout the graphs in our present study, z = 0, Re = 7, $K_0 = 1$, Sc = 0.22, Gr = 3, Gm = 1, M = 1, Pr = 0.71

The results of the graphs are discussed below:

Figures (2), (3) and (4) exhibit the behaviour of velocity with respect to various parameters. The increase in Reynolds number (Re), Grashof number for heat transfer (Gr) and Soret number (Sr) increases the velocity profile.

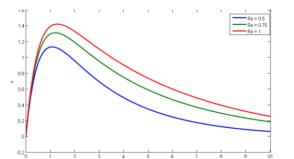


Figure 2. Effects of Reynolds number (Re) on velocity profile

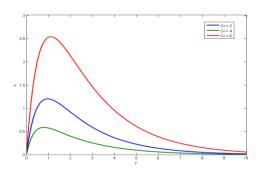


Figure 3. Effects of Grashof number for heat transfer (Gr) on velocity profile

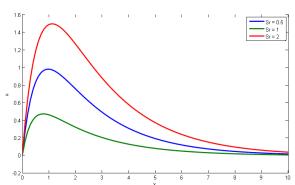


Figure 4. Effects of Soret number (Sr) on velocity profile

Figures (5) and (6) shows that decrease in Hartmann number (M) and Grashof number for mass transfer (Gm) increases the velocity profile of the fluid flow.

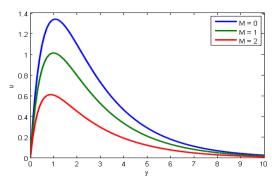


Figure 5. Effects of Hartmann number (M) on velocity Profile

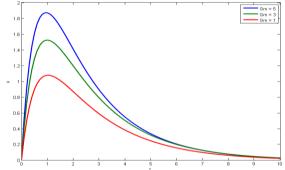


Figure 6. Effects of Grashof number for mass number (Gm) on velocity profile

Figures (7) - (9) depict the effect of Schmidt number (Sc), Prandtl number (Pr) and permeability parameter (K0) on velocity profile. The velocity gradually rises due to increase in Schmidt number (Sc) and Prandtl number (Pr). Increasing Permeability parameter (K0) tends to decrease the velocity profile.

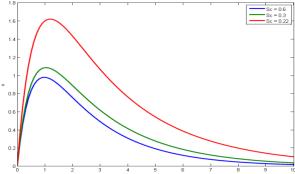


Figure 7. Effects of Schmidt number (Sc) on velocity profile

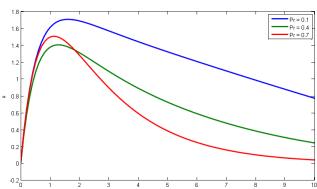


Figure 8. Effects of Prandtl number (Pr) on velocity profile

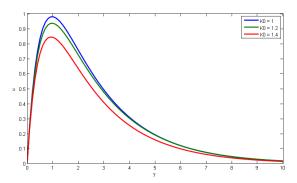


Figure 9. Effects on Permeability (K_0) on velocity profile

Increase in Hartmann number (M) and Prandtl number (Pr) tend to decrease the temperature profile which can be seen from Figures (10) and (11).

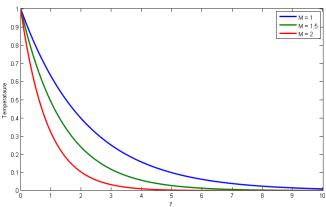


Figure 10. Effects of Hartmann number M on temperature profile

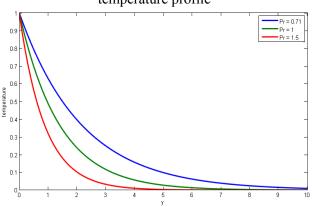


Figure 11. Effects of Prandtl number (Pr) on temperature profile

Figure (12) exhibit the concentration profile for various Schmidt number. It is clear that the increase in Schmidt number (Sc) tends to decrease the concentration profile.

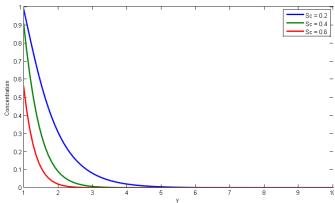


Figure 12. Effects of Schmidt number (Sc) on concentration profile

Figure (13) shows that decrease in Reynolds number (Re) increase the concentration profile of the fluid.

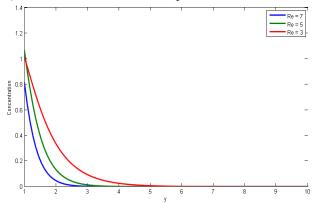


Figure 13. Effects of Reynolds number (Re) on concentration profile

The concentration profile for various Prandtl number (Pr) and Soret number (Sr) is shown in Figures (14) and (15). From these figures, it is clear that the concentration profile increases gradually with increase of Prandtl number (Pr) and Soret number (Sr).

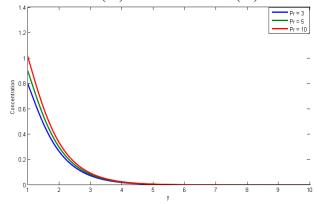


Figure 14. Effects of Prandtl number (Pr) on concentration profile

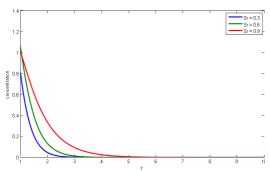


Figure 15. Effects of Soret number (Sr) on concentration profile

IV. CONCLUSION

A theoretical analysis is performed to study the effects of Thermal Diffusion and Permeability of the medium on three dimensional MHD free convective flow through a porous plate. A Perturbation technique is used to solve the equations governing the flow. The solutions for velocity, temperature and concentration are obtained and also illustrated the results of flow characteristics for the fluid fields. The analysis is made and the results are summarized as follows:

- ✓ The increase of Reynolds number (Re), Grashof number for heat transfer (Gr), Soret number (So), Schmidt number (Sc), Prandtl number (Pr) increases the velocity of the fluid.
- ✓ As the Hartmann number (M) and Grashof number for mass transfer (Gm) decreases, the velocity of the fluid will increase.
- \checkmark As the Permeability parameter (K_0) increases, the velocity profile decreases.
- ✓ The increase of Hartmann number (M) and Prandtl number (Pr) decreases the temperature of the flow field.
- ✓ The increase of Prandtl number (Pr) and Soret number (So) increases the concentration of the fluid.
- ✓ As the Schmidt number (Sc) increases, the concentration of the fluid decreases and the concentration of the fluid increases with respect to the decrease in the Reynolds number.

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VI. APPENDIX

$$R_{0} = \frac{\text{Re} + \sqrt{\text{Re}^{2} + 4\left(\frac{1}{K_{0}} + \text{MRe}\right)}}{2}$$

$$R_{1} = \frac{\text{Re} + \sqrt{\text{Re}^{2} + 4\text{MRe}}}{2}$$

$$R_{2} = \frac{R_{1} - \sqrt{R_{1}^{2} + 4\pi^{2}}}{2}$$

$$R_{3} = \frac{\text{Re} - \sqrt{\text{Re}^{2} + 4\text{MRe}}}{2}$$

$$R_{4} = \frac{R_{3} - \sqrt{R_{3} + 4\pi^{2}}}{2}$$

$$h_{1} = \frac{\text{RePr} + \sqrt{(\text{RePr})^{2} + 4\pi^{2}}}{2}$$

$$h_{2} = \frac{\text{ReSc} + \sqrt{(\text{ReSc})^{2} + 4\pi^{2}}}{2}$$

$$\begin{split} & h_3 = \frac{\text{Re} + \sqrt{\text{Re}^2 + 4 \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)}}{2} \\ & B_0 = \frac{\text{ScSrPr}}{\text{Sc-Pr}} \\ & B_1 = \frac{-\text{Gm}}{\text{ReSC(Sc-1)} - \left(\frac{1}{K_0} + \text{MRe}\right)} \left[1 + \frac{\text{ScSrPr}}{\text{Sc-Pr}}\right] \\ & B_2 = \frac{\text{GmSCSr}}{\text{Re(Pr-1)} - \left(\frac{1}{K_0} + \text{MRe}\right)} - \frac{\text{Gr}}{\text{RePr(Pr-1)} - \left(\frac{1}{K_0} + \text{MRe}\right)} \\ & B_3 = \frac{R_2}{r_4(r_4 + \text{RePr}) - \pi^2} \\ & B_4 = \frac{R_2}{R_2(R_2 + \text{RePr}) - \pi^2} \\ & B_5 = \frac{(1 - B_0) \text{ReSCR}_2}{R_2(R_2 + \text{ReSC}) - \pi^2} \\ & B_6 = \frac{(1 - B_0) \text{ReSCR}_2}{R_2(R_2 + \text{ReSC}) - \pi^2} \\ & B_7 = \frac{B_0 \text{PrRe}R_4}{R_2(R_2 + \text{ReSC}) - \pi^2} \\ & B_8 = \frac{B_0 \text{PrRe}R_4}{R_2(R_2 + \text{ReSC}) - \pi^2} \\ & B_{10} = \frac{(R_4 + \text{PrRe})^2 \text{Sr} B_3 \text{RePr}^2}{(R_4 + \text{RePr})^2 - \text{ReSC}(R_4 + \text{Re Pr}) - (\pi^2)} \\ & B_{11} = \frac{(R_2 + \text{PrRe})^2 \text{Sr} B_3 \text{RePr}^2}{(R_4 + \text{RePr})^2 - \text{ReSC}(R_2 + \text{Re Pr}) - (\pi^2)} \\ & B_{12} = \frac{\pi^2 \text{ReSrPr}^2 (B_4 - B_3)}{h_1^2 - \text{ReSCh}_1 - \pi^2} \\ & B_{13} = \frac{\pi^2 \text{ReSrPr}^2 (B_4 - B_3)}{R_2(R_2 + \text{ReSC}) - \pi^2} \\ & B_{14} = \frac{\pi^2 \text{ReSrPr}^2 B_3}{R_2(R_4 + \text{ReSC}) - \pi^2} \\ & B_{15} = \frac{R_4 R_0}{(R_2 + \text{Re})^2 - \text{Re}(R_2 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{16} = \frac{R_4 (B_1 + B_2)}{(R_2 + \text{Re})^2 - \text{Re}(R_2 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{18} = \frac{R_2 R_0}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{19} = \frac{R_2 R_0}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{20} = \frac{R_2 R_0 (B_1 + B_2)}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{21} = \frac{R_2 R_0 (B_1 + B_2)}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{22} = \frac{R_2 R_0 (B_1 + B_2)}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{23} = \frac{R_2 R_0 (B_1 + B_2)}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{24} = \frac{R_2 R_0 (B_1 + B_2)}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)} \\ & B_{24} = \frac{R_2 R_0 (B_1 + B_2)}{(R_4 + \text{Re})^2 - \text{Re}(R_4 + \text{Re}) - \left(\pi^2 + \frac{1}{K_0} + \text{MRe}\right)}$$

$$\begin{split} C_0 &= \frac{\pi^2 \text{ScSrRe}^2 \text{Sc}^2}{R_2 - R_4} (B_{24} - B_{25} + B_{26}) \ + \frac{\text{ScSrRe}^2 \text{Sc}^2}{R_2 - R_4} (B_{21} - B_{22} - B_{23}) + \frac{\text{Re}}{R_2 - R_4} ((B_{15} - B_{16}) + B_{17} - (B_{18} - B_{18})) + \frac{\text{Re}}{R_2 - R_4} (B_{18} - B_{18}) + \frac{\text{Re}}{R_2 - R_4} (B_{18} - B_$$