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Changing Behavior of Vertices of Some Graphs

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ABSTRACT

Let G be a (p,q) graph and $f: V(G) \to \{1,2,\dots,p+q-1,p+q+2\}$ be an injection. For each edge $e = uv$, the induced edge labeling f^* is defined as follows:

$$f^*(e) = \begin{cases} \dfrac{|f(u)-f(v)|}{2} & \text{if } |f(u)-f(v)| \text{ is even} \\ \dfrac{|f(u)-f(v)|+1}{2} & \text{if } |f(u)-f(v)| \text{ is odd} \end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and are from $\{1,2,3,\dots.q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, a new parameter V^+ is introduced and verified for some graphs.

Keywords: Fan Graph, Jewel Graph, Octopus Graph, Near Skolem Difference Mean Labeling.

I. INTRODUCTION

All graphs considered in this paper are finite, undirected and simple. The vertex set and the edge set of a graph G are denoted by V(G) and E(G) respectively. For standard terminology and notations, we follow Harary (1) and for graph labeling, we refer to Gallian (2).

In this paper, a non-Near skolem difference mean graph is investigated and a new parameter is introduced to check whether addition of minimum number of vertices to G converts this non-Near skolem difference mean graph G into a Near skolem difference mean graph. The following definitions are used in the subsequent section:

Definition 1.1: The fan graph $F_n (n \geq 2)$ is obtained by joining all vertices of P_n (path of n vertices) to a further vertex called the center and contains $(n+1)$ vertices and $(2n-1)$ edges. i.e., $F_n = (P_n + K_1)$.

Definition 1.2: The Jewel J_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vv_i, 1 \leq i \leq n\}$.

Definition 1.3: An octopus graph O_n, $(n \geq 2)$ can be constructed by joining a fan graph $F_n (n \geq 2)$ to a star graph $K_{1,n}$ by with sharing a common vertex, where n is any positive integer. i.e., $O_n = F_n + K_{1,n}$.

Definition 1.4: The graph $\overline{K_2} \vee P_n$, which is the join of the complementary of K_2 and the path graph P_n is the double fan graph and is denoted by Df_n. In other words, the double fan graphs can be considered as the join of two similar fan graphs at the path.

II. MAIN RESULT

Definition 2.1: A graph $G = (V, E)$ with p vertices and q edges is said to have Nearly skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $\{1,2,\dots,p+q-1,p+q+2\}$ in such a way that each edge $e = uv$, is labeled as $f^*(e) = \frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $f^*(e) = \frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and are from $\{1, 2, \dots\dots, q\}$. A graph that admits a Near skolem difference mean labeling is called a Near skolem difference mean graph.

Definition 2.2: Let G be a non-Near skolem difference mean graph. Then the parameter V^+ of a graph G is defined as the minimum number of isolated vertices to be added to G, so that the resulting graph is Near skolem difference mean.

Theorem 2.3: $V^+(P_n+K_1) = n-4$, for $n \geq 5$.

Proof: Let F_n be the graph P_n+K_1.

Let $V(F_n)=\{v, u_i / \ 1 \leq i \leq n\}$.

and $E(F_n) = \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{u_iv/ 1 \leq i \leq n\}$.

Then $|V(F_n)| = n+1$ and $|E(F_n)| = 2n-1$.

Suppose, F_n is Near skolem difference mean for $n \geq 5$.

Let $f: V(F_n) \rightarrow \{1,2,\ldots,3n-1, 3n+2\}$.

Let $uv \in E(F_n)$ such that, $f(u) < f(v)$.

Then $1 \leq f(u) < f(v) \leq 3n+2$.

These are two cases:

Case(i) Suppose $\frac{|f(v)-f(u)|}{2} = 2n-1$.

This implies $f(v) = 4n-2+f(u)$.

$\geq 4n-2+1$

$= 4n-1$.

Case (ii) Suppose, $\frac{|f(v)-f(u)|+1}{2} = 2n-1$.

This implies $f(v) = 4n-2-1+f(u)$.

$= 4n-3+f(u)$

$\geq 4n-3+1$

$= 4n-2$.

Thus, in both cases, for every Near skolem difference mean labeling of F_n,

$f(v) \geq 4n-2 > 3n+2$ as $n \geq 5$.

But by definition, $f(v) \leq 3n+2$.

This is a contradiction.

Then the graph F_n is not Near skolem difference mean.

Thus, in order to make F_n a Near skolem difference mean graph, at least $4n-2-3n-2$ isolated vertices should to be added to $F_n = P_n + K_1$.

Then $V^+ = (P_n+K_1) \geq n-4$.

Claim: $V^+(F_n) = n-4$.

Let F_n^+ be the graph obtained from F_n by adding $(n-4)$ isolated vertices

Let $V(F_n^+) = \{v, u_i, w_j / \ 1 \leq i \leq n, \ 1 \leq j \leq n-4\}$

and $E(F_n^+) = \{u_iu_{i+1}/ \ 1 \leq i \leq n-1\} \cup \{u_iv/1 \leq i \leq n\}$.

Then $|V(F_n^+)| = 2n-3$ and $|E(F_n^+)| = 2n-1$

Let $f: V(F_n^+) \rightarrow \{1,2,\ldots,4n-5, 4n-2\}$ be defined as follows:

$f(v) = 1$

$f(w_i) = 2i+1, \ 1 \leq i \leq n-4$.

$$f(u_i) = \begin{cases} 4n-2i, & i \equiv 1(\text{mod}2), \quad 1 \leq i \leq n \\ 2i-2, & i \equiv 0(\text{mod}2), \quad 1 \leq i \leq n \end{cases}$$

Let f^* be the induced edge labeling. Then,

$f^*(u_iu_{i+1}) = 2n-2i, 1 \leq i \leq n-1$

$$f^*(u_iv) = \begin{cases} 2n-i, & i \equiv 1(\text{mod}2), \quad 1 \leq i \leq n \\ i-1, & i \equiv 0(\text{mod}2), \quad 1 \leq i \leq n \end{cases}$$

Therefore, the induced edge labels are all distinct and are $\{1,2,\ldots,2n-1\}$.

Hence F_n^+ is Near Skolem Difference Mean for $n \geq 5$.

Example 2.4: The Near Skolem Difference Mean labeling of F_8^+ and F_9^+ are given in figure 1 and figure 2 respectively.

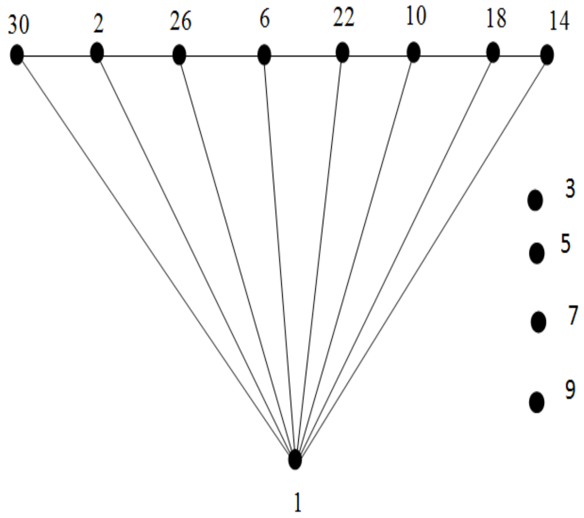

Figure 1

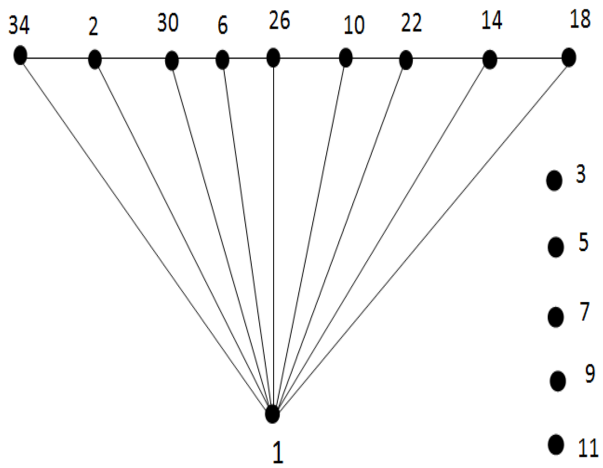

Figure 2

Theorem 2.5: $V^+(G) = n-2$, for $n \geq 3$; where G is the Jewel graph.

Proof: Let G be the Jewel graph with $n \geq 3$.

Let $V(G) = \{u, v, x, y, w_i / 1 \leq i \leq n\}$ and

$E(G) = \{ux, vx, uy, vy, uw_i, vw_i / 1 \leq i \leq n\}$.

Then $|V(G)| = n+4$ and $|E(G)| = 2n+4$.

Suppose, G is Near skolem difference mean for $n \geq 3$.

Define $f: V(G) \to \{1,2,3,..,3n+7, 3n+10\}$.

Let $uv \in E(G)$ such that $f(u) < f(v)$.

Then $1 \leq f(u) < f(v) \leq 3n+10$,

There are two cases:

Case (i): Suppose, $\frac{|f(v)-f(u)|}{2} = 2n+4$

Then $f(v) = 4n+8+f(u)$

$\geq 4n+8+1$

$= 4n+9$

$> 3n+10$.

Case (ii): Suppose, $\frac{|f(v)-f(u)|+1}{2} = 2n+4$

Then $f(v) = 4n+8+f(u)-1$

$\geq 4n+7+1$

$= 4n+8$

$> 3n+10$.

Thus, in both cases, we conclude that for any Near Skolem Difference Mean labeling of G.

$f(v) \geq 4n+8 > 3n+10$ as $n \geq 3$.

But, by definition $f(v) \leq 3n+10$.

This implies the graph G is not Near Skolem Difference Mean.

Therefore, at least $4n+8-(3n+10)$ isolated vertices should be added to the graph G to make it a Near skolem difference mean graph.

Then $V^+(G) \geq n-2$.

Claim: $V^+(G) = n-2$.

Let G^+ be the graph obtained from G by adding $(n-2)$ isolated vertices to it.

Let $V(G^+) = \{u, v, x, y, w_i, u_j / 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and

$E(G^+) = \{ux, uy, vx, vy, uw_i, vw_i, /1 \leq i \leq n\}$.

Then $|V(G^+)| = 2n+2$ and $|E(G^+)| = 2n+4$.

Let $f: V(G^+) \to \{1,2,\dots,4n+5, 4n+8\}$ be defined as follows:

$f(x) = 4n+8$.

$f(y) = 4n+5$.

$f(u) = 1$.

$f(v) = 3$.

$f(w_i) = 4n+5-4i$, $1 \leq i \leq n$.

$f(u_j) = 2i$, $1 \leq j \leq n-2$.

Let f^* be the induced edge labeling of f. Then,

$f^*(ux) = 2n+4$.

$f^*(uy) = 2n+2$.

$f^*(vx) = 2n+3$.

$f^*(vy) = 2n+1$.

$f^*(uw_i) = 2n+2-2i$, $1 \leq i \leq n$.

$f^*(vw_i) = 2n+1-2i$, $1 \leq i \leq n$.

The induced edge labeling are all distinct and are $\{1,2,\dots,2n+4\}$.

Hence, the graph $G^+ = G \cup (n-2)K_1$ is Near Skolem Difference Mean for $n \geq 3$.

Example 2.6: The Near Skolem Difference Mean labeling of the Jewel graph with $n = 5$ and $n = 6$ are given in figure 3 and figure 4 respectively.

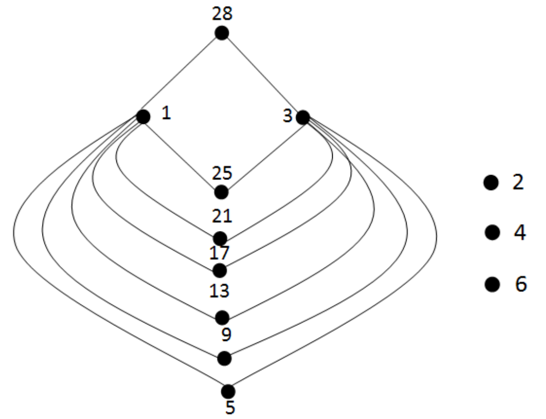

Figure 3

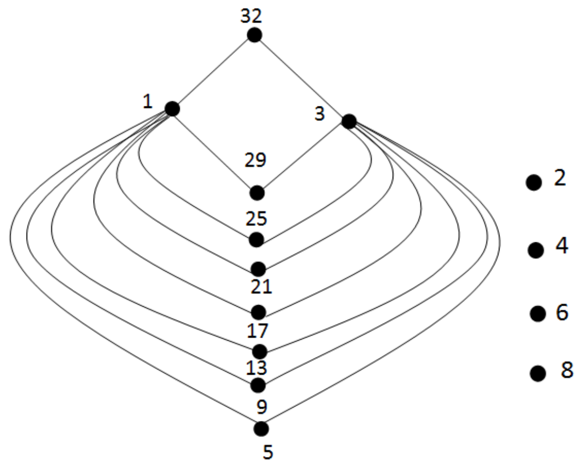

Figure 4

Theorem 2.7: $V^+(O_n) = n-4$ for $n \geq 5$ where O_n is octopus graph.

Proof: Let G be the octopus graph O_n.

For $n \leq 5$, the graph G satisfies the condition for Near skolem difference mean.($p \leq q-2$)

Consider the graph G for $n \geq 6$.

Let $V(G)=\{u_i, v_i, v / \ 1 \leq i \leq n\}$ and

$$E(G)=\{u_iu_{i+1}, vu_j, vv_j / \ 1 \leq i \leq n-1, \ 1 \leq j \leq n\}.$$

Hence, $|V(G)| = 2n+1$ and $|E(G)| = 3n-1$

Suppose, G is Near skolem difference mean for $n \geq 6$.

Define a labeling $f: V(G) \rightarrow \{1,2,..,5n-1, 5n+2\}$

Let $uv \in E(G)$ such that $f(u) < f(v)$.

Then, $1 \leq f(u) < f(v) \leq 5n+2$ provided neither of them equals $5n$ or $5n+1$.

There are two cases:

Case(i): Suppose, $\frac{|f(v)-f(u)|}{2} = 3n-1$

This implies $|f(v)-f(u)| = 6n-2$.

$f(v) = 6n-2+f(v)$

$\geq 6n-2+1$

$= 6n-1$

Case(ii): Suppose, $\frac{|f(v)-f(u)|}{2} = 3n-1$

Then $|f(v)-f(u)| = 6n-2-1$

$f(v) = 6n-3+f(v)$

` $\geq 6n-3+1$

$= 6n-2$

Thus, in both cases, we concluded that for any Near skolem difference mean labeling of $G = O_n$,

$f(v) \geq 6n-2 > 5n+2$; as $n \geq 6$

But by definition, $f(v) \leq 5n+2$.

This implies that the graph G is not Near skolem difference mean graph, therefore, in order to make G a Near skolem difference mean graphs we have to add at least $6n-2-5n-2$ vertices to G.

Then $V^+(G) \geq n-4$.

Claim: $V^+(G) = n-4$.

Let G^+ be the graph obtained by adding $(n-4)$ isolated vertices to G.

Let $V(G^+) = \{u_i, v_i, v, w_j / \ 1 \leq i \leq n, \ 1 \leq j \leq n-4\}$

and $E(G^+) = \{u_iu_{i+1}, vu_j, vv_j / \ 1 \leq i \leq n-1, \ 1 \leq j \leq n\}$.

Then, $|V(G^+)| = 3n-3$ and $|E(G^+)| = 3n-1$

Let $f: V(G^+) \rightarrow \{1,2,\ldots,6n-5, 6n-2\}$ be defined as follows:

There are two cases:

Case(i) When n is odd:

$f(v) = 1$

$f(u_{2i+1}) = 6n-2-4i, \quad 0 \leq i \leq \frac{n-1}{2}.$

$f(u_{2i}) = 4i-2, \quad 1 \leq i \leq \frac{n-1}{2}.$

$f(v_i) = 4i+1, \quad 1 \leq i \leq n-1$

$f(v_n) = 2n+1$

$f(w_j) = 6n-3-2j, \quad 1 \leq j \leq n-4.$

Case(ii): When n is even:

$f(v) = 1$

$f(u_{2i+1}) = 6n-2-4i, \quad 0 \leq i \leq \frac{n-2}{2}.$

$f(u_{2i}) = 4i-2, \quad 1 \leq i \leq \frac{n}{2}.$

$$(v_i) = \begin{cases} 4i+1, & 1 \leq i \leq \frac{n}{2} \\ 4i-1, & \frac{n+2}{2} \leq i \leq n \end{cases}.$$

$f(w_j) = 6n-3-2j, \quad 1 \leq j \leq n-4.$

Let f^* be the induced edge labeling. Then,

Case (i) When n is odd:

$f^*(u_iu_{i+1}) = 3n-2i, \quad 1 \leq i \leq n-1$

$f^*(vu_{2i+1}) = 3n-1-2i, \quad 0 \leq i \leq \frac{n-1}{2}/$

$f^*(vu_{2i}) = 2i-1, \quad 1 \leq i \leq \frac{n-1}{2}.$

$f^*(vv_i) = 2i, \quad 1 \leq i \leq n-1.$

$f(vv_n) = n,$

Case(ii) When n is even:

$f^*(u_iu_{i+1}) = 3n-2i, \quad 1 \leq i \leq n-1.$

$f^*(vu_{2i+1}) = 3n-1-2i, \quad 0 \leq i \leq \frac{n-2}{2}.$

$f^*(vv_{2i}) = 2i-1, \quad 1 \leq i \leq \frac{n}{2}.$

$$f^*(vv_i) = \begin{cases} 2i, & 1 \leq i \leq \frac{n}{2} \\ 2i-1, & \frac{n+2}{2} \leq i \leq n \end{cases}.$$

The induced edge labels are all distinct and are $\{1,2,\ldots,3n-1\}$.

Hence G^+ admits Near skolem difference mean labeling.

Example 2.8: The Near Skolem Difference Mean labeling of $O_8 \cup (n-4)K_1$ and $O_9 \cup (n-4)K_1$ are given fig 5 and fig 6 respectively.

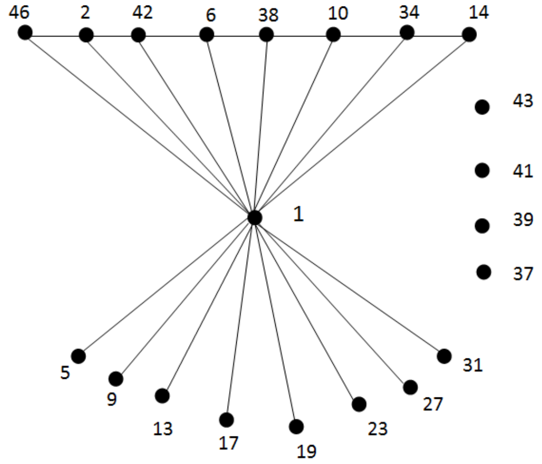

Figure 5

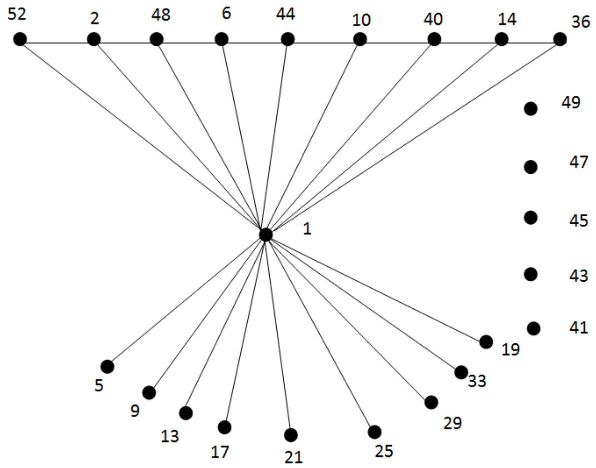

Figure 6

Theorem 2.9: $V^+(Df_n) = 2n - 5$ for $n \geq 3$.

Proof: Let G be the graph Df_n with $n \geq 3$.

Let $V(G) = \{v, w, u_i / \ 1 \leq i \leq n\}$ and

$E(G) = \{u_i u_{i+1}, u_j v, u_j w / \ 1 \leq i \leq n-1, 1 \leq j \leq n\}$.

Then $|V(G)| = n + 2$ and $|E(G)| = 3n - 1$

Suppose, G is Near Skolem Difference Mean For $n \geq 3$.

Let $f: V(G) \rightarrow \{1,2,3, \dots, 4n, 4n+3\}$.

Let $uv \in E(G)$ such that $f(u) < f(v)$.

Then $1 \leq f(u) < f(v) \leq 4n + 3$.

There are two cases:

Case(i): Suppose, $\frac{|f(v)-f(u)|}{2} = 3n - 1$

Then, $f(v) = 6n - 2 + f(u_i)$.

$\geq 6n - 2 + 1$

$= 6n - 1$

$> 4n + 3$.

Case (ii): Suppose, $\frac{|f(v)-f(u)|+1}{2} = 3n - 1$

Then, $f(v) - f(u) = 6n - 2 - 1$

Therefore $f(v) = 6n - 3 + f(u)$

$\geq 6n - 3 + 1$

$= 6n - 2$

$> 4n + 3$

Thus, in both cases, for any Near Skolem Difference Mean labeling of G,

$f(v) \geq 6n - 2 > 4n + 3$ as $n \geq 3$.

But, by definition $f(v) \leq 4n + 3$.

This implies that the graph G is not Near Skolem Difference Mean

Therefore at least $(6n-2) - (4n+3)$ isolated vertices should be added to the graph G to make it a Near skolem difference mean graph.

Then $V^+(G) \geq 2n - 5$.

Claim: $V^+(G) = 2n - 5$.

Let G^+ be the graph obtained from G by adding $(2n-5)$ isolated vertices.

Let $V(G^+) = \{v, w, u_i, x_j / \ \ 1 \leq i \leq n, \ 1 \leq j \leq 2n-5\}$ and

$E(G) = \{u_i u_{i+1}, \ vu_j, \ wu_j / \ 1 \leq i \leq n-1, \ 1 \leq j \leq n\}$.

Then $|V(G)| = 3n - 3$ and $|E(G)| = 3n - 1$

Let $f: V(G^+) \rightarrow \{1,2, \dots, 6n-5, 6n-2\}$ be defined as follows:

Case(i): When n is odd:

$f(v) = 1$

$f(u_{2i+1}) = 4i + 2, \qquad 0 \leq i \leq \frac{n-1}{2}$.

$f(u_{2i}) = 6n + 2 - 4i, 1 \leq i \leq \frac{n-1}{2}$.

$f(w) = 4n + 1$

$f(x_j) = 2j + 1, \qquad 1 \leq j \leq 2n - 5$

Case(ii) When n is even:

$f(v) = 1$

$f(u_{2i+1}) = 6n - 2 - 4i, \quad 0 \leq i \leq \frac{n-2}{2}$.

$f(u_{2i}) = 4i - 2, \qquad 1 \leq i \leq \frac{n}{2}$.

$f(w) = 2n + 1$

$f(x_j) = 2j + 1, \qquad 1 \leq j \leq 2n - 5$.

Let f^* be the induced edge labeling. Then,

Case(i) When n is odd:

$f^*(u_i u_{i+1}) = 3n - 2i, \qquad 1 \leq i \leq n - 1$

$f^*(vu_{2i+1}) = 2i + 1, \qquad 0 \leq i \leq \frac{n-1}{2}.$
$f^*(vu_{2i}) = 3n + 1 - 2i, \quad 1 \leq i \leq \frac{n-1}{2}.$
$f^*(wu_{2i+1}) = 2n - 2i, \qquad 0 \leq i \leq \frac{n-1}{2}.$
$f^*(wu_{2i}) = n + 1 - 2i, \qquad 1 \leq i \leq \frac{n-1}{2}.$

Case (ii) When n is even:
$f^*(u_i u_{i+1}) = 3n - 2i, \qquad 1 \leq i \leq n - 1.$
$f^*(vu_{2i+1}) = 3n - 1 - 2i, \quad 0 \leq i \leq \frac{n-2}{2}.$
$f^*(vu_{2i}) = 2i - 1, \qquad 1 \leq i \leq \frac{n-2}{2}.$
$f^*(wu_{2i+1}) = (2n + 1) - 2i; 0 \leq i \leq \frac{n-2}{2}.$
$f^*(wu_{2i}) = n + 2 - 2i, \qquad 1 \leq i \leq \frac{n}{2}.$
The induced edge labels are all distinct and are $\{1,2,\ldots,3n-1\}$.
Hence, G^+ is Near skolem difference mean.

Example 2.8: The Near skolem difference mean of $Df_8 \cup 11K_1$ and $Df_9 \cup 13K_1$ are given in figure 7 and figure 8 respectively.

III. CONCLUSION

In this paper, we investigated a non-Near skolem difference mean graph and introduced a new parameter to check whether addition of minimum number of vertices to G converts a non-Near skolem difference mean graph G into a Near skolem difference mean graph. We have planned to investigate this property for some special cases of graphs in our next paper.

IV. REFERENCES

[1]. F.Harary, Graph Theory, Narosa Publishing House, New Delhi, (2001).
[2]. J. A.Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 15(2008),DS6.
[3]. S. Shenbaga Devi, A. Nagarajan, Near Skolem Difference Mean Labeling of cycle related Graphs, International Journal for Science and Advance Research in Technology, Volume 3 Issue 12 – December 2017, pages 1037-1042
[4]. S. Shenbaga Devi and A. Nagarajan, Near Skolem Difference Mean labeling of some special types of trees, International Journal of Mathematics Trends and Technology, Volume 52 Number 7 December 2017, pages 474-478
[5]. S. Shenbaga Devi and A. Nagarajan, Near Skolem Difference Mean Labeling of some Subdivided graphs. (Communicated)
[6]. S. Shenbaga Devi and A. Nagarajan, Some Results on Duplication of Near Skolem Difference Mean graph C_n. (Communicated)
[7]. S. Shenbaga Devi and A. Nagarajan, On Near Skolem Difference Mean Graphs (Communicated).
[8]. S. Shenbaga Devi and A. Nagarajan, On Changing behavior of edges of some special classes of graphs I (Communicated)

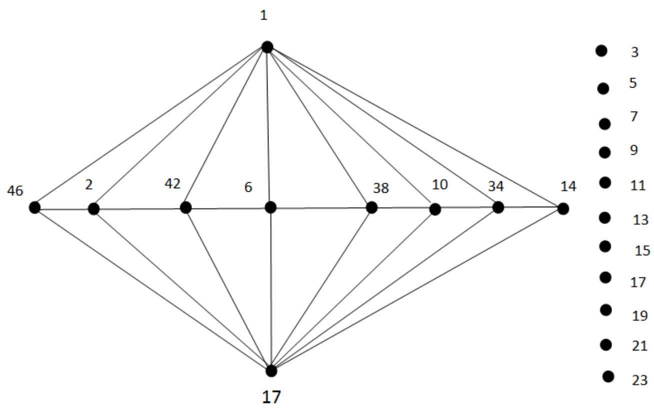

Figure 7

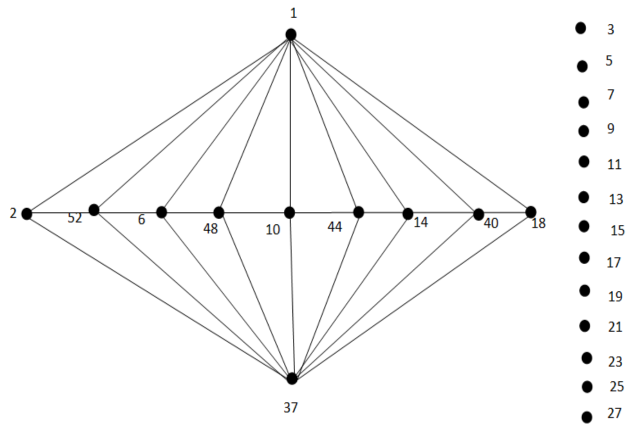

Figure 8