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Robust Estimator Based $\overline{X}$ Control Chart for Specified Capability Index Cpk

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ABSTRACT

Statistical process Control is a tool used to improve the quality of a product by achieving process capability. Control Charts are used to confirm whether the process is statistically control. In general, process performance is measured through the four basic process capability indices C_p, C_{pk}, C_{pm} and C_{pmk}. To calculate the basic capability indices, it is assumed that the quality characteristics follow normal distribution. In practice most of the key quality characteristics fails normality. MAD based capability index is an alternative for heavily skewed data. In this paper, a robust $\bar{X}$ chart for a specified capability index is proposed. The proposed control chart and basic Shewhart control chart are compared using a simulated process data from normal distribution.

Keywords: Capability indices, Median Absolute Deviation, Control chart, Normal Distribution.

I. INTRODUCTION

W. A. Shewhart initialized the concept of statistical process control. The variations in the process are due to chance cause and assignable cause. The variation due to chance cause is termed as allowable variation.Assignable causes arise due to non-random causes and are rectifiable. Control charts are used to assess the presence of assignable causes. A typical control chart consists of a central line (CL), Upper control limit (UCL), Lower control limit (LCL). The $\bar{X}$ and R control charts are widely used to monitor the mean and variability of a process. A process that is operating with only chance causes is said to be in statistical control and the presence of assignable causes is said to be out of control. Control chart is an on-line process monitoring technique widely used to detect the occurrence of assignable causes. Mean and variability are the two vital characteristics used to understand the process. The process mean is controlled using $\bar{X}$ chart. In this study $\bar{X}$ chart is considered. The control limits for Shewhart variable control charts are given by,

The control limit for the $\overline{X}$ chart :

$$Upper\ Control\ Limit = \bar{\bar{X}} + A_2\bar{R} \qquad (1)$$

$$Centrl\ Line = \bar{\bar{X}} \qquad (2)$$

$$Lower\ Control\ Limit = \bar{\bar{X}} - A_2\bar{R} \qquad (3)$$

The estimator of σ is given by

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \qquad (4)$$

Janacek et. al [4], proposed a modified control chart based on median to overcome the non-normality problems. Chen et. al [2] constructed the control chart of unilateral specification index Cpl and Cpu to monitor the stability of process and process capability. Subramaniet. al [10] proposed $\bar{X}$ and R control chart based on process capability indices Cp and Cpk. Liaquat Ahmad et. al [6] proposed a $\bar{X}$ control chart based on process capability index (Cp) using repetitive sampling and the performance of the proposed chart was evaluated by ARL_1 and the performance is efficient for the quick detection of false alarms. Moustafa [8] proposed a univariate robust control chart for location by modifying the control limits using robust estimators. The modified control limits are:

$$UCL = \overline{MD} + R_1\overline{MAD} \quad (5)$$

$$CL = \overline{MD} \quad (6)$$

$$UCL = \overline{MD} - R_1\overline{MAD} \quad (7)$$

where

$$R_1 = \frac{3.759 b_n}{\sqrt{n}}$$

II. Basic Process Capability Indices

The four basic process capability indices Cp, Cpk, Cpm and Cpmk are used for measuring the capability of a process. The index Cp only measures the variability of the process. Cpk consider the location of mean. According to Taguchi [12] there is a loss to society associated with missing the target and developed the concept of the quadratic loss function. Thus Chan et.al [1] introduced Taguchi capability indexCpm. Pearn et al. [8] proposed the capability index C_{pmk}.

The capability index Cpmeasures the variability alone, where

$$C_P = \frac{USL - LSL}{6\sigma} \quad (8)$$

Kane [13] discussed the index Cpk, which attempts to measure the variability and shift in process mean simultaneously and it is defined as

$$C_{pk} = min\left\{\frac{USL - M}{3\sigma}, \frac{M - \text{LSL}}{3\sigma}\right\} \quad (9)$$

Chan et al. [1] introduced so called Taguchi capability index Cpmconsidering specification range, process variation and variation of mean from the target, given by

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \quad (10)$$

Pear et al. [8] proposed the index C_{pmk} by modifying the numerator and denominator of the index C_{pm} as

$$C_{pmk} = \frac{min(USL - \mu, \mu - \text{LSL})}{3\sqrt{\sigma^2 + (\mu - T)^2}} \quad (11)$$

When the data are non-normal or are skewed, the Median Absolute Deviation (MAD) is a robust estimator of variability. Kayode S. Adekeye [5] modified the four basic process capability indices using median absolute deviation and is defined as

$$C_P = \frac{USL - LSL}{6(b_n\overline{MAD})} \quad (12)$$

$$C_{pk} = \frac{(USL - LSL) - |M - m|}{6(b_n\overline{MAD})} \quad (13)$$

$$C_{Pm} = \frac{USL - LSL}{6\sqrt{(b_n\overline{MAD})^2 + (M - T)^2}} \quad (14)$$

$$C_{pmk} = \frac{(USL - LSL) - |M - m|}{6\sqrt{(b_n\overline{MAD})^2 + (M - T)^2}} \quad (15)$$

Subramaniet. al [10] proposed $\bar{X}$ and R chart based on the process capability indices Cp and Cpk and presented the control chart constants D^*, D_3^*, D_4^*, A_2^* for the sample size n (2≤ n ≤ 25). In this paper, a robust estimator based $\bar{X}$ control chart for specified capability index is proposed and the quality control constant R_2 is tabulated.

III. A Robust Estimator Based $\bar{X}$ Control Chart for Specified Capability Index Cpk

i) Cpk Based Robust Control Chart

The proposed control chart can be used to calculate the control limits for any specified values of the capability index Cpk.

$\overline{MAD}$is obtained by simplifying (13), thus

$$\overline{MAD} = \frac{(USL - LSL) - |M - m|}{6 b_n C_{pk}}$$

$$UCL = \overline{MD} + \left(3 * \left(\frac{1.253 * b_n * \left(\frac{(USL-LSL)-|M-m|}{6b_n C_{pk}}\right)}{\sqrt{n}}\right)\right)$$

$$UCL = \overline{MD} + \left(\frac{0.6265}{\sqrt{n}}\right)\left(\frac{(USL-LSL)-|M-m|}{C_{pk}}\right)$$

Capability index - Cpk based Control Limits for Robust $\overline{X}$ – Chart:

$$LCL = \overline{MD} - (R_2)\left(\frac{(USL-LSL)-|M-m|}{C_{pk}}\right) \quad (20)$$

$$CL = \overline{MD} \quad (21)$$

$$UCL = \overline{MD} + (R_2)\left(\frac{(USL-LSL)-|M-m|}{C_{pk}}\right) \quad (22)$$

where

$$R_2 = \frac{0.6265}{\sqrt{n}}$$

Table 1.represent the quality control constant (R_2) for the proposed control charts.

n	R2
2	0.44300
3	0.36171
4	0.31325
5	0.28018
6	0.25577
7	0.23679
8	0.22150
9	0.20883
10	0.19812
11	0.18890
12	0.18085
13	0.17376
14	0.16744
15	0.16176
16	0.15663
17	0.15195
18	0.14767
19	0.14373
20	0.14009
21	0.13671
22	0.13357
23	0.13063
24	0.12788
25	0.12530

Table 1

IV. Simulation Study

The proposed control charts are tested with a simulated normally distributed data with mean μ = 20 and standard deviation σ = 1. The following parameters, USL = 22, LSL = 18 and T = 20 are considered for comparing Shewhart control chart and the proposed capability index based control chart.

$\overline{\overline{X}} = 20.014$; $\overline{R} = 2.330$; Cp=0.67, Cpk = 0.66 and Cpm = 0.69.

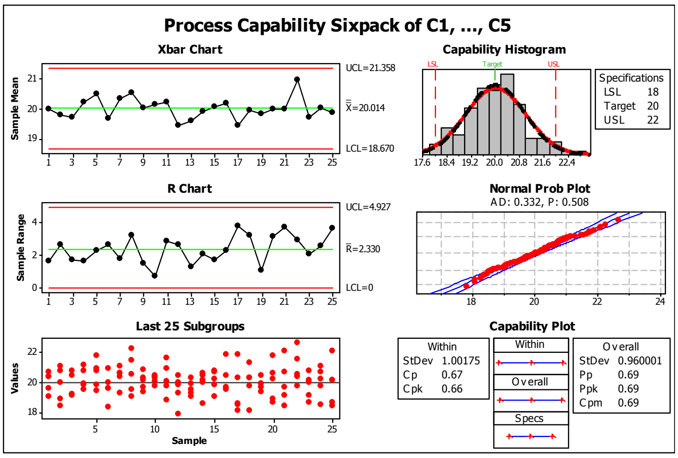

Figure 1. Shewhart$\overline{X}$, R Control Chart and the Capability Indices

The statistical software MINITAB 16 is used to calculate various capability indices. Figure 1 represents the Shewhart$\overline{X}$, R control chart and the various capability indices. Since all the points in the control charts are within the control limits it is inferred that the process is statistically control. But from the capability analysis of the sample of size 25 with subgroup size 5, since all the capability values are less than 1, we conclude that the process is not capable to meet the specification.

Table 2 represents the control limits of $\overline{X}$ chart for the specified value (1.33) of the four basic capability indices.

Table 2

Capability Index based Robust Control chart			Robust Control chart		
Capability Index					
Cpk = 1.33					
UCL	LCL	CL	UCL	LCL	CL
20.55271	19.4404	19.99656	21.76878	18.22433	19.99656

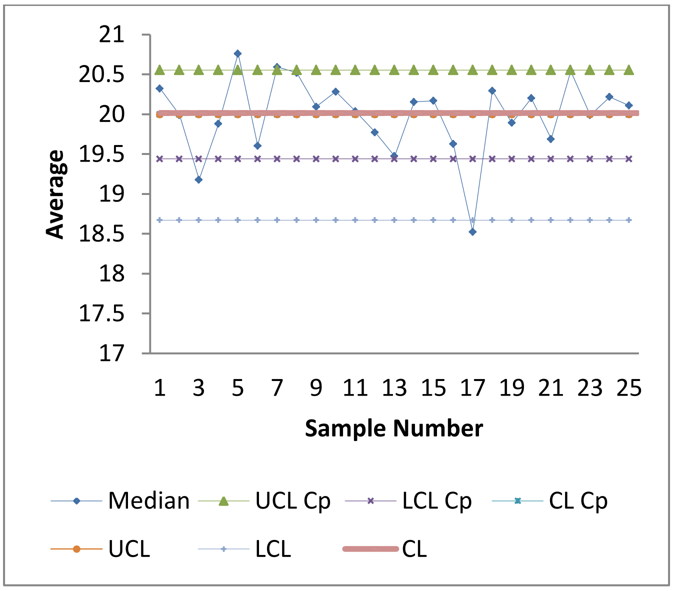

Figure 2. Cpk = 1.33 based $\bar{X}$ chart

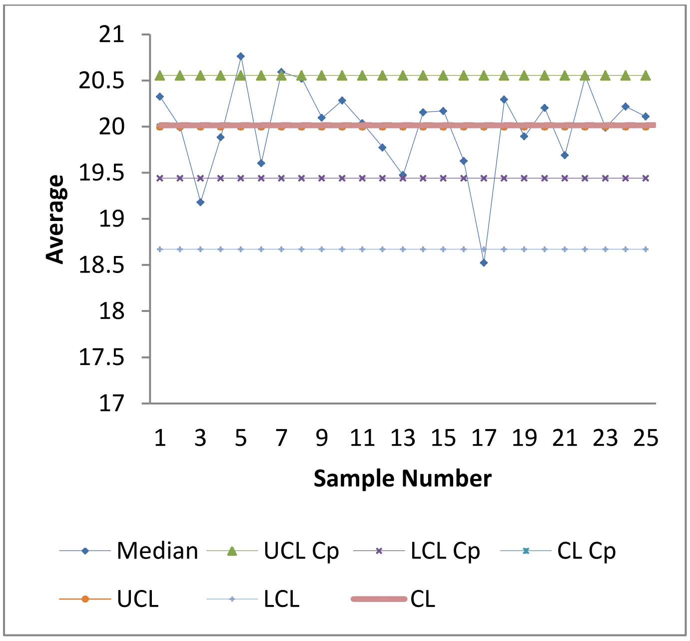

Figure 3. Cpk = 1.5 based $\bar{X}$ chart

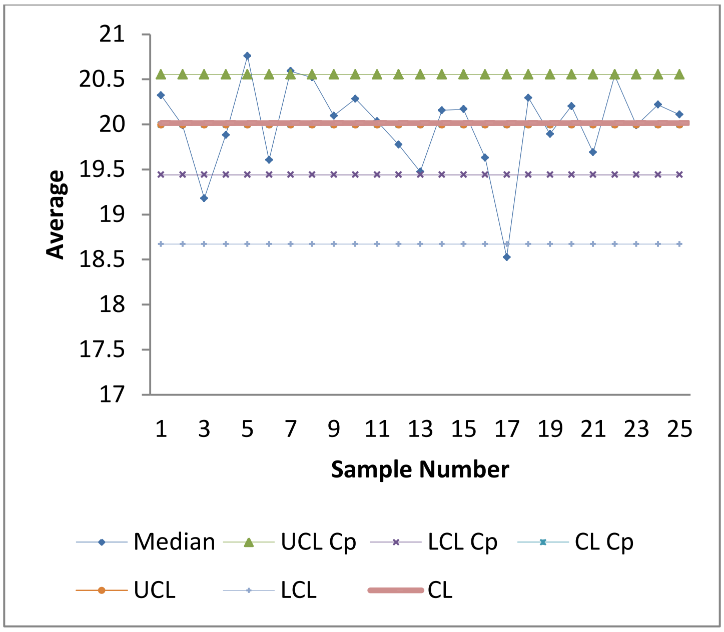

Figure 4. Cpk = 2 based $\bar{X}$ chart

Fig 1, 2, 3, 4 and 5 represents the $\bar{X}$ control chart based on Cpk for the specified capability index value 1.33, 1.5 and 2. In all the cases, it can be inferred that the process is statistically control to meet the specification since the process capability index values of the data considered are very closer to each other.

		Width of the control limits of $\bar{X}$ chart	
		Shewhart	**Cpk**
Capability Value	**1.33**	2.688	1.672656
	1.5	2.688	1.483088
	2	2.688	1.112316

Table 4 Width of the Control limits

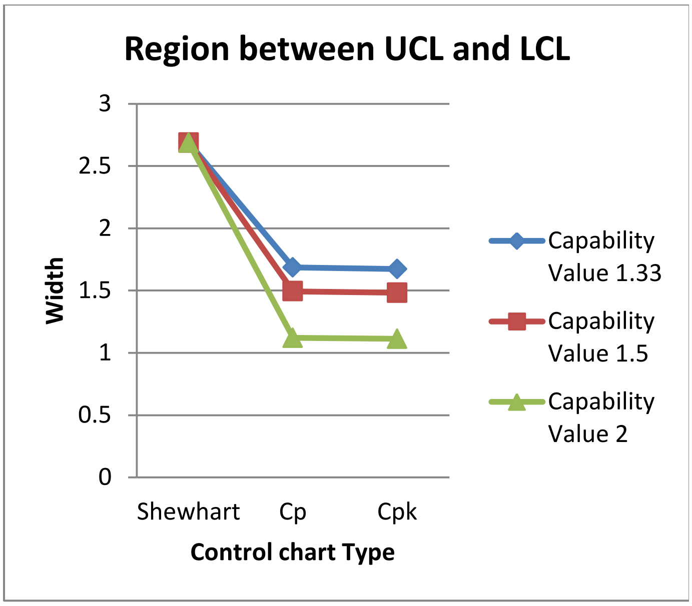

Figure 5. Comparison of Region between UCL and LCL for Shewhart Control Chart and capability index based $\bar{X}$ control chart

Table 3 and Table 4 provides the width between UCL and LCL of R chart and $\bar{X}$ chart respectively. From both the tables it is clear that the region between UCL and LCL of R chart and $\bar{X}$ chart reduces when the capability value increases and also when the capability index is changed.

		Ratio
		Cpk
Capability Value	**1.33**	0.622268
	1.5	0.551744
	2	0.413808

$$Ratio = \frac{Width\ of\ Process\ Capability\ Index\ based\ control\ limit}{Width\ of\ Shewhart\ control\ limit} \quad (20)$$

Equation (20) is used to calculate the ratio between the width of Shewhart control limits and Capability index based control limits for R chart and $\bar{X}$ chart and presented in Table 5 and Table 6 respectively. If the widths are equal, the ratio will be equal to 1. But from the tables it is clear that the region between UCL and LCL of R chart and $\bar{X}$ chart reduces when the capability value increases and also when the capability index is changed.

V. Conclusion

Control charts are used to check whether the process is statistically control. When the process is statistically control, Capability indices are used to measure whether the process is capable to meet the customer specifications.

A unified approach to capability index based variable control chart is proposed which extends the variable control chart with specified capability index. A simulated data that follows normal distribution have been used to construct the Shewhart control chart and the proposed capability index based control chart. It is clear from the result that the width of the control limits is reduced for the proposed control charts as compared with Shewhart control chart. The ratio is also calculated for Shewhart control limits and the proposed various capability index based control limits for three specified values 1, 1.33 and 2. Since the ratio is less than 1, it is clear that the control limit width of the proposed chart is less than the Shewhart control limit width. The proposed control chart can be extended other capability indices.

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