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Adomian Decomposition Analysis of Displacement through Homogeneous Porous Media

Patel Anilkumar J.*[1], Khatri Ronak G.*[1], Dr. Bhathawala P. H.[2]
[1]Research Scholar, PACIFIC University, Udaipur, Rajasthan, India
[2]Department of Mathematics, UGC Visiting Professor, HNGU Patan, Gujarat, India

ABSTRACT

In ground water flow in porous media givens some models solve by nonlinear partial differential equations considering Adomian decomposition method. We create the being of accurate solutions for those models. The numerical results show the ability and correctness of this method.

Keywords : Adomian decomposition method, nonlinear partial differential equations, Imbibition, Drainage.

I. INTRODUCTION

Nonlinear partial differential equations can be found in wide variety scientific and engineering applications. Many important mathematical models can be expressed in terms of nonlinear partial differential equations. The most general form of nonlinear partial differential equation is given by:

$$F(u, u_t, u_x, u_y, x, y, t) = 0 \quad \textbf{(1a)}$$

With initial and boundary conditions

$$u(x, y, 0) = \emptyset(x, y), \forall x, y \in \Omega, \Omega \in R^2 \quad \textbf{(1b)}$$

$$u(x, y, t) = f(x, y, t), \forall x, y \in \partial\Omega \quad \textbf{(1c)}$$

WhereΩ is the solution region and $\partial\Omega$ is the boundary of Ω.

In recent years, much research has been focused on the numerical solution of nonlinear partial equations by using numerical methods and developing these methods (Al-Saif, 2007; Leveque, 2006; Rossler & Husner, 1997; Wescot & Rizwan-Uddin, 2001). In the numerical methods, which are commonly used for solving these kind of equations large size or difficult of computations is appeared and usually the round-off error causes the loss of accuracy.

The Adomian decomposition method which needs less computation was employed to solve many problems (Celik et al., 2006; Javidi & Golbabai, 2007). Therefore, we applied the Adomian decomposition method to solve some models of nonlinear partial equation, this study reveals that the Adomian decomposition method is very efficient for nonlinear models, and it results give evidence that high accuracy can be achieved.

II. MATHEMATICAL METHODOLOGY

The principle of the Adomian decomposition method (ADM) when applied to a general nonlinear equation is in the following form(Celik et al., 2006; Seng et al., 1996):

$$Lu + Ru + Nu = g \quad \textbf{(2)}$$

The linear terms decomposed into $Lu+Ru$, while the nonlinear terms are represented by Nu , where L is an easily invertible linear operator, R is the remaining linear part. By inverse operator L, with $L^{-1}(\cdot) = \int_0^t (\cdot)dt$. Equation (2) can be hence as;

$$u = L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu) \quad \textbf{(3)}$$

The decomposition method represents the solution of equation (3) as the following infinite series:

$$u = \sum_{n=0}^{\infty} u_n \quad \textbf{(4)}$$

The nonlinear operator $Nu = \Psi(u)$ is decomposed as:

$$Nu = \sum_{n=0}^{\infty} A_n \quad \textbf{(5)}$$

Where A_n are Adomian's polynomials, which are defined as (Seng et al., 1996):

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[\psi(\sum_{i=1}^{n} \lambda^i u_i)]_{\lambda=0} \qquad n = 0,1,2,\dots \quad \textbf{(6)}$$

Substituting equations (4) and (5) into equation (3), we have

$$u = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}\left(R(\sum_{n=0}^{\infty} u_n)\right) - L^{-1}(\sum_{n=0}^{\infty} A_n) \quad \textbf{(7)}$$

Consequently, it can be written as:

$$\left.\begin{aligned}
u_0 &= \varphi + L^{-1}(g)\\
u_1 &= -L^{-1}(R(u_0)) - L^{-1}(A_0)\\
u_2 &= -L^{-1}(R(u_1)) - L^{-1}(A_1)\\
&\;\;.\\
&\;\;.\\
&\;\;.\\
u_n &= -L^{-1}(R(u_{n-1})) - L^{-1}(A_{n-1})
\end{aligned}\right\} \quad \textbf{(8)}$$

where φ is the initial condition,

Hence all the terms of u are calculated and the general solution obtained according to ADM as $u = \sum_{n=0}^{\infty} u_n$. The convergent of this series has been proved in (Seng et al., 1996). n=0

However, for some problems (Celik et al., 2006) this series can't be determined, so we use an approximation of the solution from truncated series

$$U_M = \sum_{n=0}^{M} u_n \, with \lim_{M\to\infty} U_M = u$$

III. STATEMENT OF THE PROBLEM

When a fluid contained in porous medium is displaced by another fluid of lesser viscosity, instead of regular displacement of the whole front, protuberance may occur which shoot through the porous medium at relatively great speed. Those protuberances are called fingers [10] and the phenomenon is called fingering.

We consider that a finite cylindrical piece of homogeneous porous medium of length L, fully saturated with oil, which is displaced by injecting water which give rise to fingers (protuberance). Since the entire oil at the initial boundary $\boldsymbol{x} = \mathbf{0}$ (x being measured in the direction of displacement), is displaced through a small distance due to water injection, therefore, it is assumed that complete saturation exists at the initial boundary.

Here, an analytical expression for the cross – sectional area occupied by fingers has been obtained. For the mathematical formulation, we consider the governing law which id Darcy's law, here, as valid for the investigated flow system and assumed further that the macroscopic behavior of fingers is governed by statistical treatment.

In the statistical treatment of fingers only the average behavior of the two fluids involved is taken into consideration. It was shown by Scheideger and Johnson [12], that this treatment of motion with the introduction of the concept of fictitious relative permeability become formally identical to the Buckley – Leverett description of two immiscible of injected fluid flow through porous media. The saturation of injected fluid (S_i) is then defined in average cross-sectional area occupied by the injected fluid level x at time t, i.e. $S_i(x,t)$. Thus the saturation of injected fluid in porous medium represent the average cross-sectional area occupied by fingers [13].

IV. MATHEMATICAL MODELING

The equation for motion for saturation can be obtained by substituting the values of V_w and V_o

$$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x}\right] \tag{9}$$

$$\varphi \frac{\partial S_0}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_0}{\mu_0} k \frac{\partial P_0}{\partial x}\right] \tag{10}$$

Eliminating $\frac{\partial P_w}{\partial x}$ from equations (10) and $P_c = P_0 - P_w$, we get

$$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w} k \left\{\frac{\partial P_0}{\partial x} - \frac{\partial P_c}{\partial x}\right\}\right] \tag{11}$$

Combining equation (10) and (11) by using equation $S_w + S_0 = 1$, we obtain

$$\frac{\partial}{\partial x}\left[\left\{\frac{k_w}{\mu_w} + \frac{k_0}{\mu_0}\right\} k \frac{\partial P_0}{\partial x} - \frac{k_w}{\mu_w} k \frac{\partial P_c}{\partial x}\right] = 0 \tag{12}$$

Integrating equation (12) with respect to x, we get

$$\left[\left\{\frac{k_w}{\mu_w} + \frac{k_0}{\mu_0}\right\} k \frac{\partial P_0}{\partial x} - \frac{k_w}{\mu_w} k \frac{\partial P_c}{\partial x}\right] = -A \tag{13}$$

Where A is the constant of integration (negative sign on right hand side is considered for our convenience). Simplifying equation (13), we get

$$\frac{\partial P_0}{\partial x} = \frac{-A}{k\left\{\frac{k_w}{\mu_w} + \frac{k_0}{\mu_0}\right\}} + \frac{k_w}{\mu_w\left\{\frac{k_w}{\mu_w} + \frac{k_0}{\mu_0}\right\}} \frac{\partial P_c}{\partial x} \tag{14}$$

Now substituting the value of $\frac{\partial P_0}{\partial x}$ from equation (14) into equation (11), we have

$$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w} k \left\{\frac{-A}{k\left\{\frac{k_w}{\mu_w} + \frac{k_0}{\mu_0}\right\}} + \frac{k_w}{\mu_w\left\{\frac{k_w}{\mu_w} + \frac{k_0}{\mu_0}\right\}} \frac{\partial P_c}{\partial x} - \frac{\partial P_c}{\partial x}\right\}\right]$$

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w} k \frac{\partial P_c}{\partial x} \frac{1}{1+\frac{k_0\mu_w}{k_w\mu_0}} + \frac{A}{1+\frac{k_0\mu_w}{k_w\mu_0}}\right] = 0 \tag{15}$$

Now from $\frac{\partial P_0}{\partial x} = \frac{1}{2}\frac{\partial P_c}{\partial x}$

Substituting this value in (13), we have

$$A = \left\{\frac{k_w}{\mu_w} - \frac{k_0}{\mu_0}\right\} \frac{k}{2} \frac{\partial P_c}{\partial x} \tag{16}$$

Substituting the value of A from (16) into (15), we get

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[k \frac{k_w}{2\mu_w} \frac{\partial P_c}{\partial x}\right] = 0$$

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[k \frac{k_w}{2\mu_w} \frac{\partial P_c}{\partial S_w} \frac{\partial S_w}{\partial x}\right] = 0 \tag{17}$$

From equation $k_w = S_w$, $k_0 = S_0 = 1 - S_w$, $P_c = -\beta S_w$, and equation (17), we have

$$\varphi \frac{\partial S_w}{\partial t} - \frac{\beta k k_w}{2\mu_w} \frac{\partial^2 S_w}{\partial x^2} = 0 \tag{18}$$

Let $k = c_0\tau\frac{\varphi^3}{M_s(1-\varphi)^2}$, $\tau = (\frac{L}{L_e})^2$ **(19)**

Where φ is porousity

M_s is specific surface area

$\boldsymbol{c_0}$ Kozeny constant

$\boldsymbol{L_e}$ Effective length of the path of the fluid

From (18) and (19), we have

$$\frac{\partial \mathrm{S_w}}{\partial \mathrm{t}} - \frac{\beta k_w}{2\mu_w}c_0\tau\frac{\varphi^3}{M_s(1-\varphi)^2}\frac{\partial^2 \mathrm{S_w}}{\partial x^2} = 0 \quad \textbf{(20)}$$

This is a non-linear partial different equation of motion for the saturation of the injected fluid through the homogeneous porous media.

Here, the capillary pressure co-efficient, from our assumption, it is small enough to consider. It is perturbation parameter. Again $\boldsymbol{\beta}$ is multiplied to the highest derivative in equation (20), therefore, the problem (20) is a singular perturbation problem. Such problem together with appropriate conditions has been solved analytically or numerically.

A set of boundary condition are written as

$$\mathrm{S_w}(0,\mathrm{t}) = \mathrm{S_{W_0}}, \mathrm{S_w}(\mathrm{L},\mathrm{t}) = \mathrm{S_{W_1}} \quad \textbf{(21)}$$

$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial x}}\mathrm{S_w}(\mathrm{L},\mathrm{t}) = 0, \quad 0 \le x \le L \quad \textbf{(22)}$$

Where $\mathrm{S_{W_0}}$ is the saturation at x = 0

$\mathrm{S_{W_1}}$ is the saturation at x = L

Also we assume that there is no flow across the face x = L (because the face at x = L is assumed to be impermeable), that is

Let is $X = \frac{x}{L}$, $T = \frac{\boldsymbol{k_w}}{\boldsymbol{2\mu_w}}c_0\tau\frac{\varphi^3}{M_s(1-\varphi)^2L^2}t$

Solution of the Problem:

Consider the Problem

$$\frac{\partial \mathrm{S_w}}{\partial T} - \beta\frac{\partial^2 \mathrm{S_w}}{\partial X^2} = 0$$

with the initial Condition

$$\mathrm{S_w}(0,\mathrm{t}) = \mathrm{S_{W_0}}, \mathrm{S_w}(\mathrm{L},\mathrm{t}) = \mathrm{S_{W_1}}$$

$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial x}}\mathrm{S_w}(1,\mathrm{t}) = 0, \quad 0 \le X \le 1$$

Solution: In this problem, we have

$N(\mathrm{S_w}) = \Psi(\mathrm{S_w}) = -\beta\frac{\partial^2 \mathrm{S_w}}{\partial X^2}$

$g(\xi, t) = 0$

$R(\mathrm{S_w}) = 0$

$L(\mathrm{S_w}) = \frac{\partial \mathrm{S_w}}{\partial T}$

And $\phi = \mathrm{S_w}(0,\mathrm{t}) = \mathrm{S_{W_0}}$

By using equation(6) Adomain's polynomials can be derived as follows:

$$
\left.\begin{aligned}
A_0 &= -\beta_0 \frac{\partial^2 S_{w_0}}{\partial X^2} \\
A_1 &= -\beta_1 \frac{\partial^2 S_{w_0}}{\partial X^2} - \beta_0 \frac{\partial^2 S_{w_1}}{\partial X^2} \\
A_2 &= -\beta_2 \frac{\partial^2 S_{w_0}}{\partial X^2} - \beta_1 \frac{\partial^2 S_{w_1}}{\partial X^2} - \beta_0 \frac{\partial^2 S_{w_2}}{\partial X^2} \\
A_3 &= -\beta_3 \frac{\partial^2 S_{w_0}}{\partial X^2} - \beta_2 \frac{\partial^2 S_{w_1}}{\partial X^2} - \beta_1 \frac{\partial^2 S_{w_2}}{\partial X^2} - \beta_0 \frac{\partial^2 S_{w_3}}{\partial X^2} \\
&\vdots
\end{aligned}\right\} \quad (23)
$$

And so on. The rest of the polynomials can be constructed in similar manner.

By using Equation (8), we have

$$
\left.\begin{aligned}
\phi &= S_w(0,t) = S_{W_0} \\
S_{w_1} &= S_{W_0} \frac{\partial^2 S_{w_0}}{\partial X^2} t \\
S_{w_2} &= S_{W_0} \left(\frac{\partial^2 S_{w_0}}{\partial X^2}\right)^2 \frac{t^2}{2} \\
S_{w_3} &= S_{W_0} \left(\frac{\partial^2 S_{w_0}}{\partial X^2}\right)^3 \frac{t^3}{6} \\
\mathbf{S_{w_4}} &= \mathbf{S_{W_0}} \left(\frac{\partial^2 S_{w_0}}{\partial X^2}\right)^4 \frac{t^4}{24} \\
&\vdots
\end{aligned}\right\} \quad (24)
$$

Substituting these individual terms in equation (4) obtain

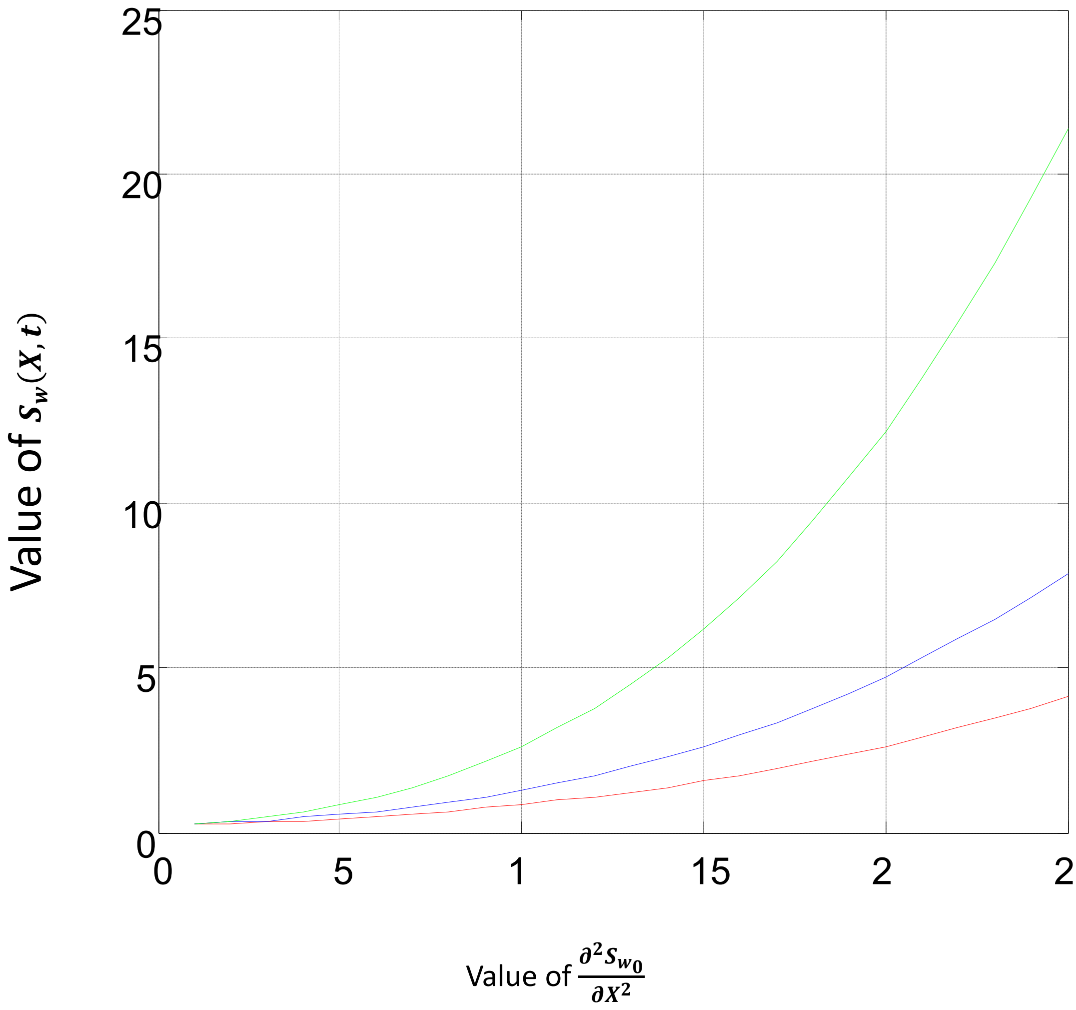
$$
S_w(X,t) = S_{W_0}\left(1 + \frac{\partial^2 S_{w_0}}{\partial X^2} t + \left(\frac{\partial^2 S_{w_0}}{\partial X^2}\right)^2 \frac{t^2}{2} + \left(\frac{\partial^2 S_{w_0}}{\partial X^2}\right)^3 \frac{t^3}{6} + \left(\frac{\partial^2 S_{w_0}}{\partial X^2}\right)^4 \frac{t^4}{24} + \cdots\right)
$$

This gives the exact solution.

$$\mathbf{S_{W_0} = 0.2}$$

Table1

t	$\frac{\partial^2 S_{w_0}}{\partial X^2}$	$S_w(X,t)$	t	$\frac{\partial^2 S_{w_0}}{\partial X^2}$	$S_w(X,t)$	t	$\frac{\partial^2 S_{w_0}}{\partial X^2}$	$S_w(X,t)$
0.15	1	0.2324	0.2	1	0.2443	0.3	1	0.2699
	4	0.3632		4	0.4411		4	0.6416
	7	0.5588		7	0.7675		7	1.3697
	10	0.8375		10	1.2667		10	2.6000
	13	1.2174		13	1.9819		13	4.4783
	16	1.7168		16	2.9563		16	7.1504
	19	2.3539		19	4.2331		19	10.7621
	22	3.1469		22	5.8555		22	15.4592
	25	4.1141		25	7.8667		25	21.3875



V. CONCLUSION

In this paper, we have applied the Adomian decomposition method for solving problem of nonlinear partial equations. We demonstrated that the decomposition procedure is quite efficient to determine the exact solutions. We can say that as time (t) increases saturation (S_w) increases. Keeping S_{W_0} constant. It is clear that the curves are parabolic type. In addition, the numerical results which obtained by this method indicate a high degree of accuracy.

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