

# Analysis of a Passive Linear Quarter Car Suspension System using Matlab/Simulink

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## ABSTRACT

This paper discusses the suspension of a two-degree-of-freedom (2-DOF), Linear Quarter Car, vehicle system. MATLAB/Simulink environment is used to analyze response of the quarter car. The quarter car model is subjected to various inputs like step input, sine, random, saw tooth, ramp etc... To study the system, first the governing equations for a linear quarter car model are derived. Then a Differential Equation approach, State Space approach and Transfer Function approach was used using Simulink blocks. For all the approaches the results matches to each other.

**Keywords:** Linear Quarter car, MATLAB/Simulink, State Space, Transfer Function.

## I. INTRODUCTION

The automobile is a combination of variety of complex systems. Each system has its own functions to perform and how good an automobile turns out to be depends on the proper synthesis of these systems. One such system is the suspension system. Suspension system has been widely used, to the vehicles from the horse drawn carriages with flexible leaf springs, to the modern automobiles with complex control algorithms.

The conventional system i.e. passive suspension system, which comes *as is*, is a system of springs, shock absorbers, bushings, rods, linkages and arms. Vehicles generally have two suspension systems – one for the front wheel and other for rear. These two systems work together to control driving and braking forces to provide smooth ride for driver and passengers.

The passive suspension systems are trade off between

ride comfort and performance. A case with a nice cushy ride usually wallows through the corners whereas a car with high performance suspension, like F1 cars, will hang on tight through the corners but will make the passengers feel very every little dip and bump in the road.

## II. VEHICLE MODEL

Modeling of suspension system is done in the vertical plane. Longitudinal or transverse deflection of the suspension components is considered negligible in comparison to vertical deflections. A linear 2-DOF system is used as a model for the road vehicle. The two masses  $m_1$  and  $m_2$ , of the vehicle model represent the wheel (and axle if there is any) and the vehicle body (often called the unsprung and the sprung mass, respectively). These masses representing one half or a quarter of a real car. The spring stiffness are  $k_1$ ,  $k_2$  and the damper stiffness are  $c_1$ ,  $c_2$ .

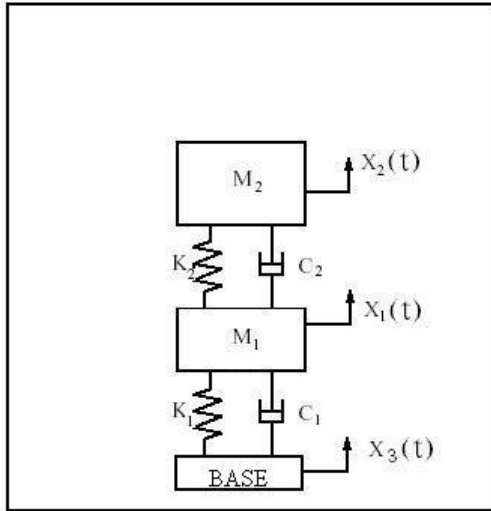


Figure 1. 2-DOF (Linear) system with base excitation (Quarter Car Model).

### III. MATHEMATICAL MODELING

Differential equations (DE) to account for the moving base become,

$$m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2)x_1 - c_2 \dot{x}_2 - k_2 x_2 - c_1 \dot{x}_3 - k_1 x_3 = 0$$

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - c_2 \dot{x}_1 - k_2 x_1 = 0 \quad \text{---- (1)}$$

#### Transfer Function (TF)

Taking the Laplace transform of the eq.1, we get,  
 $m_1 (s^2 X_1 - s x_1(0) - \dot{x}_1(0)) + (c_1 + c_2) (s X_1 - x_1(0)) + (k_1 + k_2) X_1$   
 $= c_2 (s X_2 - x_2(0)) + k_2 X_2 + c_1 (s X_3 - x_3(0)) + k_1 X_3$   
 &  
 $m_2 (s^2 X_2 - s x_2(0) - \dot{x}_2(0)) + (c_2 s X_2 - c_2 x_2(0)) +$   
 $k_2 X_2 = (c_2 s X_1 - c_2 x_1(0)) + k_2 X_1$   
 ---- (2)

Putting initial conditions,  $x(0) = 0$  and  $\dot{x}(0) = 0$ ,  
 $m_1 (s^2 + (c_1 + c_2) s + k_1 + k_2) X_1 = (c_2 s + k_2) X_2 + (c_1 s + k_1) X_3$   
 &  
 $(m_2 s^2 + c_2 s + k_2) X_2 = (c_2 s + k_2) X_1$  ---- (3)

The second equation from (3) can be written as,  
 $X_2 / X_1 = (c_2 s + k_2) / (m_2 s^2 + c_2 s + k_2)$  ---- (4)

The equation, (4), is known as *transmissibility function*.

eliminating  $X_1$  from above equations,  
 $X_1 = (m_2 s^2 + c_2 s + k_2) X_2 / (c_2 s + k_2)$

Now,  
 $m_1 (s^2 + (c_1 + c_2) s + k_1 + k_2) (m_2 s^2 + c_2 s + k_2) X_2 / (c_2 s + k_2) =$

$$(c_2 s + k_2) X_2 + (c_1 s + k_1) X_3$$

solving and rearranging, we have

$$\therefore X_2 / X_3 = [c_1 c_2 s^2 + (k_1 c_2 + k_2 c_1) s + k_1 k_2] / \{ [m_1 m_2 s^4 + [m_1 c_2 + m_2 (c_1 + c_2)] s^3 + [m_2 (k_1 + k_2) + c_1 c_2 + m_1 k_2] s^2 + (k_1 c_2 + k_2 c_1) s + k_1 k_2 \}$$
 ---- (5)

Similarly we can compute,  
 $X_1 / X_3 = \{ m_2 c_1 s^3 + (c_1 c_2 + m_2 k_1) s^2 + (c_1 k_2 + c_2 k_1) s + k_1 k_2 \} / \{ [m_1 m_2 s^4 + [m_1 c_2 + m_2 (c_1 + c_2)] s^3 + [m_2 (k_1 + k_2) + c_1 c_2 + m_1 k_2] s^2 + (k_1 c_2 + k_2 c_1) s + k_1 k_2 \}$  [1, 2, 4, 5, 6] ---- (6)

#### State Space Approach (SS)

For the given system we define 4 state variables, two gives the displacement of the two masses and the other two gives the velocities of the respective masses. [1, 4, 5, 6, 8]

We have,  
 $m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2)x_1 - c_2 \dot{x}_2 - k_2 x_2 - c_1 \dot{x}_3 - k_1 x_3 = 0$   
 $m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - c_2 \dot{x}_1 - k_2 x_1 = 0$  ---- (Refer eq. 1)

Let,  
 $x_1(t) = x_2 \quad x_2(t) = \dot{x}_1$   
 $x_3(t) = \dot{x}_1'(t) \quad x_4(t) = \dot{x}_2'(t)$   
 $x_3'(t) = \dot{x}_1''(t) = x_3'' \quad x_4'(t) = \dot{x}_2''(t) = x_4''$  ---- (7)

Putting equation (6) in equation (1), we can get  
 $x_4'(t) = (c_2/m_1) x_3(t) + (k_2/m_1) x_1(t) + (c_1/m_1) x_3'(t) + (k_1/m_1) x_3 - [(c_1 + c_2)/m_1] x_4(t) - [(k_1 + k_2)/m_1] x_2(t)$

$$x_3'(t) = (c_1/m_2) x_4(t) + (k_2/m_2) x_2(t) - (c_2/m_2) x_3(t) - (k_2/m_2) x_1(t)$$

And  
 $x_1'(t) = x_3(t)$   
 $x_2'(t) = x_4(t)$

Finally re-arranging above equations in the following form of equation

$$x'(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

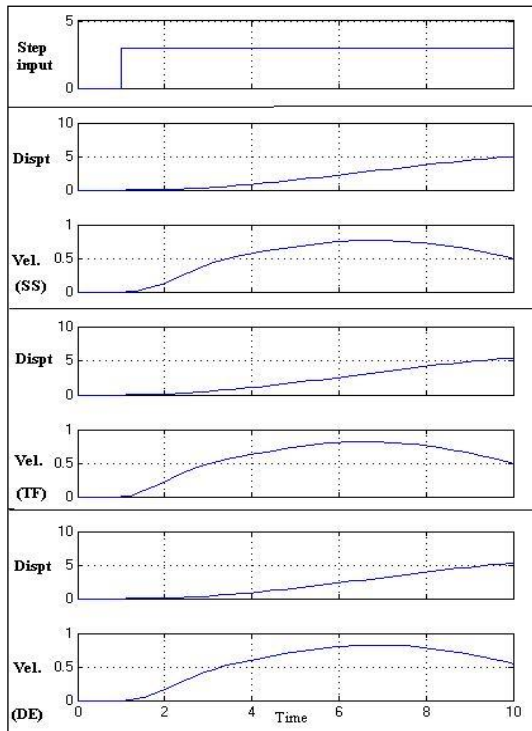
---- (8)

where  $x'(t)$ ,  $x(t)$ ,  $y(t)$ ,  $A$ ,  $B$ ,  $C$  and  $D$  are the matrices of various order.



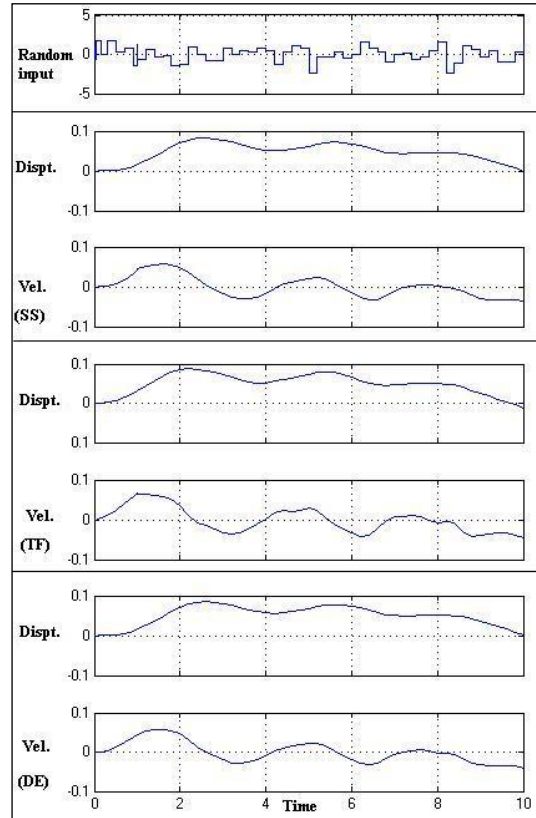
**MATLAB/Simulink Results for Linear Quarter Car Model:**

**Step Input: (Step = 3units)**



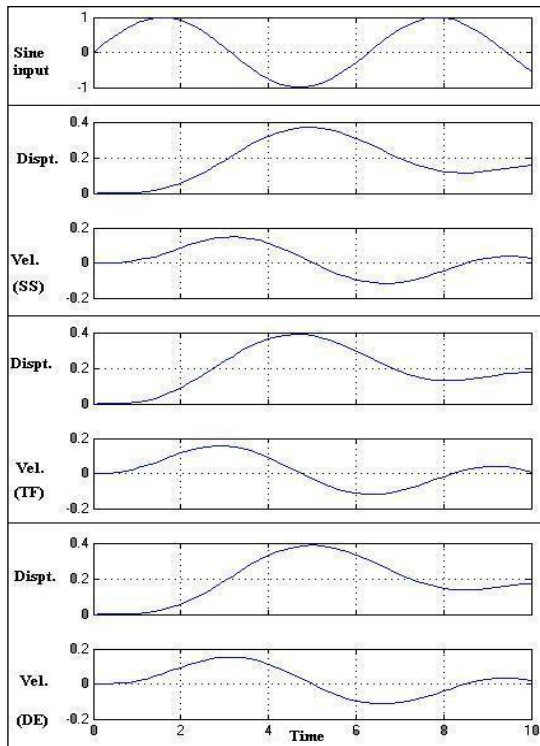
**Figure 4.** Step response for SS, TF, DE approach.

**Random Input:**



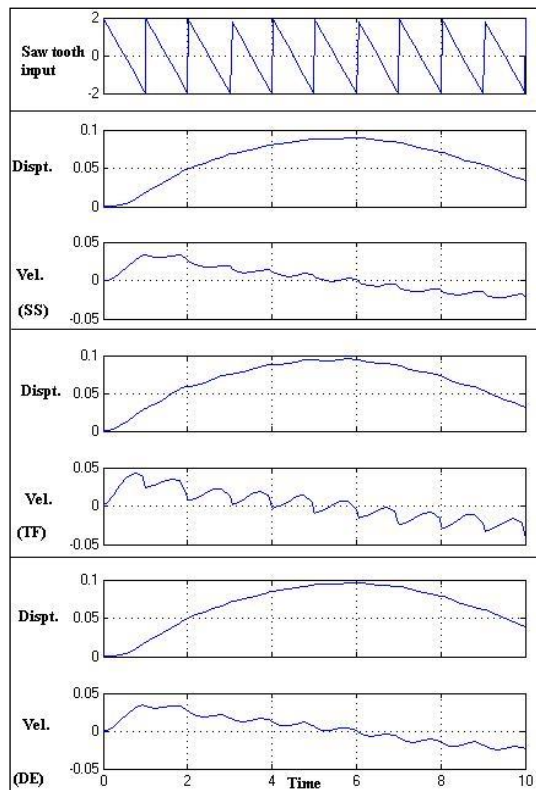
**Figure 6.** Random response – SS, TF, DE approach.

**Sine Input:**



**Figure 5.** Sine response for SS, TF, DE approach.

**Saw Tooth Input:**



**Figure 7.** Saw Tooth response – SS, TF, DE approach.

## V. CONCLUSION

Here, MATLAB/Simulink environment is used to simulate the Linear Quarter Car Model. Mathematically obtained FRFs are simulated using MATLAB. When input is known, the FRFs are computed for  $x_1/x_2$  and  $x_2/x_3$ . If input is not known, generally transmissibility is computed between sprung and unsprung mass. There is only one peak in transmissibility function. This peak occurs near the sprung mass resonance, but it is not natural mode of system because the transmissibility only indicate how the sprung mass is responding relative to the unsprung mass (the inverse shows the opposite relation).

In SIMULINK, three different approaches, Differential Equation (DE), Transfer Function (TF) and State Space (SS) are used for the analysis of a Linear Quarter Car Model. Responses obtained shows that the approaches used give same results.

Here only standard test signals (i.e. Unit Step, Sinusoidal and Random etc.) are used to obtain the system responses. In reality the road disturbances are random in nature and hence if available the actual road load data, will be utilized. Simulink can be proved to be the best tool from the analysis and the interpretation point of view.

## VI. REFERENCES

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