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Mathematical Modelling and Analysis of Tire-Vehicle Suspension System Using Matlab

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ABSTRACT

The aim of this paper is to introduce the mathematical modeling of a tire-vehicle suspension system for a quarter car model. This paper discusses the 2-DOF system as a quarter car model with base excitation. To study the system, first derive the dynamic equations of the vehicle model. The Laplace transform approach is selected with assumption that the displacement of the base is a half-sine wave. To analyse the responses, a interactive system MATLAB® is used. The simulation results shows that a two degree-of-freedom system with appropriately chosen parameters can be an effective isolator of ground vibrations compared to a single degree-of-freedom system.

Keywords : Tire-Vehicle Suspension System, Mathematical Modelling, Laplace transform, Two-Degree-of-Freedom

I. INTRODUCTION

Since the days of horse-drawn spring carriages people have strived for making rides com¬fortable by isolating the car body from road irregularities. Today's "carriage" isolation could consist of passive and/or active spring and dashpot elements. The aim of this paper is to optimize a passive linear spring-dashpot road vehicle suspension system with respect to both ride comfort and road holding.

Since the 1950s the theory of stochastic processes has been applied to road vehicle response problems. The road profile is taken as a one-dimensional stationary Gaussian stochastic process in space. The road vehicle is modeled as a linear two-degree-of-freedom (2-DOF) system. The road-induced vehicle responses studied will then come out as stochastic processes of the same type. Criteria for ride comfort and road holding are formulated on the basis of the vertical acceleration response of the vehicle and the wheel-road force, respectively. The vehicle suspension working space is limited and the limitation is formulated in probabilistic terms.

The ride comfort criterion is based on the vertical acceleration response process pi of the vehicle. This process is filtered through the passenger's seat and then also weighted in the frequency domain according to human sensitivity to vertical acceleration. The power spectral density of a process so obtained could serve as a base for subjective judgment of the ride comfort, or the standard deviation of the process could be used. It is supposed in what follows here that the largest maxima of a weighted stochastic acceleration process are mainly responsible for ride discomfort and the comfort criterion is based on these maxima only.

When studying road holding and limited working space (under stochastic excitation) the quantities most commonly used hitherto are the standard deviations of the road-wheel contact force and of the distances. In what follows here, however, optimal road holding is defined as a minimum probability that the randomly varying part of the road-wheel contact force will exceed a given level during a specified time period.

The suspension of a two-degree-of-freedom (2-DOF) vehicle traveling on a forlornly corrugated road is optimized with respect to both road holding and ride comfort. [3,6]

Optimal comfort is defined as a minimum mean value of the latest maxima of a stationary Gaussian random process. This process is the vertical vehicle seat acceleration weighted with respect to human sensitivity (ISO 2631). [6]

Optimal road holding is defined as a minimum probability that the road-wheel contact force will be smaller than a given level. This contact force is conceived as another stationary Gaussian random process. [6]

The two criteria are synthesized and the suspension system is optimized with respect to the joint criterion obtained. One restriction accounted for is the limited working space of the vehicle suspension.

II. VEHICLE MODEL

A linear 2-DOF system is used as a model for the road vehicle (Refer Fig.1). The two (fixed-base) eigen frequencies of the model should represent the lowest two eigen frequencies of the Vehicle. The lowest eigen frequency of a road vehicle pertains to the whole-body vibration. For a medium-size passenger car this frequency is about 1.0-1.5 Hz (6-10 radians/s). The second lowest eigen frequency is due to wheel vibration. Normally this frequency is about 10 Hz (60 radians/s).

The two masses m1 and m2, of the vehicle model represent the wheel (and axle if there is any) and the vehicle body (often called the unsprung and the sprung mass, respectively). These masses (representing one half or a quarter of a real car). The spring stiffness are k1, k2 and the damper stiffness are c1, c2 . (Refer Fig. 2.1) [1,5,6]

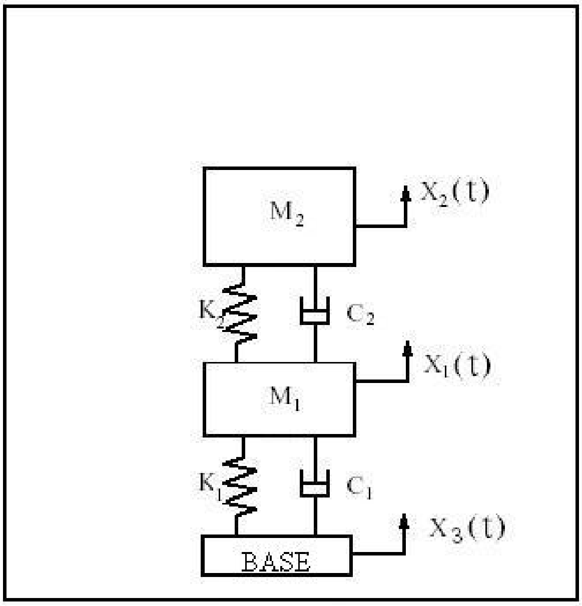

Figure 1. 2-DOF system with base excitation.

III. DYNAMIC EQUATIONS OF VEHICLE MODEL

Another way to determine the efficacy of a two degree-of-freedom isolation system is to compare the magnitude of the peak (maximum) displacement of m2 to the magnitude of the peak displacement of the ground. To analyze this type of situation, we consider the base supporting the two degree-of-freedom system to be a moving base as shown in Figure1. In analyses of such systems, one usually assumes that the masses are initially at rest and that there are no applied forces directly on the inertial elements, and x3(t) is given. Eqs. to account for the moving base become,

$$\left.\begin{array}{l} m_1\ddot{x}_1+(c_1+c_2)\dot{x}_1+(k_1+k_2)x_1 - c_2\dot{x}_2 \\ -k_2x_2 - c_1\dot{x}_3 - k_1x_3 = 0 \end{array}\right\} \quad \text{------1}$$

$m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2 - c_2\dot{x}_1 - k_2x_1 = 0$

After using the nondimensional quantities, Eqs.(1) are rewritten as

$$\left.\begin{aligned}&\ddot{x}_1+(2\zeta_1 + 2\zeta_2 m_r\omega_r)\dot{x}_1+(1+ m_r\omega_r^2)x_1 - \\ &2\zeta_2 m_r\omega_r \dot{x}_2 - m_r\omega_r^2 x_2 - 2\zeta_1\dot{x}_3 - x_3 = 0\end{aligned}\right\} \qquad \text{---------2}$$

$$\ddot{x}_2+2\zeta_2\omega_r \dot{x}_2+ \omega_r^2 x_2 - 2\zeta_2 m_r \dot{x}_1 - \omega_r^2 x_1 = 0$$

Then, taking the Laplace transform of Eqs. (2) and solving for $X_1(s)$
and $X_2(s)$, which are the transforms of $x_1(\tau)$ and $x_2(\tau)$, respectively, we find that

$$\left.\begin{aligned}X_1(s) &= K_3(s)E_2(s) / D_2(s) \\ X_2(s) &= K_3(s)C(s) / D_2(s)\end{aligned}\right\} \qquad \text{----------3}$$

Where,

$$\left.\begin{aligned}K_3(s) &= (2\zeta_1 s + 1) X_3(s) - 2\zeta_1 X_3(0) \\ E_2(s) &= s^2 + 2\zeta_2\omega_r s + \omega_r^2 \\ C(s) &= 2\zeta_2\omega_r s + \omega_r^2\end{aligned}\right\} \qquad \text{-------- -4}$$

$$D_2(s) = s^4 + [2\zeta_1+ 2\zeta_2\omega_r m_r+ 2\zeta_2\omega_r] s^3 + [1+ m_r\omega_r^2 +\omega_r^2 + 4\zeta_1\zeta_2\omega_r] s^2 + [2\zeta_2\omega_r + 2\zeta_1\omega_r^2] s + \omega_r^2$$

To compare the responses of the single degree-of-freedom system with a moving base to that of a two degree-of-freedom system with a moving base, we assume that the displacement of the base is a half-sine wave. [1]

$x_3(\tau) = Xo \sin(\Omega o\tau)[u(\tau) - u(\tau - \tau o)]$ ---------5

where, $\Omega o = \omega_o / \omega_{n1}$

$\tau o = \omega_{n1}\, to$ & $to = \Pi /\omega_o$

Assuming that, $x_3(o) = 0$ ---- for convenience [1]

Taking Laplace Transform of $x_3(\tau)$,

$x_3(s) = Xo\, \Omega o(1 + e^{-\Pi s/ \Omega o}) / (s^2 + \Omega o^2)$ --------6

$\therefore K_3(s) = (2\zeta_1 s + 1) X_3(s) - 2\zeta_1 X_3(0)$

$= Xo\, \Omega o\, (2\zeta_1 s + 1) (1 + e^{-\Pi s/ \Omega o}) / (s^2 + \Omega o^2)$

---------7

putting this value in $X_2(s)$,

$X_2(s) = K_3(s)C(s) / D_2(s)$

$X_2(s) = Xo\, \Omega o\, (2\zeta_1 s + 1) (1 + e^{-\Pi s/ \Omega o}) / (s^2 + \Omega o^2) D_2(s)$

$\therefore X_2(s) / Xo = \Omega o\, (2\zeta_1 s + 1) (1 + e^{-\Pi s/ \Omega o}) / (s^2 + \Omega o^2) D_2(s)$ -------8

Taking Laplace Transform of above equation, we can find the response of the mass m_2. [1]

IV. SIMULATION USING MATLAB

4.1 Introduction:

The name MATLAB® stands for matrix laboratory. MATLAB® is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses includes –

- ✓ Math and computation
- ✓ Algorithm development
- ✓ Data acquisition
- ✓ Modeling, simulation, and prototyping
- ✓ Data analysis, exploration, and visualization
- ✓ Scientific and engineering graphics
- ✓ Application development, including graphical user interface building

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows you to solve many technical computing problems, especially those with matrix and vector formulations.

This is a vast collection of computational algorithms ranging from elementary functions, like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigenvalues, Laplace, inverse Laplace and Fourier transforms.

MATLAB has extensive facilities for displaying vectors and matrices as graphs, as well as annotating and printing these graphs. It includes high-level functions for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. [7]

4.2 MATLAB Functions:

syms : Construct symbolic numbers, variables and objects.

poly2sym : returns a symbolic representation of the polynomial whose coefficients are in the numeric vector c. The default symbolic variable is x.

vpa : uses variable-precision arithmetic (VPA) to compute each element to decimal digits of accuracy. Each element of the result is a symbolic expression.

ilaplace : is the inverse Laplace transform of the scalar symbolic object with default independent variable s. The default return is a function of t. The inverse Laplace transform is applied to a function of s and returns a function of t. [7]

4.3 MATLAB Program:

4.3.1 For specific input: ($m_r = 0.1$, $\zeta_1 = \zeta_2 = 0.1$, $\omega_r = 0.05$, 0.2 and $\Omega o = 0.05, 0.1, 0.2, 0.4$)

```
clear
clc
syms s
mr=0.1;
wr1=[0.05 0.2];
Om=[0.05 0.1 0.2 0.4];
z1=0.1; z2=0.1;
time=150;
for n=1:2
 wr=wr1(n);
 D2=poly2sym([1 2*(z1+z2*mr*wr+z2*wr) 1+mr*wr^2+4*z1*z2*wr+wr^2 2*(z2*wr+z1*wr^2) wr^2],s);
   for k=1:4
       fact=wr*Om(k);
       x1sn=(2*z2*s+wr)*(2*z1*s+1);
       xsd=(s^2+Om(k)^2)*D2;
```

```
        fbase=vpa(ilaplace(x1sn/xsd),5);
        X1t=inline(vectorize(fbase),'t');
        t=linspace(0,time,250);
        pfun=fact*real(X1t(t))+fact*real(X1t(t-pi/Om(k))).*(t>=pi/Om(k));
         subplot(4,2,2*(k-1)+n)
         ts=linspace(0,pi/Om(k),50);
        plot(t,pfun,'-',ts,sin(Om(k)*ts),'--',[0 time],[0 0],'-');
ylabel('x_2(\tau)/X_o');
if k==4
xlabel('\tau');
end
az=axis; az(2)=time;
axis(az);
text(.7*az(2),.7*az(4),['\Omega_o=' num2str(Om(k))] );
   end
end
```

4.3.2 A general program:

```
clear
clc
syms s
mr=input('Enter mr : ');
wr=input('Enter wr : ');
Om=input('Enter Omega : ');
z1=input('Enter Zeta1 : ');
z2=input('Enter Zeta2 : ');
time=150;
D2=poly2sym([1 2*(z1+z2*mr*wr+z2*wr) 1+mr*wr^2+4*z1*z2*wr+wr^2 2*(z2*wr+z1*wr^2) wr^2],s);
fact=wr*Om;
x1sn=(2*z2*s+wr)*(2*z1*s+1);
xsd=(s^2+Om^2)*D2;
fbase=vpa(ilaplace(x1sn/xsd),5);
X1t=inline(vectorize(fbase),'t');
t=linspace(0,time,250);
pfun=fact*real(X1t(t))+fact*real(X1t(t-pi/Om)).*(t>=pi/Om);
ts=linspace(0,pi/Om,50);
plot(t,pfun,'-',ts,sin(Om*ts),'--',[0 time],[0 0],'-');
ylabel('x_2(\tau)/X_o');
xlabel('\tau');
az=axis; az(2)=time;
axis(az);
text(.85*az(2),.85*az(4),['\Omega_o=' num2str(Om)] );
```

Here user can enter the user defined values of m_r, ζ_1, ζ_2, and ω_r and Ωo.

V. RESULTS & DISCUSSION

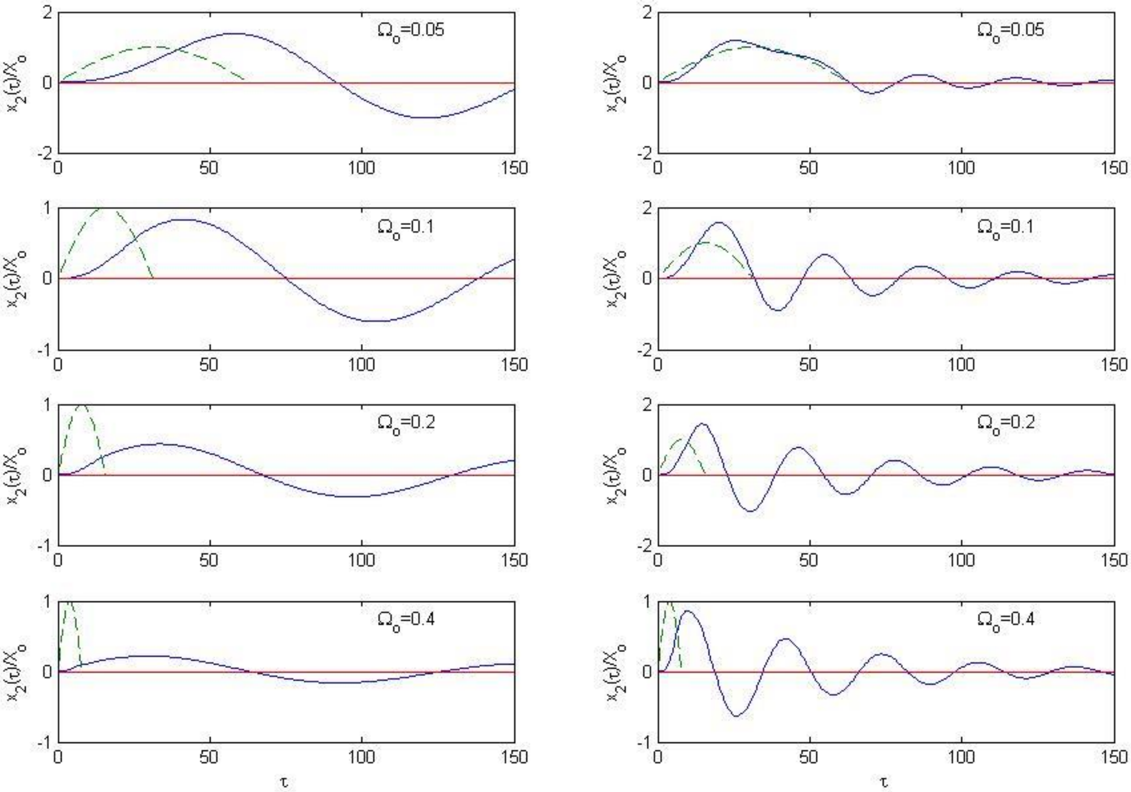

Figure 2. The response of the system for $m_r = 0.1$, $\zeta_1 = \zeta_2 = 0.1$, $\omega_r = 0.05$, 0.2 and $\Omega o = 0.05, 0.1, 0.2, 0.4$

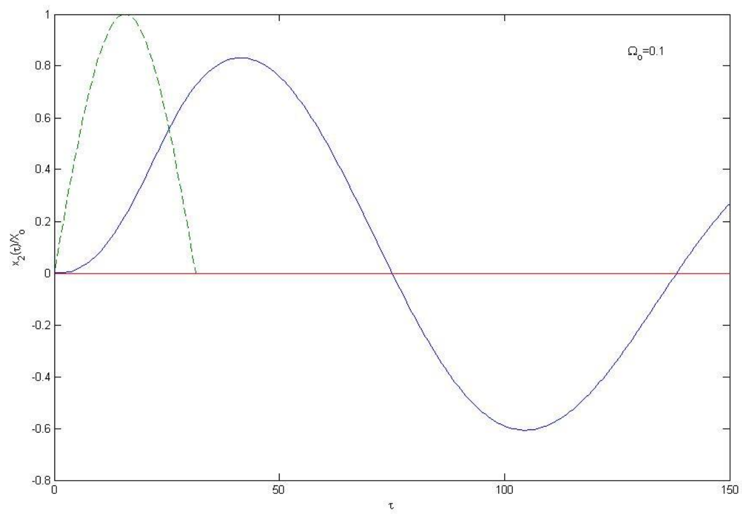

Figure 3. The response of the system for $m_r = 0.1$, $\zeta_1 = 0.1$, $\zeta_2 = 0.1$, $\omega_r = 0.05$and $\Omega o=0.1$

VI. CONCLUSIONS AND FUTURE WORK

The numerically obtained inverse Laplace transforms is shown in Figure 2 and 3. We see that as the duration of the half-sine wave pulse decreases, the amplitude of m_2 decreases. This behavior is opposite to what takes place during the base excitation of a single degree-of- freedom system. Where as the pulse duration decreased the peak displacement of the mass increased. We see, then, that the interposition of m_1 and its spring and damper act as a mechanical filter, decreasing the amount of relatively high frequency energy generated by the half-sine wave pulse from being transferred to m_2. Thus, a two degree-of-freedom system with appropriately chosen parameters can be an effective isolator of ground vibrations compared to a single degree-of-freedom system.

In future, SIMULINK can be used for the different approaches for the analysis of a Model. Here non-linearity is not considered so that model and its equations become simpler. In the practical case every system is non-linear and hence will be used for the further work. In reality the road disturbances are random in nature and hence if available the actual road load data will be utilized MATLAB can be used to make a program to make the system more generalized. For the sake of simplicity, the simple numeric values of the parameters are used. Once the generalized mathematical model is prepared any values can be tasted and hence the optimized design can be obtained.

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