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Analysis of a Passive Linear Quarter Car Suspension System using Matlab/Simulink

Dr. K.R. Borole[1], N. P. Sherje[2], M. P.Nagarkar[3]
[1]Professor, Mechanical Engineering, STES's SKNCoE, Pune, Maharashtra, India
[2] Asst. Professor, Mechanical Engineering, STES's SKNCoE, Pune, Maharashtra, India
[3] 52, Datta Nagar Pipe Line Road, Ahmednagar, Maharashtra, India

ABSTRACT

This paper discusses the suspension of a two-degree-of-freedom (2-DOF), Linear Quarter Car, vehicle system. MATLAB/Simulink environment is used to analyze response of the quarter car. The quarter car model is subjected to various inputs like step input, sine, random, saw tooth, ramp etc... To study the system, first the governing equations for a linear quarter car model are derived. Then a Differential Equation approach, State Space approach and Transfer Function approach was used using Simulink blocks. For all the approaches the results matches to each other.

Keywords: Linear Quarter car, MATLAB/Simulink, State Space, Transfer Function.

I. INTRODUCTION

The automobile is a combination of variety of complex systems. Each system has its own functions to perform and how good an automobile turns out to be depends on the proper synthesis of these systems. One such system is the suspension system. Suspension system has been widely used, to the vehicles from the horse drawn carriages with flexible leaf springs, to the modern automobiles with complex control algorithms.

The conventional system i.e. passive suspension system, which comes *as is*, is a system of springs, shock absorbers, bushings, rods, linkages and arms. Vehicles generally have two suspension systems – one for the front wheel and other for rear. These two systems work together to control driving and breaking forces to provide smooth ride for driver and passengers.

The passive suspension systems are trade off between ride comfort and performance. A case with a nice cushy ride usually wallows through the corners whereas a car with high performance suspension, like F1 cars, will hang on tight through the corners but will make the passengers feel very every little dip and bump in the road.

II. VEHICLE MODEL

Modeling of suspension system is done in the vertical plane. Longitudinal or transverse deflection of the suspension components is considered negligible in comparison to vertical deflections. A linear 2-DOF system is used as a model for the road vehicle. The two masses m1 and m2, of the vehicle model represent the wheel (and axle if there is any) and the vehicle body (often called the unsprung and the sprung mass, respectively). These masses representing one half or a quarter of a real car. The spring stiffness are k1, k2 and the damper stiffness are c1, c2.

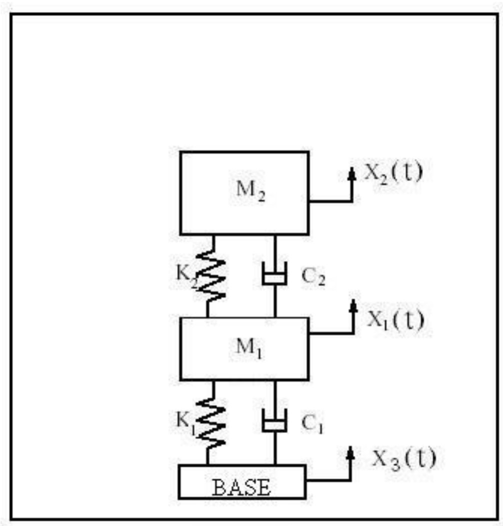

Figure 1. 2-DOF (Linear) system with base excitation (Quarter Car Model).

III. MATHEMATICAL MODELING

Differential equations (DE) to account for the moving base become,

$m_1\ddot{x}_1+(c_1+c_2)\dot{x}_1+(k_1+k_2)x_1-c_2\dot{x}_2-k_2x_2-c_1\dot{x}_3-k_1x_3 = 0$

$m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2 - c_2\dot{x}_1 - k_2x_1 = 0$

---- (1)

Transfer Function (TF)

Taking the Laplace transform of the eq.1, we get,

$m_1 (s^2 X_1 - sx(o)-\dot{x}(o)) + (c_1+c_2) (sX_1 - x(o)) + (k_1+k_2)X_1$

$= c_2 (s X_2 - x(o)) + k_2X_2 + c_1 (s X_3 - x(o)) + k_1X_3$

&

$m_2 (s^2 X_2 - sx(o)-\dot{x}(o)) + (c_2s X_2 - c_2 x(o)) +$

$k_2X_2 = (c_2s X_1 - c_2 x(o)) + k_2X_1$

---- (2)

Putting initial conditions, x (o) =0 and

$\dot{x}$ (o) = 0,

$m_1 (s^2+ (c_1+c_2) s + k_1+k_2) X_1$

$= (c_2 s + k_2)X_2 + (c_1 s + k_1)X_3$

&

$(m_2s^2+c_2s+k_2)X_2=(c_2s+k_2)X_1$ ---- (3)

The second equation from (3) can be written as,

$X_2/X_1=(c_2s+k_2)/(m_2s^2+ c_2s + k_2)$ ---- (4)

The equation, (4), is known as *transmissibility function.*

eliminating X_1 from above equations,

$X_1 = (m_2 s^2 + c_2s + k_2)X_2 / (c_2s + k_2)$

Now,

$m_1 (s^2+ (c_1+c_2) s \quad k_1+k_2) (m_2 s^2 + c_2s + k_2)X_2 / (c_2s + k_2) = (c_2 s + k_2)X_2 + (c_1 s + k_1)X_3$

solving and rearranging, we have

$\therefore X_2 / X_3$

$= [c_1c_2s^2+(k_1c_2+k_2c_1)s+k_1k_2]$

$/\{[m_1m_2s^4+[m_1c_2+m_2(c_1+c_2)]s^3+[m_2(k_1+k_2)$

$+c_1c_2+m_1k_2]s^2+(k_1c_2+k_2c_1)s+k_1k_2\}$ ---- (5)

Similarly we can compute,

X_1 / X_3

$= \{m_2c_1 s^3+ (c_1c_2+ m_2 k_1)s^2 + (c_1 k_2 +c_2k_1)s + k_1k_2\}$

$/\{m_1m_2s^4+[m_1c_2+m_2(c_1+c_2)]s^3+[m_2(k_1+k_2)+c_1c_2$

$+m_1k_2]s^2+(k_1c_2+k_2c_1)s+k_1k_2\}$ [1, 2, 4, 5, 6] ---- (6)

State Space Approach (SS)

For the given system we define 4 state variables, two gives the displacement of the two masses and the other two gives the velocities of the respective masses. [1, 4, 5, 6, 8]

We have,

$m_1\ddot{x}_1+(c_1+c_2)\dot{x}_1+(k_1+k_2)x_1 - c_2\dot{x}_2 - k_2x_2 - c_1\dot{x}_3 - k_1x_3 = 0$

$m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2-c_2\dot{x}_1-k_2x_1=0$ ----(Refer eq. 1)

Let,

$x_1(t) = x_2$ $\quad$ $x_2(t) = x_1$

$x_3(t) = \dot{x}_1(t)$ $\quad$ $x_4(t) = \dot{x}_2(t)$

$\dot{x}_3(t) = \ddot{x}_1(t) = \ddot{x}_2$ $\quad$ $\dot{x}_4(t) = \ddot{x}_2(t) = \ddot{x}_1$ ----(7)

Putting equation (6) in equation (1), we can get

$\dot{x}_4(t) = (c_2/m_1)x_3(t) + (k_2/m_1)x_1(t) + (c_1/m_1)\dot{x}_3 +(k_1/m_1)x_3-[(c_1+c_2)/m_1] x_4(t) - [(k_1+k_2)/m_1] x_2(t)$

$\dot{x}_3(t) = (c_1/m_2)x_4(t) + (k_2/m_2)x_2(t) - (c_2/m_2)x_3(t) - (k_2/m_2) x_1(t)$

And

$\dot{x}_1(t) = x_3(t)$

$\dot{x}_2(t) = x_4(t)$

Finally re-arranging above equations in the following form of equation

$\dot{x}(t) = Ax(t)+ B u(t)$

$y(t) = Cx(t) + D u(t)$

---- (8)

where $x^{\cdot}(t)$, $x(t)$, $y(t)$, A, B, C and D are the matrices of various order.

Here matrix A is called the State Matrix, B is the Input Matrix, C is the Output Matrix and D is the Direct Transmission Matrix.

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k2/m2 & k2/m2 & -c2/m2 & c2/m2 \\ k2/m1 & -(k1+k2)/m1 & c2/m1 & -(c1+c2)/m1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k1/m1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

IV. RESULTS

Simulation using Simulink

Simulink® is a software package for modeling, simulating, and analyzing dynamic systems. It supports linear and nonlinear systems, modeled in continuous time, sampled time, or a hybrid of the two. Systems can also be multirate, i.e., have different parts that are sampled or updated at different rates.

Simulink is a graphical extension to MATLAB for modeling and simulation of systems. In Simulink, systems are drawn on screen as block diagrams. Many elements of block diagrams are available, such as transfer functions, summing junctions, etc., as well as virtual input and output devices such as function generators and oscilloscopes. Simulink is integrated with MATLAB and data can be easily transferred between the programs. Here, Simulink is used to model the simple linear spring mass 2 DOF base excitation systems, and is used to simulate the system. Simulink is supported on UNIX, Macintosh, and Windows environments; and is included in the student version of MATLAB for personal computers. [4, 7]

Input Data – Quarter Car Model

$m_1 = 40kg$	$m_2 = 240kg$
$c_1 = 49KN\ s/m$	$c_2 = 7\ KN\ s/m$
$k_1 = 140\ KN/m$	$k_2 = 20KN/m$ [2]

Frequency Response Function (FRF) plot

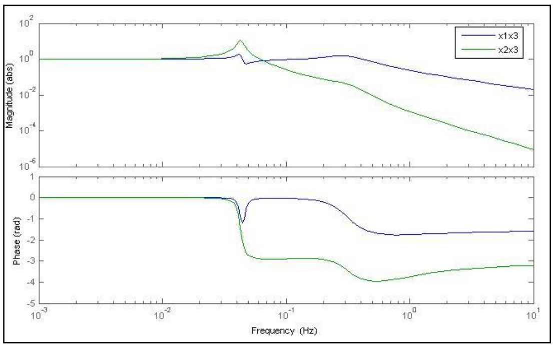

Figure 2. Frequency Response plot for Linear quarter car model.

Transmissibility Function plot

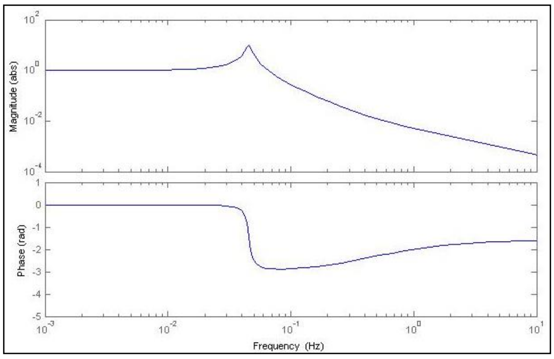

Figure 3. Transmissibility Function plot for Linear quarter car model.

The sprung and unsprung mass frequency responses are shown here, when input is known. The FRF are computed for x1/x2 and x2/x3. If input is not known, generally transmissibility is computed between sprung and unsprung mass.

The peak gains are as follows:

FRF:

For x1/x3, maximum gain is 1.85(absolute) which occurs at 0.0414 Hz.

For x2/x3, maximum gain is 10.08(absolute) at 0.0428 Hz frequency.

Transmissibility Function:

Maximum gain is 9.96 (absolute) at 0.0458 Hz.

MATLAB/Simulink Results for Linear Quarter Car Model:

Step Input: (Step = 3units)

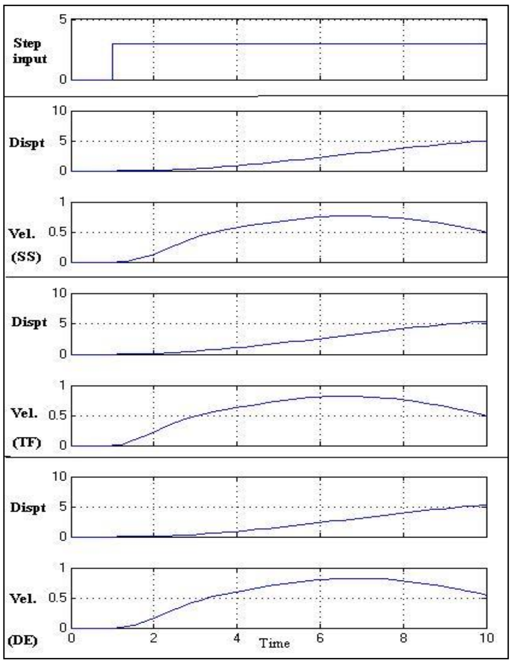

Figure 4. Step response for SS, TF, DE approach.

Sine Input:

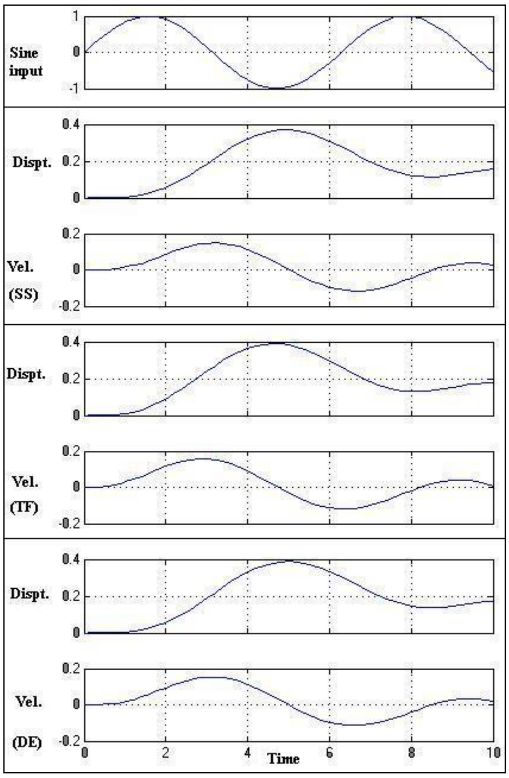

Figure 5. Sine response for SS, TF, DE approach.

Random Input:

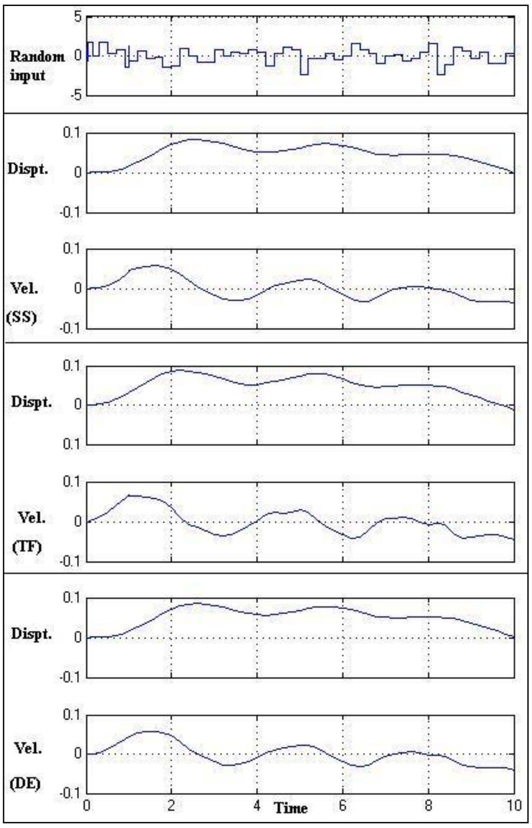

Figure 6. Random response – SS, TF, DE approach.

Saw Tooth Input:

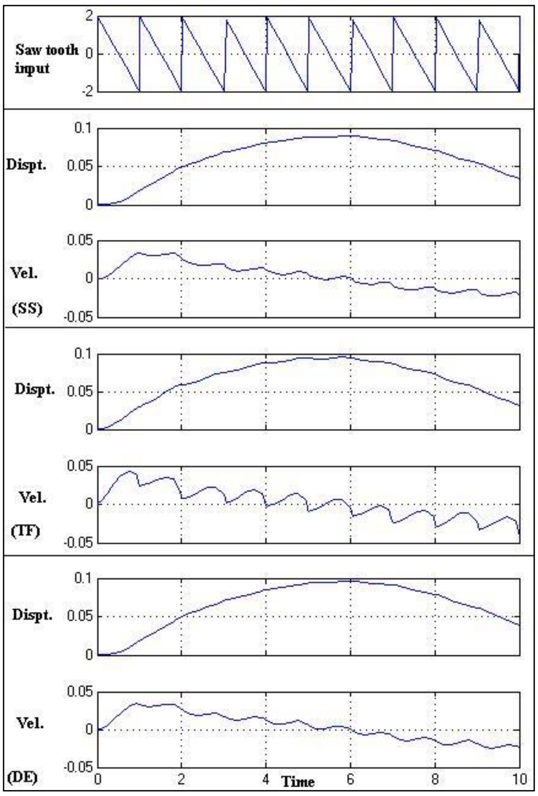

Figure 7. Saw Tooth response – SS, TF, DE approach.

V. CONCLUSION

Here, MATLAB/Simulink environment is used to simulate the Linear Quarter Car Model. Mathematically obtained FRFs are simulated using MATLAB. When input is known, the FRFs are computed for x1/x2 and x2/x3. If input is not known, generally transmissibility is computed between sprung and unsprung mass. There is only one peak in transmissibility function. This peak occurs near the sprung mass resonance, but it is not natural mode of system because the transmissibility only indicate how the sprung mass is responding relative to the unsprung mass (the inverse shows the opposite relation).

In SIMULINK, three different approaches, Differential Equation (DE), Transfer Function (TF) and State Space (SS) are used for the analysis of a Linear Quarter Car Model. Responses obtained shows that the approaches used give same results.

Here only standard test signals (i.e. Unit Step, Sinusoidal and Random etc.) are used to obtain the system responses. In reality the road disturbances are random in nature and hence if available the actual road load data, will be utilized. Simulink can be proved to be the best tool from the analysis and the interpretation point of view.

VI. REFERENCES

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