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MHD Free Convection flow in a Micropolar fluid past an Inclined Stretching sheet with considering Viscous Dissipation and Radiation

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ABSTRACT

In this paper analyzed the heat and mass transfer of the free convection flow in a micropolar fluid past an inclined stretching sheet. A uniform magnetic field is applied normal to the sheet with considering viscous dissipation and Radiation effect. The governing boundary value partial differential equation for micropolar fluid is converted into the system of ordinary differential equation by using similarity transformation and to solved numerically by using Fourth order classical Runge kutta method. The Dimensionless velocity, micro rotation, temperature, concentration profile find out graphically by using MATLab with different parameter values. In addition the Nusselt number, skin friction, Sherwood number and the wall couple stress are displayed in the table.

Keywords: Micropolar fluid, viscous dissipation, Radiation effect, Nusselt number, skin friction, Sherwood number,wall couple stress.

I. INTRODUCTION

In recent years Micropolar fluids has get a great attention, because the Newtonian fluids cannot accurately defines the characteristic of fluid with suspended particles. Micropolar fluids are a subset of the micromorpic fluid theory introduced in a pioneering paper by Eringen[11] . Eringen theory has provided a good model for studying a number of complicated fluids such as colloidal fluids, polymeric fluids and blood. Micro fluids are supports the stress moments and body moments and are influenced by the spin inertia. Micro fluids are defines certain microscopic effects occurs from the local structure and micro motion of the fluid particle. Micropolar fluids are the subclass of micro fluid, which exhibit the micro rotational effects and micro rotational inertia. These fluids are supports couple stress and body couple only.

In MHD flow and heat transfer is a important effects of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluid such as MHD power generators, the cooling of nuclear reactors, plasma studies, purification of molten metals from non-metallic inclusion, geothermal energy extraction. MHD Free convection flow past an inclined stretching sheet with considering viscous dissipation and radiation have been extensively studied by M. Hasan et al.[13].

Ostrach[15] analyzed the Laminar natural convection flow and heat transfer of fluid with and without heat source in channels with constant wall temperature. Hossain and Takhar[8] studied the effects of Radiation Effects on Mixed Convection Along a Vertical Plate with Uniform Surface Temperature. Following Ali et al.[1] and Mansour[11] analyzed the interaction of mixed convection with thermal radiation in laminar boundary layer flow over a horizontal ,continuous moving sheet with suction or injection. Chen[3] made a study of the natural convection flow over a permeable surface with variable wall temperature and concentration. S.P.Anjali Devi and B.Ganga[2] have studied the effect of viscous dissipation on non-linear MHD flow in a porous medium over a stretching porous surface. Jha and Ajibade[9] analyzed the effect of viscous dissipation on natural convection flow between vertical parallel plates with time periodic boundary conditions.

Ferdows et al[6] describes that in the presence of uniform magnetic field with viscous dissipation at the wall, the thermophoretic parameter is one of the most useful parameter to control the boundary layer of the fluid. D.Srinivasacharya, and Mendu Upendar[18] investigate the effect of double stratification on MHD free convection in a micropolar fluid. M. F. El-Amin[4] examined the Magnetohydroynamics free convection and mass transfer flow in micropolar fluid with constant section. M. M Rahman et al.[17] made a study of heat transfer in micropolar fluid along an inclined permeable plate with variable fluid properties. B.L.Olajuwon and J.I.Oahimire[14] studied the unsteady Heat and mass transfer in an MHD micro polar fluid in the presents of thermo diffusion and thermal radiation. They conclude that the problem of unsteady free convection MHD heat and mass transfer flow of an incompressible micropolar fluid past a semi infinite vertical permeable moving plate embedded in a uniform porous medium with suction in the presence of thermo diffusion and thermal radiation. Wange et al.[18] studied peristaltic motion of a MHD micropolar fluid in a tube. They observed that for micropolar fluid the peristalsis works as a pump against greater pressure rise compared with a Newtonian fluid well there exists no considerable difference in free pumping flex for Newtonian fluid and micropolar fluid. Hayat et al[7] explained MHD flow of micropolar fluid near a stagnation point towards a non linear stretching surface. They developed that the serious solution of highly non linear problem described the MHD flow of micropolar fluid over a nonlinear stretching surface.

Manjoolatha et al[12] examined radiation and mass transfer effect on MHD flow of micropolar fluid towards a stagnation point on a vertical stretching sheet. They conclude that the buoyancy assisting flow region, the skin friction decreases with increasing value of the radiation parameter. Whereas the opposite behavior buoyancy opposing flow region. For both assisting and opposing flow the rate of heat transfer increases with an increasing in the radiation parameter.

The aim of present study is analyze the considering the combined effects of viscous dissipation and radiation on free convection flow in micro polar fluid an inclined stretching sheet. Using similarity transformation, the governing equations are transformed into a set of self-similar non-linear ordinary differential equations, which are then solved numerically solved. The numerical results are plotted in some figures and the variations in velocity and temperature distributions for several physical parameters involved in the equations are discussed in detail.

II. MATHEMATICAL ANALYSIS

We consider Steady state Two Dimensional, Incompressible free convection heat and mass transfer flow of an electrically conducting micro polar fluid along an isothermal stretching perimeter inclined sheet with angle α with heat generation and abortion is considered. It is consider that sheet is stretched along the xy-plane. A strong magnetic field is applied in the y-axis direction. The magnetic field is assumed to be small so that induced magnetic field can be neglected in comparison with the applied magnetic field. The radiative heat flux in the x-direction is negligible to the flux in the y-direction. The plate temperature and concentration are initially raised to T_w and C_w respectively which aer there after maintained constant. The ambient temperature of the flow is T_∞ and the concentration of the uniform flow is C_∞ . nder the usual boundary layer and Boussinesq approximation and using the Darcy-Forchhemier model, the flow and heat transfer in the presences of radiation are governed by the following eqations.

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentom Equuation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu+\kappa}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial \omega}{\partial y}+ g\beta(T-T_\infty)\cos\alpha+g\beta(C-C_\infty)-\frac{\sigma B^2 u}{\rho}-\frac{bu^2}{\kappa} \qquad (2)$$

Angular momentum

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 \omega}{\partial y^2}-\frac{\kappa}{\rho j}(2\omega+\frac{\partial u}{\partial y}) \qquad (3)$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \frac{\kappa}{\rho C_p}\frac{\partial^2 \omega}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty)+\left(\frac{\mu+\kappa}{\rho j C_p}\right)(\frac{\partial u}{\partial y})^2-\frac{1}{\rho C_p}(\frac{\partial q_r}{\partial y}) \qquad (4)$$

Concentration Equuation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \qquad (5)$$

Where u and v are the components of velocity along x and y directions respectively, ω is the components of micro rotation whose direction of rotating line in the xy-plane. g is the acceleration due to gravity, T and T_∞ are the fluid temperature within the boundary layer and in the free stream respectively, β is the volumetric co-effcient of thermal expansion, α is the angle of inclination. While C is denoted by the concentration of the fluid within the boundary layer, σ is the electric conductivity, B_0 is the uniform magnetic field strength. ρ is the density of fluid, C_p is the specific heat at constant pressure. Q_0 is the volumetric rate of heat generation and absorption and D_m is the chemical molecular diffusivity. μ is the dynamic co-effcient of viscosity of the fluid , j is the micro inertia density, γ is the spin gradient viscosity.

The appropriate initial and boundary conditions for the problem are given by

$$u = 0 \;\; v_w = 0 \;\; \omega = 0 \;\; T = T_w \;\; C = C_w \;\; \text{at} \;\; y = 0$$
$$u \to 0 \;\; \omega \to 0 \;\; T \to T_\infty \;\; C \to C_\infty \;\; \text{at} \;\; y \to 0 \qquad (6)$$

The wall velocity component v_w having positive value. The uniform temperature and concentration of the fluid are T_w and C_w respectively. By using the term of radiative heat flux by the Rosseland estimation is given by

$$q_r=-\frac{4\sigma_1}{3\kappa_1}\frac{\partial T^4}{\partial y} \qquad (7)$$

Where the constant σ_1 is a Stefan Boltzmann constant, and the mean absorption coeffcient is κ_1, T^4 is expressed as a linear function of temperature because to assume the temperature different within the flow. Thus

$$T^4 \cong 4T_\infty^3-3T_\infty^3 \qquad (8)$$

Using Equ (6) and (7) in Equation (4), We get

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \frac{\kappa}{\rho C_p}\frac{\partial^2 \omega}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty)+\left(\frac{\mu+\kappa}{\rho j C_p}\right)(\frac{\partial u}{\partial y})^2-\frac{16\sigma_1 T_\infty^3}{3\rho C_p \kappa_1}\frac{\partial^2 T}{\partial y^2} \qquad (9)$$

2.1. Methods of Solution

Now we introduce the stream function ψ. The Continuity Equation which satisfies the stream function, which is defined by,

$$u=\frac{\partial \psi}{\partial y},\; v=\frac{\partial \psi}{\partial x} \qquad (10)$$

$$\eta=y\sqrt{\frac{D}{v}},\; \psi=\sqrt{Dvx}f(\eta),\; \omega=\sqrt{Dvx}g(\eta)$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\; \phi=\frac{C-C_\infty}{C_w-C_\infty} \qquad (11)$$

Where u,v are the velocity components and f, θ, ϕ are the dimensionless stream function, temperature, concentration respectively. Where the prime represents differentiation with respect to η . In

Equation (3.11) gives the similarity variables, and using these variables in Equation (10),(2),(9), and (5) are reduced to get the given dimensionless equations ,

$$\frac{1}{1-N}f''' + ff' + \gamma(\theta\cos(\alpha)+1) + \frac{N}{1-N}g' - Mf' - \left(1+\frac{Fs}{Da}\right)(f')^2 = 0 \quad (12)$$

$$f'g - g'f = \lambda g'' - \frac{N}{1-N}J(2g+f'') \quad (13)$$

$$\theta'' + Pnf\theta' + \frac{1}{1-N}PnEc(f'')^2 + PnQ\theta = 0 \quad (14)$$

$$\phi'' + Scf\phi' = 0 \quad (15)$$

Where $\gamma = \frac{Gr_x}{Re_x^2} = \frac{g_0\beta(T_{w-T_\infty})}{D^2x}$ is the Buuoyancy parameter, $M = \frac{\sigma B_0^2}{\rho D}$ is the Magnetic field parameter, $Da = \frac{k}{x^2}$ is the local Darcy number, $Re_x = \frac{x^2D}{\vartheta}$ is the Reynolds number, $Fs = \frac{b}{x}$ is the Forchhemier number, $Pr = \frac{\mu c_p}{\kappa}$ is the Parndlt number, $Q = \frac{Q_0}{\rho c_p D}$ is the heat source $(\frac{source(Q>0)}{Sink(Q<0)parameter})$, $Ec = \frac{D^2x^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $R = \frac{\kappa\kappa_1}{4\sigma_1 T_\infty^3}$ is the radiation parameter, $Sc = \frac{\vartheta}{D_m}$ is the Schmidt number, $N = \frac{\kappa}{\mu+\kappa}$ is the Coupling number, $J = \frac{1}{jB^2}$ is the Micro inertia density, $\lambda = \frac{\gamma}{j\rho\vartheta}$ is the Spin Gradient viscosity, $Pn = \frac{3RPr}{4+3R}$.

The corresponding boundary conditions are transformed in given below,

$$f = Fw \quad f' = 1 \quad \theta = 1 \quad g = 0 \quad \phi = 1 \text{ at } y=1$$
$$f' = 0 \quad \theta = 0 \quad g = 0 \quad \phi = 0 \text{ at } y \to 0 \quad (16)$$

2.2 Skin Friction, Rate of heat and mass transfer

The local Skin Friction Coefficient(C_f) is used to indicate the wall shear stress, the local Nusselt number (N_{u_x}) is indicate the rate of heat and mass transfer and the Sherwood number is indicate the local surface mass flux.

$$\tau_w = [(\mu+\kappa)\frac{\partial u}{\partial y} + \omega\kappa]_{y=0}, q_w = -[k\left(\frac{\partial T}{\partial y}\right)]_{y=0}$$

$$M_w = \gamma[\frac{\partial \omega}{\partial y}]_{y=0}, \; q_w = -D\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Where the μ is the viscosity, κ is the thermal conductivity and D_m is the mass diffusivity. The non dimenstional local wall shear stress is called skin friction, the local surface heat transfer is called Nusslelt number ,local surface mass transfer is called Sherwood number, and the wall couple stress are given below,

$$C_f = \frac{2\tau_w}{\rho U^2} = \frac{2}{1-N}(Re_x)^{-\frac{1}{2}}f''(0)$$

$$Nu_x = \frac{q_{w^x}}{\kappa(T_{w-}T_\infty)} = -\theta'(0)\,(Re_x)^{\frac{1}{2}}$$

$$Sh = \frac{M_{w^x}}{D_m(C_{w-}C_\infty)} = -\Phi'(0)\,(Re_x)^{-\frac{1}{2}}$$

$$M_w = \frac{\lambda}{j}g'(0)$$

III. RESULTS AND DISCUSSION

We have formulated the effect of viscous dissipation and radiation with suction (Fw) on MHD free convection heat and mass transfer flow of an micro polar fluid along a inclined stretching sheet. The numerical solution of non linear differential equation (12)-(16) under the boundary conditions executed by using fourth order classical Runge kutta method. The influence of the angle of inclination α of the sheet on the dimensionless velocity, micro rotation, temperature and the concentration profile with fixed values of other parameters are M, N, R, Pr, Ec, Fs, Da, Fw, λ, J, Q, Sc.

Figure.1 Figure. 4 are drawn to depict the variation of suction parameter Fw on the non dimensional velocity, micro rotation, temperature, concentration with remaining parameter are fixed. The velocity profile for several values of Suction parameter (Fw=0.1, 0.5, 1) are demonstrate in fig.1.From the figure seen that the velocity increases with the suction parameter increase, which makes the momentum boundary layer thinner. The Micro rotation profile are calculated for different values of suction parameter Fw in figure. 2. It is clear that the micro rotation component g(η) decreases near the sheet with suction value increase and increase far away from the sheet for increasing suction parameter.

The Temperature profile is plotted for different value of suction parameter in figure. 3. From the figure, we observe that the temperature θ(η) decreases according to the value of suction parameter increase, the thickness of the thermal boundary layer is reduces. In Figure.4 represents the effect of concentration profile with different suction parameter values (Fw=0.1,0.5,1), the wall concentration decreases with increasing value of suction parameter.

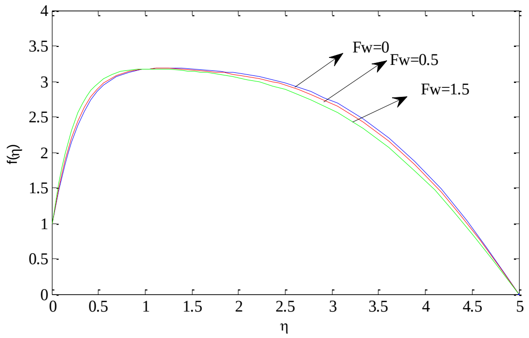

Figure 1. Velosity profile for suction parameter
α=30, Fw=1, N=0.1, Da=1, Fs=1, M=1,
λ=1, J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

Figure.5 to Figure. 8 we demonstrate the impact of the Coupling number N on the dimensionless velocity, micro rotational, temperature and the concentration profiles. In figure. 5, it is observed that when the coupling number (N = 0.1; 0.5; 0.7) increases, the velocity is decreased. The maximum of velocity decrease in amplitude and the location of the maximum velocity moves farther away the wall with N is increasing.

From Figure. 6, It can be observed that the non dimensional micro rotation increases at near the sheet and decreases far away from the sheet with increasing the coupling number N. In Figure. 7 it can be seen that the temperature profile are calculated for different values of coupling number N. It is clear that the temperature component decreases, when the coupling number is increase. Next we discuss the effect of concentration profile with different coupling number N, the concentration increases with the coupling number N is decreased.

Figure.9-12, illustrate the variation of the Non dimensional velocity, micro rotational, temperature, and concentration for the boundary layer with various values of magnetic field parameter (M=1, 1.5, 2), the remaining parameters are constants. From figure. 9, that the velocity decreases as the magnetic parameter M is increases. Figure.10. Exhibits the micro rotational profile for different values of magnetic parameter. The micro rotational component increases when the magnetic values M increases. Figure.11 is obtained by plotted the temperature profile for various value of magnetic field parameter M. It can be seen that an increasing the value M is to increases the temperature. Figure. 12, It can be observed that the dimensionless concentration profile for various magnetic parameter values, If the magnetic parameter M is increases, and the concentration will be increases.

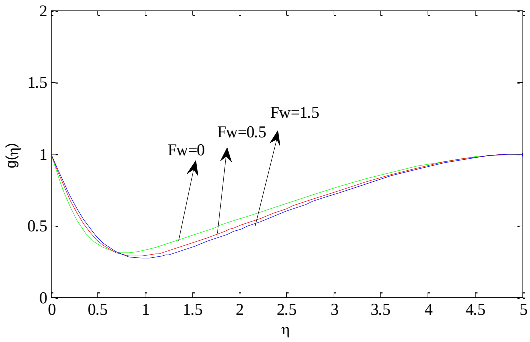

Figure 2. Microrotation profile for suction parameter Fw
α=30, N=0.1, Da=1, Fs=1, M=1, λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

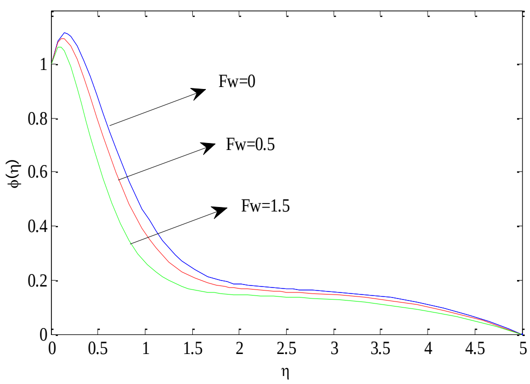

Figure 3. Temperature profile for suction parameter Fw
α=30, N=0.1, Da=1, Fs=1, M=1, λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

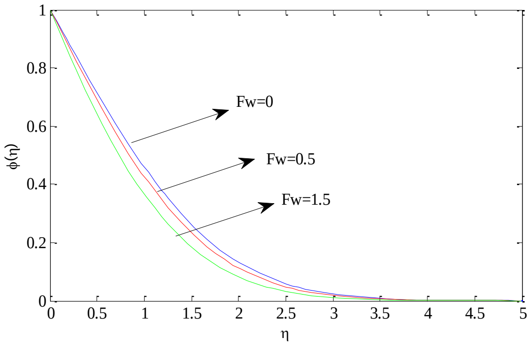

Figure 4. Concentration profile for suction parameter Fw

α=30, N=0.1, Da=1, Fs=1, M=1, λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

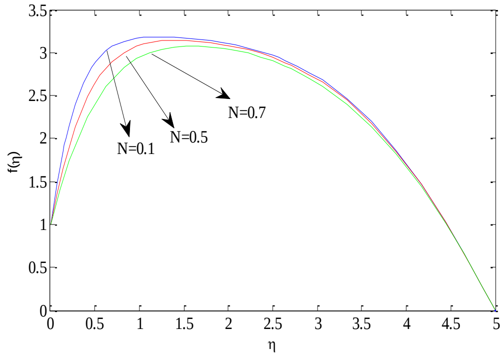

Figure 5. Velocity profile for Coupling Number N

α=30, Fw=0.1, Da=1, Fs=1, M=1,λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

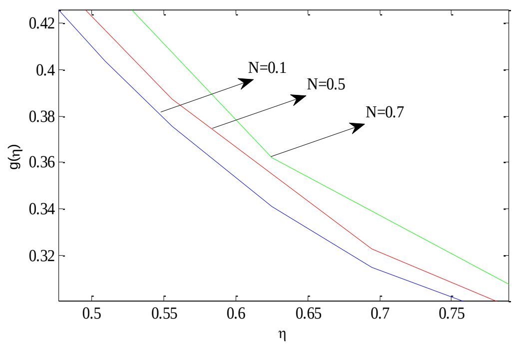

Figure 6. Micro rotation profile for Coupling Number N

α=30, Fw=0.1, Da=1, Fs=1, M=1,λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

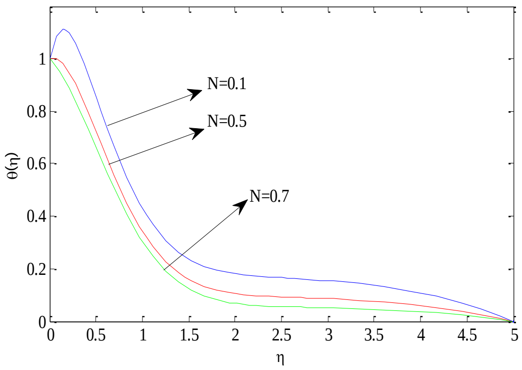

Figure 7. Temperature profile for Cooupling Nmber N

α=30, Fw=0.1, Da=1, Fs=1, M=1,λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

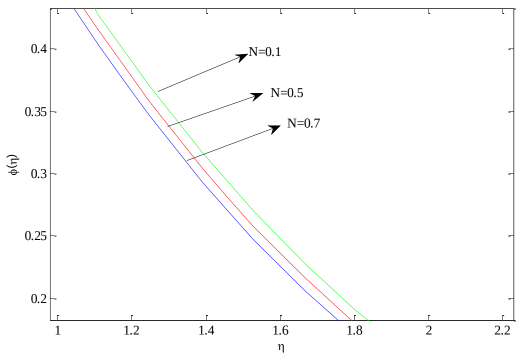

Figure 8. Concentration profile for Coupling Number N

α=30, Fw=0.1, Da=1, Fs=1, M=1, λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

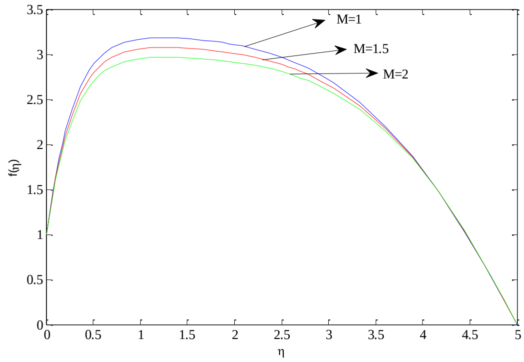

Figure 9. Velocity profile for Magnetic parameter M

α=30, Fw=0.1,N=0.1, Da=1, Fs=1, λ=1,
J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

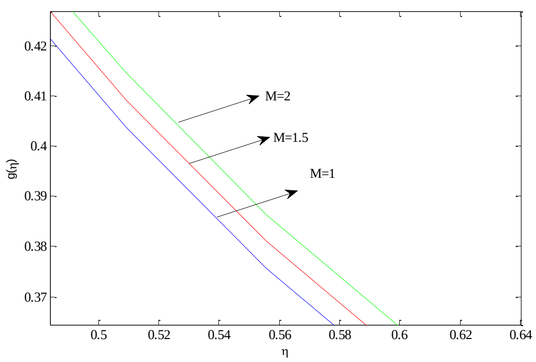

Figure 10. Micro rotation profile for Magnetic parameter M

α=30, Fw=0.1,N=0.1, Da=1, Fs=1, λ=1, J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

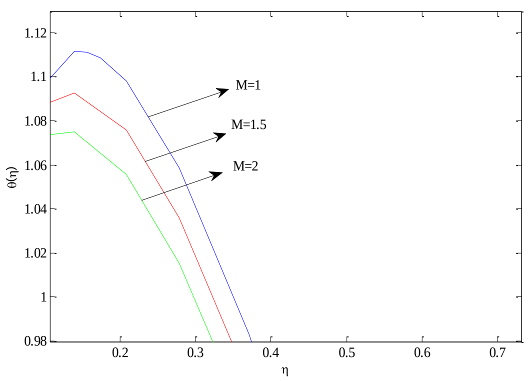

Figure 11. Temperature profile for Magnetic parameter M

α=30, Fw=0.1,N=0.1, Da=1, Fs=1,λ=1, J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

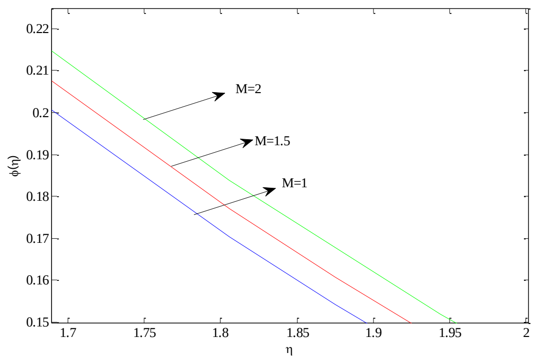

Figure 12. Concentration profile for Magnetic parameter M

α=30, Fw=0.1,N=0.1, Da=1, Fs=1, λ=1, J=1, Pn=0.71, Ec=0.5, Q=0.5, Sc=0.2

IV. CONCLUSION

The objective of this investigation is to study was the mathematical and numerical study of the viscous dissipation and radiation effect on MHD free convection flow in micropolar fluid past an inclined stretching sheet. The numerical solutions of the governing differential equations were obtained by using the classical fourth order Runge kutta method. Velocity inside the boundary layer thickness region decreases with magnetic field and the coupling number increases. The thermal boundary layer thickness will be increases with magnetic field, and the temperature will decreases. We observe that the micro rotation decreasing near the wall and simultaneously increasing with the suction parameter increasing.

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