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A Study on Prime Labeling of Some Special Graphs

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ABSTRACT

In this paper, we discussed about the prime labeling for Herschel graph and cubic graph with 8 vertices. A graph G with vertex set V is said to have a prime labeling, if its vertices are labeled with integers 1,2,3,….|V|.Such that for each xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper we also discuss prime labeling in the context of some graph operations namely Fusion, Duplication and Switching.
Keywords : Prime Labeling, Fusion, Duplication And Switching.

I. INTRODUCTION

This works deals with graph labeling. All the graphs considered here are finite and undirected. The graph $G = (V(G), E(G))$ has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is set of vertices(or edges) then the labeling is called a vertex labeling(or an edge labeling).
Following are the common features of any graph labeling problem

- ✓ A set of numbers from which vertex labels are assigned.
- ✓ A rule that assigns value to each edge.
- ✓ A condition that these values must satisfy.

The notation of prime labeling was introduced by Roger Entringer and was discussedin a paper byA.Tout (1982 P 365-368)[9]. Many researches have studied prime graph for example in H.C.Fu (1994 P 181-186) [1] have proved that path $P_n$on n vertices is a prime graph.

T.O Dertsky (1991 P 359-369) [8] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee (1998 P 59-67) [7] have proved that Wheel W_n is a prime graph iff n is even . Around 1980 Roger Entringer conjectured that all tress have prime labelling, which is not settled till today.

The prime labeling for planner grid is investigated by M. Sundaram (2006 P 205-209) [4]. In [5] S.K. Vaidhya and K.K. Kanmani have proved that the prime labeling for some cycle related graphs. In [6] S.Meena and K.Vaithilingam, Prime labeling for some Helm related graphs.

We will provide brief summary of definitions and some other information which are necessary for the present investigations.

Definition : 1
If the vertices of the graph are assigned values subject to certain conditions then it is called as (vertex) graph labeling.

Definition : 2
Let $G(V, E)$ be a graph with n vertices. A bijection $f: V \rightarrow \{1,2,3, \dots . n\}$ is called a prime labeling if for

each edge $e = uv$, $\gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition : 3

An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition : 4

Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructing by Fusing(identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u(or) v in G now incident with x in G_1.

Definition: 5

Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v'_k with $N(v'_k) = N(v_k)$. In other words, a vertex v'_k is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k.

Definition: 6

A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition :6

A bipartite undirected graph with 11 vertices and 18 edges is called as Herschel Graph, it is denoted as H_S.

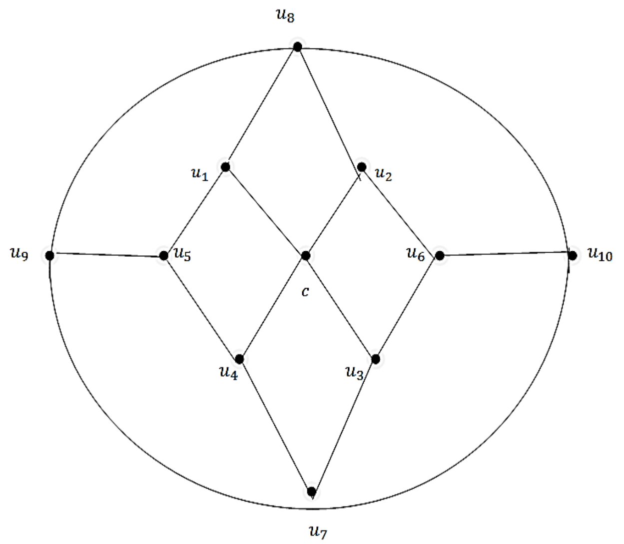


Fig 1. A Herschel graph

Theorem 1:

The Herschel graph H_S is a prime graph.

Proof:

First let us assume that H_S be the Herschel graph with 11 vertices and 18 edges. Assume that 'c' be the centre of the Herschel graph.

Then $|V(H_S)| = 11$ and $|E(H_S)| = 18$

Consider, $f(c) = 1$ and $f(u_i) = 2i$ for $1 \leq i \leq 4$, where u_i's are adjacent to c.

Therefore, $f(u_1) = 2$

$f(u_2) = 4$

$f(u_3) = 6$

$f(u_4) = 8$

After that we assign the label vertex is u_5, but $u_5$is adjacent to u_1 and u_4.

And the vertices u_1 and u_4 are having even label.

Therefore, $f(u_5) = 3$

Next we assign the label of vertex is u_6, then u_6 is adjacent to u_2 and u_3.

And the vertices u_2 and u_3 are having even label.

Therefore, $f(u_6) = 5$

In the same way, u_7 is adjacent to u_3 and u_4 also having even label.

Therefore, $f(u_7) = 7$

And u_8 is adjacent to $u_1$and u_2 having even label

Therefore,$f(u_8) = 9$

Finally, Let$f(u_9) = 10$ and $f(u_{10}) = 11$

Thus, for every edge

$e = cu_i \in H_s, \gcd(f(c), f(u_i)) = 1$ and the edge

$e = u_iu_j \in H_s, \gcd\left(f(u_i), f(u_j)\right) = 1.$

Then Herschel graph H_S admits prime labeling.

Hence Herschel graph $H_s$is a prime graph.

Example

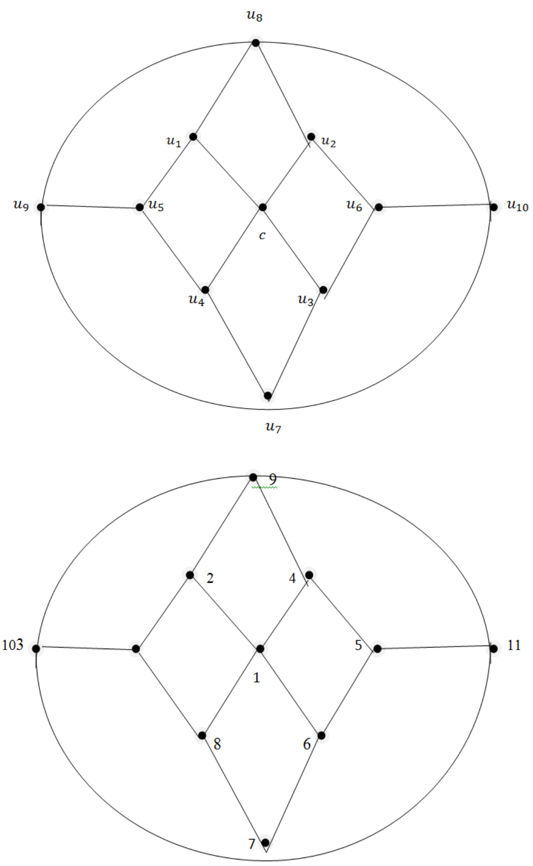


Fig 2 The Herschel graph is a prime graph

Theorem 2:

The fusion of two adjacent vertices of degree 3 in a Herschel graph is a prime graph.

Proof:

First we suppose that H_S be the Herschel graph with 11 vertices and 18 edges.

Next we assume that c be the centre of the Herschel of graph H_S and it has 8 vertices of degree 3 and 3 vertices of degree 4.

Then $|V(H_S)| = 11$ and $|E(H_S)| = 18$

Consider G be the graph obtained by fusing of two adjacent vertices of degree 3 in the Herschel graph H_S.

Therefore $|V(H_S)| = 10$

Now we define a bijective function $f: V(G) \to \{1,2,3,4, \dots \dots 10\}$

Let $f(c) = 1$ and $f(u_i) = 2i$ for $1 \le i \le 4$, where $u_i^{\prime}$s are adjacent to c.

Therefore, $f(u_1) = 2$

$f(u_2) = 4$

$f(u_3) = 6$

$f(u_4) = 8$

After that we assign the label is u_5 but it is adjacent to u_1 and u_4 and the vertices having even label.

Therefore $f(u_5) = 3$

In the same way, u_6 is adjacent to u_2 and u_3 also having even label.

Therefore, $f(u_6) = 5$

In the same way, we assign the label of the vertex is $u_7$and u_8.

Therefore, Let $f(u_7) = 7$ and $f(u_8) = 9$

Finally, we assign the label of the vertex is u_9 with the remaining label is 10,

Therefore, $f(u_9) = 10$

Now, for each edge $e = cu_i \in H_s, \gcd\big(f(c), f(u_i)\big) = 1$ and the edge $e = u_i u_j \in H_s, \gcd\left(f(u_i), f(u_j)\right) = 1.$

Then G admits prime labeling.

Hence G is a prime graph.

Example

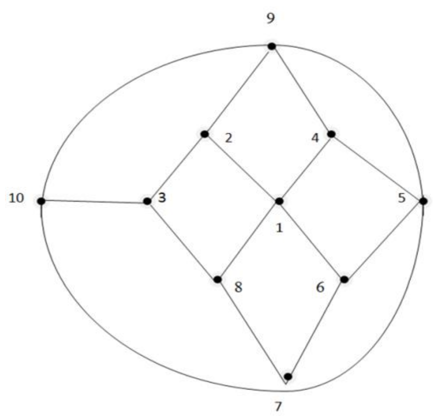


Fig 2 The Herschel graph is a prime graph

Theorem 2:
The fusion of two adjacent vertices of degree 3 in a Herschel graph is a prime graph.

Proof:
First we suppose that H_s be the Herschel graph with 11 vertices and 18 edges.

Next we assume that c be the centre of the Herschel of graph H_s and it has 8 vertices of degree 3 and 3 vertices of degree 4.

Then $|V(H_S)| = 11$ and $|E(H_S)| = 18$

Consider G be the graph obtained by fusing of two adjacent vertices of degree 3 in the Herschel graph H_s.

Therefore $|V(H_S)| = 10$

Now we define a bijective function $f: V(G) \rightarrow \{1,2,3,4, \dots \dots 10\}$
Let $f(c) = 1$ and $f(u_i) = 2i$ for $1 \leq i \leq 4$, where u_i's are adjacent to c.

Therefore, $f(u_1) = 2$
$f(u_2) = 4$
$f(u_3) = 6$
$f(u_4) = 8$

After that we assign the label is u_5 but it is adjacent to u_1 and u_4 and the vertices having even label.

Therefore $f(u_5) = 3$

In the same way, u_6 is adjacent to u_2 and u_3 also having even label.

Therefore, $f(u_6) = 5$

In the same way, we assign the label of the vertex is $u_7$and u_8.

Therefore, Let $f(u_7) = 7$ and $f(u_8) = 9$
Finally, we assign the label of the vertex is u_9 with the remaining label is 10,

Therefore, $f(u_9) = 10$

Now, for each edge $e = cu_i \in H_s, \gcd(f(c), f(u_i)) = 1$ and the edge $e = u_iu_j \in H_s, \gcd\left(f(u_i), f(u_j)\right) = 1.$

Then G admits prime labeling.

Hence G is a prime graph.

Example

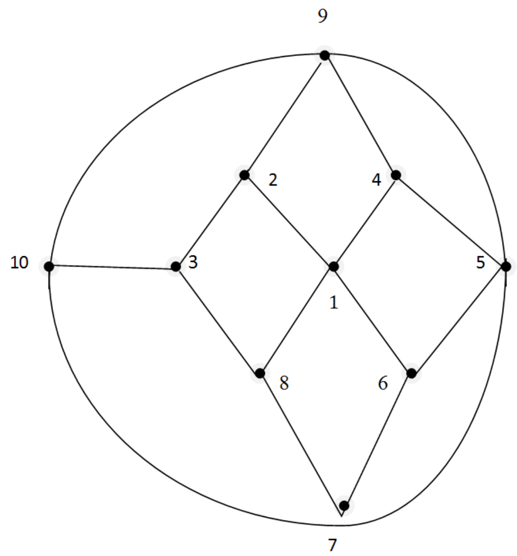


Fig 3 Fusion of the vertices u_6 and u_{10} in a Herschel graph H_s is a prime graph.

Theorem 3:
The Duplication of any vertex of 3 in a Herschel graph is a prime graph.

Proof:

Let H_s be the Herschel graph with 11 vertices and 18 edges and c be the centre vertex of the graph.

Then $|V(H_S)| = 11$ and $|E(H_S)| = 18$

Let us assume that u_k be the any vertex of degree 3 and u_k' be the duplication of the vertex u_k in the Herschel graph H_s.
Now we assume that G_k be the graph obtained by after duplicating the vertex u_k of degree 3 in Herschel graph H_s. So $|V(G_k)| = 12$.
Next we define the label of bijective function $f: V(G_k) \rightarrow \{1,2,3,4, \dots \dots 12\}$.

Let us assume that $f(c) = 1$. Then $f(u'_k) = 12$ where u'_k is the duplicating vertex of u_k. And $f(u_i) = 2i$ for $1 \leq i \leq 4$, where u'_is are adjacent to c.

Therefore, $f(u_1) = 2$

$f(u_2) = 4$

$f(u_3) = 6$

$f(u_4) = 8$

Let $f(u_5) = 3$, by reason of vertex u_5 is adjacent to u_1 and $u_4$both having an even label.

$f(u_6) = 5$, by reason of vertex u_6 is adjacent to $u_2$and u_3 both having an even label.

$f(u_7) = 7$, by reason of vertex u_7 is adjacent to $u_{3,}u_4, u_9, u_{10}$and u_k.

$f(u_8) = 11$, by reason of vertex u_8 is adjacent to $u_1, u_2, u_9, u_{10}$and u_k.

$f(u_9) = 10$, by reason of vertex u_9 is adjacent to u_5, u_7, u_8 are having even label.

$f(u_k) = 9$, Here we duplicating u_k in Herschel graph H_s as u'_k.

Now, for every edge $e = cu_i \in G_k, \gcd(f(c), f(u_i)) = 1$ and the edge $e = u_iu_j \in G_k, \gcd\left(f(u_i), f(u_j)\right) = 1$.

Then $G_k$admits prime labeling.

Hence G_k is a prime graph.

Example

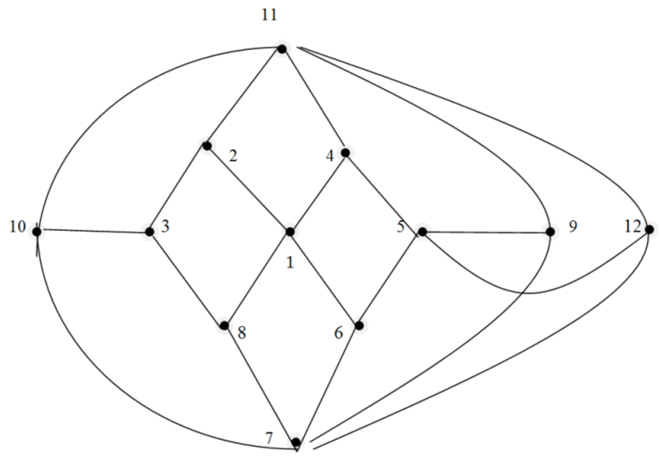


Fig 4 Duplication of the vertex $u_{10}$of degree 3 in H_s is a prime graph

Theorem 4:

Switching the centre vertex$c$ in the Herschel graph H_s is a prime graph.

Proof:

Consider H_s be the Herschel graph with 11 vertices and 18 edges and c be the centre vertex of the graph.

Then $|V(H_S)| = 11$ and $|E(H_S)| = 18$

Let us assume that c be the switching vertex and G_c be the new graph obtained by switching the centre vertex c .

Clearly $|V(G_c)| = 11$ and $|E(G_c)| = 19$

Next we define the label of bijective function $f: V(G_c) \rightarrow \{1,2,3,4, \dots \dots 11\}$.

Such that$f(c) = 1$. And $f(u_i) = 2i$ for $1 \leq i \leq 4$, where u'_is are adjacent to c.

Therefore, $f(u_1) = 2$

$f(u_2) = 4$

$f(u_3) = 6$

$f(u_4) = 8$

Now we assume that $f(u_5) = 3$,since vertex u_5 is adjacent to u_1 and $u_4$both having an even labels.

$f(u_6) = 5$, since vertexu_6 is adjacent to $u_2$and $u_3$of even labels.

$f(u_7) = 7$, since vertex u_7 is adjacent to u_3 and u_4 of even labels

$f(u_8) = 9$, since vertex u_8 is adjacent to u_1 and u_2 having an even labels.

$f(u_9) = 10$, since u_9 is adjacent to u_7 and u_8 having an even labels

Finally, $f(u_{10}) = 11$

For each edge $e = u_iu_j \in G_c$, $\gcd\left(f(u_i), f(u_j)\right) = 1$.

Then $G_c$admits a prime labeling

Hence $G_c$is a prime graph.

Example

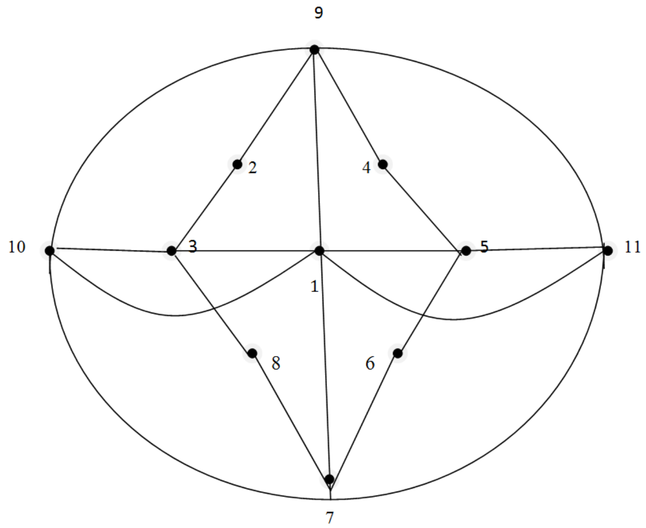
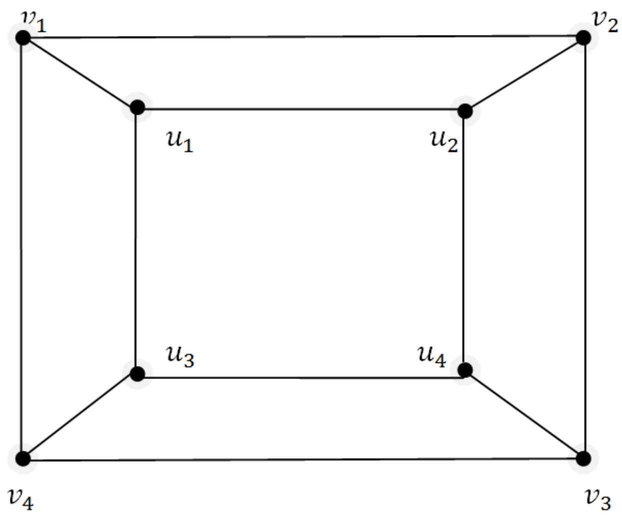


Fig 3.2.4 switching of the centre vertex c in H_s is a prime graph.

ON PRIME LABELING OF CUBIC GRAPH WITH 8 VERTICES

Cubic Graph

A Regular graph G is called a Cubic graph if all the vertices of G are of degree 3.



Cubic graph with 8 vertices

Theorem 1:

A cubic graph with 8 vertices is a prime graph.

Proof:

Let $G = (V, E)$ be a cubic graph with 8 vertices and 12 edges.

Now we assume that the vertex set is $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$ and the edge set $E(G) = \{u_i v_i \,/ 1 \le i \le 4\} \cup \{u_i u_{i+1} / \, 1 \le i \le 3, u_4 u_1\} \cup \{v_i v_{i+1} / \, 1 \le j \le 3, v_4 v_1\}$

Then$|V(G)|$=8 and $|E(G)| = 12$

Let us suppose that define a labeling function $f: V(G) \to \{1,2,3, \ldots .8\}$

Such that $f(u_i) = i$ for $1 \le i \le 4$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_4) = 4$

and $f(v_i) = f(u_i) + 5$ for $1 \le i \le 3$

(i.e.) $f(v_1) = f(u_1) + 5 = 1 + 5 = 6$

Similarly, $f(v_2) = f(u_2) + 5 = 2 + 5 = 7$

$f(v_3) = f(u_3) + 5 = 3 + 5 = 8$

Finally, $f(v_4) = f(u_4) + 1 = 4 + 1 = 5$

Thus, for each edge $e = u_i u_j \in G, \gcd(f(u_i), f(v_i)) = 1$ and the edges

$e = u_i u_j, v_i v_j \in G, \gcd\left(f(u_i), f(u_j)\right) = 1$ and $\gcd\left(f(v_i), f(v_j)\right) = 1.$

Then$G$admits a prime labeling.

Hence ,G is a prime graph.

Example

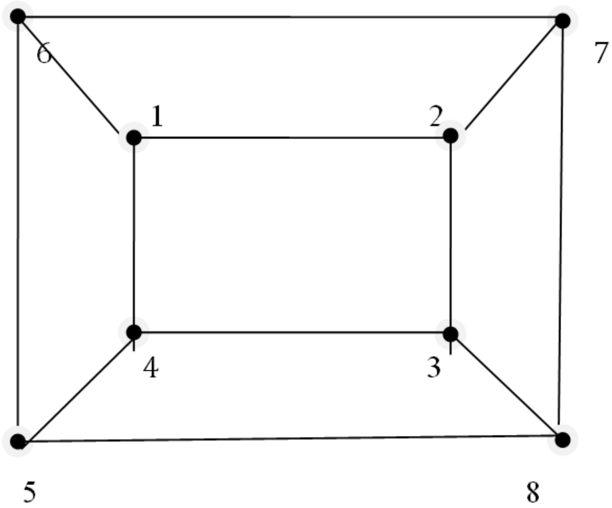


Fig 1 A cubic graph with 8 vertices is a prime graph

Theorem 2:

The fusion of two consecutive vertices in the outer cycle graph on 8 vertices is a prime graph.

Proof:

Let us assume that $G = (V, E)$ be a cubic graph on 8 vertices and G_f be the graph

Obtained by fusion (or identifying) two vertices v_1 and v_2(i.e. $v_1 = v_2$) of G.

$$(i.e). |G_f(V)| = 7$$

Now we define a label function $f: V(G_f) \to \{1,2,3, \ldots .7\}$

Such that $f(u_i) = i$ for $1 \le i \le 4$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$ and $f(u_4) = 4$

And let $f(v_i) = f(u_i) + 5$ for $1 \le i \le 3$

(i.e.) $f(v_1) = f(u_1) + 5 = 1 + 5 = 6$

Similarly, $f(v_2 = v_3) = f(u_2) + 5 = 2 + 5 = 7$

Finally , $f(v_4) = f(u_4) + 1 = 4 + 1 = 5$

Clearly, for each edge

$e = u_i u_j \in G, \gcd(f(u_i), f(v_i)) = 1$

and the edges $e = u_i u_j, v_i v_j \in G, \gcd\left(f(u_i), f(u_j)\right) = 1$ and $\gcd\left(f(v_i), f(v_j)\right) = 1$.

Then G_f admits a prime labeling.

Hence G_f is a prime graph.

Example

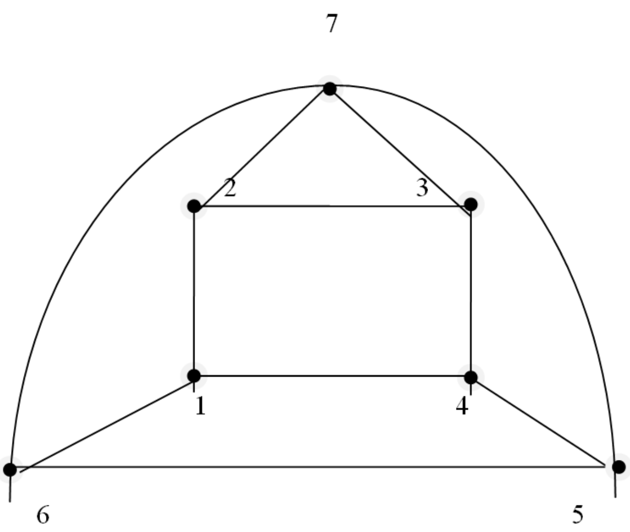


Fig 2 :Fusion of two vertices v_2 and v_3 in a cubic graph is a prime graph.

Theorem 3:

The Duplication of an arbitrary vertex of the cubic graph on 8 vertices produces a prime graph.

Proof :

Consider $G = (V, E)$ be a cubic graph with 8 vertices and 12 edges.

Now we assume that the vertex set is $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$ and the edge set $E(G) = \{u_i v_i / 1 \le i \le 4\} \cup \{u_i u_{i+1} / 1 \le i \le 3, u_4, u_1\} \cup \{v_1 v_{i+1} / 1 \le j \le 3, v_4 v_1\}$

Let G_k be the graph obtained by Duplicating and arbitrary vertex of G. Without loss of generality l et this vertex be v_1 and the newly added vetex be v_1'.

Now we define the label function $f: V(G_k) \to \{1,2,3, \ldots .9\}$

Such that $f(u_i) = i$ for $1 \le i \le 4$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_4) = 4$

And $f(v_i) = f(u_i) + 5$ for $1 \le i \le 3$

(i.e.) $f(v_1) = f(u_1) + 5 = 1 + 5 = 6$ and $f(v_1') = 9$

$f(v_2) = f(u_2) + 5 = 2 + 5 = 7$

Similarly, $f(v_3) = f(u_3) + 1 = 3 + 5 = 8$

Finally, $f(v_4) = f(u_4) + 1 = 4 + 1 = 5$

Thus,for each edge

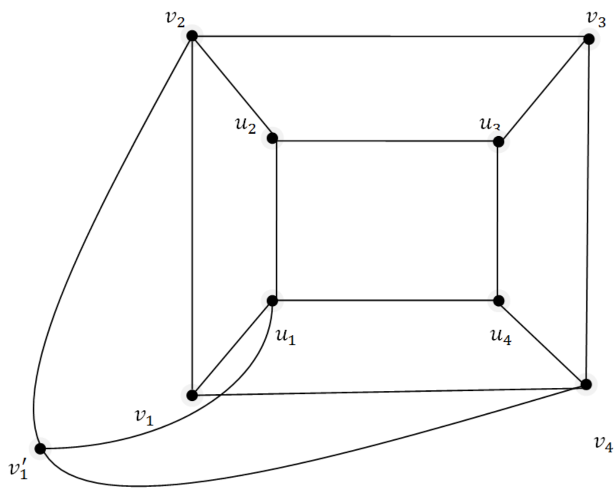
$e = u_i u_j, u_i v_i, v_i v_j \in G, \gcd(f(u_i), f(v_i)) = 1$,

$\gcd\left(f(u_i), f(u_j)\right) = 1$ and $\gcd\left(f(v_i), f(v_j)\right) = 1$.

Then G_k admits a prime labeling.

Hence G_k is a prime graph.

Example



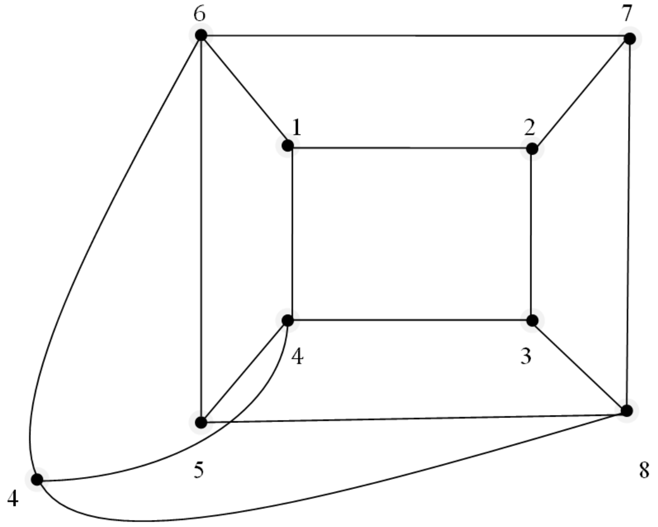


Fig 3 The Duplication of the vertex v_1 in cubic graph is a prime graph

Theorem 4:

The Switching of an arbitrary vertex in a cubic graph on 8 vertices is a prime graph.

Proof:

Consider $G = (V, E)$ be a cubic graph with 8 vertices and 12 edges.

Now we assume that the vertex set is $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$ and the edge set $E(G) = \{u_i v_i \ /1 \le i \le 4\} \cup \{u_i u_{i+1} / \ 1 \le i \le 3, u_4, u_1\} \cup \{v_1 v_{i+1} / \ 1 \le i \le 3, v_4 v_1\}$

Assume that G_s be the graph obtained by switching and arbitrary vertex of G. Without loss of generality let this vertex be $v_1$and $|V(G)|$= 8 and $|E(G)| = 12$

Now we define the label function $f: V(G_s) \to \{1,2,3, \dots .8\}$

Such that$f(u_i) = i$ for $1 \le i \le 4$

$f(v_1) = 1$, here v_1 is a switching vertex

$f(v_2) = 2$

$f(v_3) = 3$

$f(v_4) = 4$

And $f(u_i) = f(v_i) + 5$ for $1 \le i \le 3$

(i.e.) $f(u_1) = f(v_1) + 5 = 1 + 5 = 6$ and $f(v_1') = 9$

$f(u_2) = f(v_2) + 5 = 2 + 5 = 7$

Similarly, $f(u_3) = f(v_3) + 1 = 3 + 5 = 8$

Finally, $f(u_4) = f(v_4) + 1 = 4 + 1 = 5$

Thus , for every edge $e = u_i v_i \in G, \gcd(f(u_i), f(v_i)) = 1$ and the edges $e = u_i u_j, v_i v_j \in G, \gcd\left(f(u_i), f(u_j)\right) = 1$ and $\gcd\left(f(v_i), f(v_j)\right) = 1$.

Then $G_s$admits a prime labeling.

Hence $G_s$is a prime graph.

Example

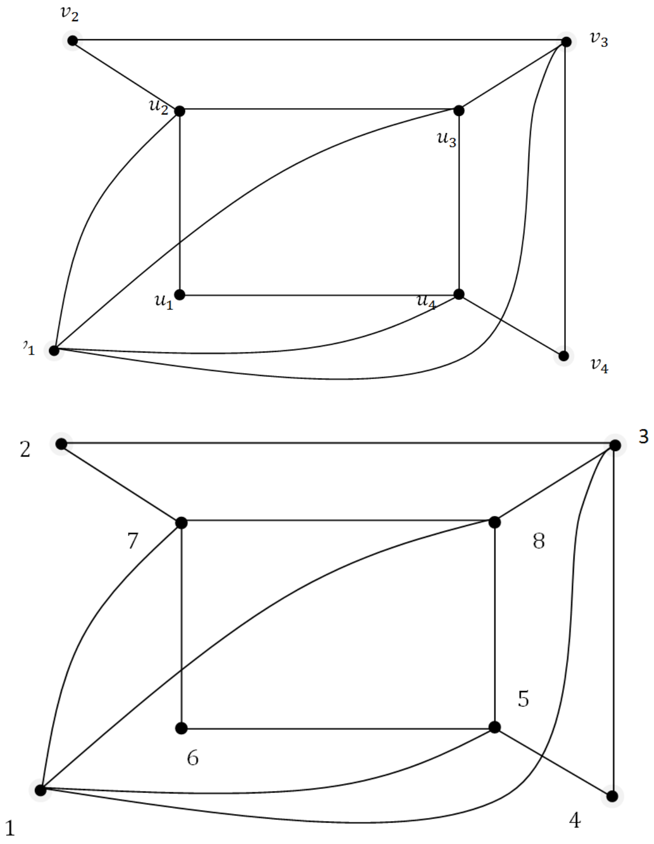


Fig 4 The Switching of $v_1$in a cubic graph is a prime graph.

II. CONCLUSION

In this dissertation ,we have investigate a four results corresponding to prime labeling on some special graphs, namely Herschel graph and cubic graph with 8 vertices. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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IV. REFERENCES

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