

Solving Fuzzy Transportation Problem Using Trapezoidal Fuzzy Numbers

K. Sangeetha¹, Priyadharshini. T², Mayuriya. S²

¹Assistant Professor Department of Mathematics, Dr.SNS Rajalakshmi college of Arts &Science, Coimbatore, Tamil Nadu, India

²B.Sc MATHS CA Department of Mathematics, Dr.SNS Rajalakshmi college of Arts &Science, Coimbatore, Tamil Nadu, India

ABSTRACT

In this paper, we introduced a Ranking method using Robust Ranking to find the fuzzy optimal solution of unbalanced fuzzy transportation problem. The modification to Vogel's Approximation method for obtaining initial solution to the unbalanced transportation problems are described by many words, the main aim of fuzzy transportation is that the least transportation cost of the commodities through the capacity of the work with the supply and demand of the consumers.

Keywords : Fuzzy Transportation problem, Robust Ranking technique, Commodity, Demand, Trapezoidal fuzzy number.

I. INTRODUCTION

The transportation problem one of the earliest applications of linear programming problems. It is assumed that decision maker is sure about the precise values of transportation cost supply and demand of the product. Fuzzy transportation problem is the problem of minimizing fuzzy valued objective function with supply and demand. Solving transportation problem where the capacities and requirements are the fuzzy set with linear or triangular membership function. Fuzzy optimal solution for a fuzzy transportation problem where transportation cost supply and demand are represented by trapezoidal fuzzy numbers.

PRELIMINARIES

In this section basic definitions and arithmetic operations are reviewed.

BASIC DEFINITIONS

1. Fuzzy set: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to be the unit interval $[0, 1]$ i.e., $A = \{x, \mu_A(x); x \in X\}$, Here $\mu_A: X \rightarrow [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A . These membership grades are often represented by real numbers ranging from $[0, 1]$.

2. Trapezoidal fuzzy numbers: A fuzzy number $\tilde{A} = (m, n, a, b)$ is said to be trapezoidal fuzzy number if its membership function given by

$$S_{\mu\bar{A}}(X) = \begin{cases} 0 & x < m \\ (x - m)/(n - m) & m \leq x \leq n \\ 1 & n \leq x \leq a \\ (b - x)/(b - a) & a \leq x \leq b \\ 0 & x > b \end{cases}$$

3. Properties of trapezoidal fuzzy numbers

1. Trapezoidal fuzzy number $\bar{A} = (m, n, a, b)$ is said to be non-negative trapezoidal fuzzy number. Iff $m-a \geq 0$
2. A trapezoidal fuzzy number. $\bar{A} = (m, n, a, b)$ is said to be zero trapezoidal fuzzy number, iff $m=0, n=0, a=0, b=0$.
3. Two trapezoidal fuzzy numbers $\bar{A}_1 = (m_1, n_1, a_1, b_1)$ and $\bar{A}_2 = (m_2, n_2, a_2, b_2)$ are said to be equal i.e., $\bar{A}_1 = \bar{A}_2$, Iff $m_1 = m_2, n_1 = n_2, a_1 = a_2, b_1 = b_2$

4. Arithmetic Operators for solving Trapezoidal fuzzy number

If $A = [m_1, n_1, a_1, b_1]$ and $B = [m_2, n_2, a_2, b_2]$ two trapezoidal fuzzy numbers then the arithmetic operations on A and B as follows

Addition $A+B = (m_1+m_2, n_1+n_2, a_1+a_2, b_1+b_2)$

Subtraction $A-B = (m_1-m_2, n_1-n_2, a_1-a_2, b_1-b_2)$

Multiplication $A.B = (t_1, t_2, t_3, t_4)$

Where $t_1 = \text{minimum} \{m_1m_2, m_1b_2, b_1m_2, b_1b_2\}$

$t_2 = \text{minimum} \{n_1n_2, n_1a_2, a_1n_2, a_1a_2\}$

$t_3 = \text{maximum} \{n_1n_2, n_1a_2, a_1n_2, a_1a_2\}$

$t_4 = \text{maximum} \{m_1m_2, m_1b_2, b_1m_2, b_1b_2\}$

5. Robust Ranking Technique

Robust ranking technique which satisfy compensation linearity, and additively properties and provides results which are consist human intuition. If \bar{a} is a fuzzy number then the Robust Ranking defined by

$$R(\bar{a}) = \int_0^1 0.5(a_{\alpha}^L a_{\alpha}^U) d\alpha$$

Where $(a_{\alpha}^L a_{\alpha}^U)$ is the α level cut of the fuzzy number \bar{a} and $\bar{a} = (\alpha a + a, b - \alpha a)$

In this paper we use this method for ranking the objective values. The Robust ranking index $R(\bar{a})$ gives the representative value of fuzzy number \bar{a} .

FUZZY TRANSPORTATION PROBLEM

In conventional transportation problems it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all these parameters of the transportation problems may not be known precisely due to uncontrollable factors. For example, in real life problems the following situation may occur:

- (a) Let a product is to be transported first times at a destination and no expert have knowledge about the transportation cost then there exist uncertainty about the transportation cost.
- (b) If a new product is launched in the market then there always exists uncertainty about the demand of that particular product.
- (c) In daily life problems, it can be seen that whenever a customer ask to a supplier that the particular product is available or not, sometimes supplier answers yes it is available but after a few seconds supplier answers sorry at this time this product is not available. Sometimes a supplier does not have any uncertainty about the

statement that the product is available or not. When a customer demands for a particular product the supplier answer yes, the product is available, but if the demand of product is large then again supplier says, I check that so much quantity is available or not i.e. their exist uncertainty about the availability of product. To deal with such situations, fuzzy set theory is applied in literature to solve the transportation problems.

Several author (Liu and Kao 2004; Dinagar and Palanivel 2009; Pandian and Natrajan 2010)7,3,11 have proposed different methods for solving balanced fuzzy transportation problems by representing the transportation cost, availability and demand as normal fuzzy numbers. The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

$$\text{Minimize } \sum_{i=1}^p \sum_{j=1}^q C_{ij} \otimes x_{ij}$$

$$\text{Subject to } \sum_{j=1}^q x_{ij} = \bar{a}_i, i = 1,2,3, \dots \dots p$$

$$\sum_{i=1}^p x_{ij} = b_j, j = 1,2,3, \dots \dots q$$

$$\sum_{i=1}^p a_i = \sum_{j=1}^q b_j$$

X_{ij} is a non- negative trapezoidal fuzzy number

Where p = total number of sources

Q = total number of destinations

a_i = the fuzzy availability of the product at i th source

b_j = the fuzzy demand of the product at j th destination

c_{ij} = the fuzzy transportation cost for unit quantity of the product from i th source to j th destination

x_{ij} = the fuzzy quantity of the product that should be transported from i th source to j th destination to minimize the total fuzzy transportation cost.

$$\sum_{i=1}^p a_i = \text{total fuzzy availability of the product}$$

$$\sum_{j=1}^q b_j = \text{total fuzzy demand of the product}$$

$$\sum_{i=1}^p \sum_{j=1}^q C_{ij} * x_{ij} = \text{total fuzzy transportation cost}$$

If $\sum_{i=1}^p a_i = \sum_{j=1}^q b_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

Numerical Example :

A company has four sources S_1, S_2, S_3 and S_4 and four destinations D_1, D_2, D_3 and D_4 ; the fuzzy transportation cost for unit quantity of the product from i th source to j th destinations is C_{ij} where $[C_{ij}]_{4 \times 4} =$

$$\left\{ \begin{array}{cccc} (0,1,3,5) & (3,4,5,6) & (7,8,9,11) & (4,6,7,8) \\ (1,2,3,4) & (0,1,2,4) & (7,8,9,10) & (8,9,12,15) \\ (9,10,11,12) & (14,15,17,18) & (5,6,9,10) & (2,3,4,6) \\ (6,7,9,11) & (10,11,13,15) & (4,6,8,10) & (11,13,14,15) \end{array} \right\}$$

And the fuzzy availability of the product at source are $((4,5,7,10), (2,3,5,7), (7,10,13,14), (5,7,10,14))$ and the fuzzy demand of the product at destination are $((4,10,13,14), (3,7,9,13), (2,3,10,14), (3,7,10,13))$ respectively.

The fuzzy transportation problems are

Table 1

	FS_1	FS_2	FS_3	FS_4	Supply
FD_1	(0,1,3,5)	(3,4,5,6)	(8,7,9,11)	(4,6,8,10)	(4,5,7,10)
FD_2	(1,2,3,4)	(0,1,2,4)	(7,8,9,10)	(8,9,10,11)	(2,3,5,7)
FD_3	(9,10,11,12)	(14,15,17,18)	(5,6,9,10)	(2,3,4,6)	(7,10,13,14)
FD_4	(6,7,9,11)	(10,11,13,15)	(4,6,8,10)	(11,13,14,15)	(5,7,10,14)
Demand	(4,10,13,14)	(3,7,9,13)	(2,3,10,14)	(3,7,10,13)	

Table 2

	FS_1	FS_2	FS_3	FS_4	FS_5	Supply
FD_1	(0,1,3,5)	(3,4,5,6)	(8,7,9,11)	(4,6,8,10)	(0,0,0,0)	(4,5,7,10)
FD_2	(1,2,3,4)	(0,1,2,4)	(7,8,9,10)	(8,9,10,11)	(0,0,0,0)	(2,3,5,7)
FD_3	(9,10,11,12)	(14,15,17,18)	(5,6,9,10)	(2,3,4,6)	(0,0,0,0)	(7,10,13,14)
FD_4	(6,7,9,11)	(10,11,13,15)	(4,6,8,10)	(11,13,14,15)	(0,0,0,0)	(5,7,10,14)
FD_5	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,2,7,10)
Demand	(4,10,13,14)	(3,7,9,13)	(2,3,10,14)	(3,7,10,13)	(6,0,0,1)	(18,27,42,55)

$$\begin{aligned} \text{Min } Z = & R(0,1,3,5)X_{11} + R(3,4,5,6)X_{12} + R(8,7,9,11)X_{13} + R(4,6,8,10)X_{14} + R(1,2,3,4)X_{21} + R(0,1,2,4)X_{22} \\ & + R(7,8,9,10)X_{23} + R(8,9,10,11)X_{24} + R(9,10,11,12)X_{31} + R(14,15,17,18)X_{32} \\ & + R(5,6,9,10)X_{33} + R(2,3,4,6)X_{34} + R(6,7,9,11)X_{41} + R(10,11,13,15)X_{42} + R(4,6,8,10)X_{43} \\ & + R(11,13,14,15)X_{44} \end{aligned}$$

$$R(\bar{a}) = \int_0^1 0.5(a\alpha + b - a\alpha) d\alpha$$

$$\begin{aligned} R(0,1,3,5) &= \int_0^1 0.5(0\alpha + 0,1 - 0\alpha) d\alpha \\ &= \int_0^1 0.5(0\alpha + 0 + 1 - 0\alpha) d\alpha \\ &= \int_0^1 0.5(1) d\alpha \\ &= (0.5\alpha)_0^1 \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} R(3,4,5,6) &= 3.5, R(8,7,9,11) = 7.5, R(4,6,8,10) = 5, R(1,2,3,4) = 1.5, R(0,1,2,4) = 0.5, R(7,8,9,10) = 7.5, \\ R(8,9,10,11) &= 8.5, R(9,10,11,12) = 9.5, R(14,15,17,18) = 14.5, R(5,6,9,10) = 5.5, R(2,3,4,6) = 2.5, \\ R(6,7,9,11) &= 6.5, R(10,11,13,15) = 10.5, R(4,6,8,10) = 5, R(11,13,14,15) = 12 \end{aligned}$$

RANK OF SUPPLY:

$$\begin{aligned} R(4,5,7,10) &= 4.5, R(2,3,5,7) = 2.5, R(7,10,13,14) = 8.5, \\ R(5,7,10,14) &= 6, R(0,2,7,10) = 1 \end{aligned}$$

RANK OF DEMAND:

$R(4,10,13,14) = 7$, $R(3,7,9,13) = 5$, $R(2,3,10,14) = 2.5$,
 $R(3,7,10,13) = 5$, $R(6,0,0,1) = 3$

Table 3

	FS ₁	FS ₂	FS ₃	FS ₄	FS ₅	Supply
FD ₁	0.5	3.5	7.5	5	0	4.5
FD ₂	1.5	0.5	7.5	8.5	0	2.5
FD ₃	9.5	14.5	5.5	2.5	0	8.5
FD ₄	6.5	10.5	5	12	0	6
FD ₅	0	0	0	0	0	1
Demand	7	5	2.5	5	3	

AFTER APPLYING THE VOGEL APPROXIMATION METHOD:

Table 4

	FS ₁	FS ₂	FS ₃	FS ₄	FS ₅	Supply
FD ₁	0.5 2	3.5 2.5	7.5	5	0	4.5
FD ₂	1.5	0.5 2.5	7.5	8.5	0	2.5
FD ₃	9.5 2	14.5	5.5 1.5	2.5 5	0	8.5
FD ₄	6.5	10.5	5	12	0 3	6
FD ₅	0	0	0 1	0	0	1
Demand	7	5	2.5	5		3

The transportation cost

$= (0.5)2 + (3.5)(2.5) + (0.5)(2.5) + (5.5)(1.5) + (2.5)(5) + (0)1 + (3)0 + (9.5)2$
 $= 1 + 8.75 + 1.25 + 8.25 + 12.5 + 0 + 0 + 19$
 $= 50.75$

Therefore the transportation cost is **Rs.50.75**

II. CONCLUSION

In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of trapezoidal numbers has been transformed into crisp transportation problem using Robust’s ranking indices. By using Robust’s ranking method we have shown

that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust’s ranking method effectively. This technique can also be used in solving other types of problems like, project schedules, assignment problems and network flow problems.

III. REFERENCES

- [1]. PonnamsHungani, S.H. Abbas and Vijay Gupta (2012) : "Unbalanced fuzzy transportation problem with robust ranking technique" pp 94-97 .
- [2]. A.Gani and K.A.Razak(2006): "Two stage fuzzy transportation problem" Journal of physics sciences,pp 63-69.
- [3]. D.Dubois and H.Prade (1980) : "Fuzzy sets and system" Theory and Applications, Academic press, New York.
- [4]. D.S.Dingar, K.Palanivel (2009) : "The transportation problem in fuzzy environment" International journal of algorithms, computing and mathematics pp 65-71.
- [5]. F.L.Hitchcock (1941) : "The distribution of product from several sources to numerous localities" journal of mathematical physics pp 224-230.
- [6]. H.J.Zimmermann (1978) : "Fuzzy programming and linear programming with several objective function" Fuzzy sets and system pp 45-55.
- [7]. Lin .Feng_Tse and Tsai, Tzong _Ru : "A two stage genetic algorithm for solving the transportation problem with fuzzy demands and fuzzy supplies", Int. J. of innovative computing information and control. Vol.5.(2009).