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Some Classes of Operators of Order 'N' on Hilbert Space and Complex Hilbert Space

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ABSTRACT

In this paper we introduce n-power hypo-normal operator of order-n, n-power quasi-normal operator of order-n, quasi parahyponormal operator of order-n on a Hilbert space H. we give some properties of these operators.

Keywords: n-power hypo-normal, n-power quasi-normal,parahyponormal ,quasi parahyponormal operator.

I. INTRODUCTION

SECTION 1.0

Let B(H) denotes the algebra of all bounded linear operators acting on a complex Hilbert space H. An operator $T \epsilon$B(H) is said to be self adjoint if $T^* = T$, isometry if $T^*T = I$ .The operator $T\epsilon$ B(H) is called normal if $TT^* = T^*T$, Quasinormal if, $T(T^*T) = (T^*T)T$.An operator T on H is called hyponormal if $TT^* \leq T^*T$.

SECTION 2.0

In this section we introduce n-power quasi-normal operator of order-n.

Definition:2.1 n-power Quasi-normal operator of order-n

An operator T is called n-power Quasi-normal operator of order-n if,

$$T^n(T^{*n}T) = (T^{*n}T)T^n$$

Theorem: 2.2

Let $T_1, T_2, \dots T_k$ be n-power quasi-normal operators of order-n in B(H).Then the direct sum $(T_1 \oplus T_2 \oplus \dots \oplus T_k)$ and tensor product $(T_1 \otimes T_2 \otimes \dots \otimes T_k)$ are n-power quasi-normal operators of order-n.

Proof:

From the definition of n-power quasi-normal operators of order-n,we have

$$T^n(T^{*n}T) = (T^{*n}T)T^n$$

$(T_1 \oplus T_2 \oplus \dots \oplus T_k)^n[(T_1 \oplus T_2 \oplus \dots \oplus T_k)^{*n}(T_1 \oplus T_2 \oplus \dots \oplus T_k)] = ({T_1}^n \oplus {T_2}^n \oplus \dots . \oplus {T_k}^n)[({T_1}^{*n} \oplus {T_2}^{*n} \oplus \dots . \oplus {T_k}^{*n})\, (T_1 \oplus T_2 \oplus \dots \oplus T_k)]$

$= ({T_1}^n \oplus {T_2}^n \oplus \dots . \oplus {T_k}^n)[({T_1}^{*n}T_1 \oplus {T_2}^{*n}T_2 \oplus \dots . \oplus {T_k}^{*n}T_k)]$

$={T_1}^n({T_1}^{*n}T_1) \oplus {T_2}^n({T_2}^{*n}T_2) \oplus \dots \oplus {T_k}^n({T_k}^{*n}T_k)$

Since, $T_1, T_2, \dots T_k$ be n-power quasi-normal operator of order-n,then

$=({T_1}^{*n}T_1){T_1}^n \oplus ({T_2}^{*n}T_2){T_2}^n \oplus \dots \oplus ({T_k}^{*n}T_k){T_k}^n$

$= [({T_1}^{*n} \oplus {T_2}^{*n} \oplus \dots . \oplus {T_k}^{*n})\, (T_1 \oplus T_2 \oplus \dots \oplus T_k)]({T_1}^n \oplus {T_2}^n \oplus \dots . \oplus {T_k}^n)$

$= [(T_1 \oplus T_2 \oplus \dots \oplus T_k)^{*n}(T_1 \oplus T_2 \oplus \dots \oplus T_k)]\, (T_1 \oplus T_2 \oplus \dots \oplus T_k)^n$

Also

$(T_1 \otimes T_2 \otimes \dots \otimes T_k)^n[(T_1 \otimes T_2 \otimes \dots \otimes T_k)^{*n}(T_1 \otimes T_2 \otimes \dots \otimes T_k)]$

$=({T_1}^n \otimes {T_2}^n \otimes \dots . \otimes {T_k}^n)[({T_1}^{*n} \otimes {T_2}^{*n} \otimes \dots . \otimes {T_k}^{*n})\, (T_1 \otimes T_2 \otimes \dots \otimes T_k)]$

$=({T_1}^n \otimes {T_2}^n \otimes \dots . \otimes {T_k}^n)\, [({T_1}^{*n}T_1 \otimes {T_2}^{*n}T_2 \otimes \dots . \otimes {T_k}^{*n}T_k)]$

$={T_1}^n({T_1}^{*n}T_1) \otimes {T_2}^n({T_2}^{*n}T_2) \otimes \dots \otimes {T_k}^n({T_k}^{*n}T_k)$

Since, $T_1, T_2, \dots T_k$ be n-power quasi-normal operator of order-n,then

$=({T_1}^{*n}T_1){T_1}^n \otimes ({T_2}^{*n}T_2){T_2}^n \otimes \dots \otimes ({T_k}^{*n}T_k){T_k}^n$

$= [({T_1}^{*n} \otimes {T_2}^{*n} \otimes \dots . \otimes {T_k}^{*n})\, (T_1 \otimes T_2 \otimes \dots \otimes T_k)]\, ({T_1}^n \otimes {T_2}^n \otimes \dots . \otimes {T_k}^n)$

$= [(T_1 \otimes T_2 \otimes \dots \otimes T_k)^{*n}(T_1 \otimes T_2 \otimes \dots \otimes T_k)]\, (T_1 \otimes T_2 \otimes \dots \otimes T_k)^n$

Section 3.0:

In this section we introduce n-power Hypo-normal operator of order-n.

Definition: 3.1 n-power Hypo-normal operator of order-n

An operator T is called n-power Hypo-normal operator of order-n if,

$$T^{*n}T^{n} \geq T^{n}T^{*n}$$

Theorem 3.2:

If S&T are doubly commuting n-power hypo-normal operators of order -n and ST^{*}=T^{*}S,then ST is an n-power hypo-normal operator of order-n.

Proof:

Since ST=TS

$\Longrightarrow$ $S^{n}T^{n} = (ST)^{n}$ and $ST^{*} = T^{*}S$

$\Longrightarrow S^{n}T^{*} = T^{*}S^{n}$ $[\because ST^{*} = T^{*}S]$

$\Longrightarrow TS^{*} = S^{*}T$

$\Longrightarrow T^{n}S^{*} = S^{*}T^{n}$

We have, $(ST)^{*n}(ST)^{n} = T^{*n}S^{*n}S^{n}T^{n}$

$=T^{*n}S^{n}S^{*n}T^{n}$

$=S^{n}T^{*n}T^{n}S^{*n}$

$\leq S^{n}T^{n}T^{*n}S^{*n}$

Hence, $(ST)^{*n}(ST)^{n} \leq (ST)^{n}(ST)^{*n}$

Then ST is an n-power hypo-normal operator of order-n.

Section 4.0:

In this section we introduce parahyponormal operator of order-n.

Definition: 4.1 Parahyponormal operator of order-n

An operator $T\epsilon$B(H) is said to be parahyponormal operator of order-n ,if

$$\|Tx\|^{2} \leq \|TT^{*n}x\|$$

Theorem:4.2

If $S, T\epsilon$B(H) are doubly commuting parahyponormal operators of order-n and $ST^{*n} = T^{*n}S$ then ST is parahyponormal operator of order-n.

Proof:

$S^{n}T^{n} = (ST)^{n}$ $[\because ST = TS]$

$ST^{*n} = T^{*n}S, S^{n}T^{*n} = T^{*n}S^{n}$ $[\because ST^{*} = T^{*}S]$

Now, $ST^{*n} = T^{*n}S$

We have to prove,$(TT^{*n})^{2} - 2\lambda(T^{*n}T) + \lambda^{2} \geq 0$

$\Longrightarrow ((ST)(ST)^{*n})^{2} - 2\lambda((ST)^{*n}(ST)) + \lambda^{2} \geq 0$

$\Longrightarrow ((ST)(T^{*n}S^{*n}))^{2} - 2\lambda\big((T^{*n}S^{*n})(ST)\big) + \lambda^{2}I \geq 0$

$\Longrightarrow ((T^{*n}T)(SS^{*n}))^{2} - 2\lambda\big((T^{*n}T)(S^{*n}S)\big) + \lambda^{2}I \geq 0$

$\Longrightarrow SS^{*n} = I, S^{*n}S = I$

$\Longrightarrow (TT^{*n})^{2} - 2\lambda(T^{*n}T) + \lambda^{2} \geq 0.$

Hence,ST is parahyponormal operator of order-n.

Theorem:4.3

If a parahyponormal operator of order-n doubly commutes with a hyponormal operator S,then product TS is parahyponormal operator of order-n.

Proof:

Let E(t) be the resolution of the identity for the self-adjoint operator $S^{*}S$.

Thus $T^{*n}T$ and $T^{*2n}T^{2}$ both doubly commute with every E(t).

Since S is hyponormal.we have,

$S^{*n}S \geq SS^{*n}$

$[(TS)^{2}(T^{*n}S^{*n})^{2}]^{2} - 2\lambda[TS(TS)^{*n}] + \lambda^{2}$

$= [T^{2}S^{2}(S^{*2n}T^{*2n})]^{2} - 2\lambda[TS(S^{*n}T^{*n})] + \lambda^{2}$

$= [(T^{*2n}T^{2})(S^{*2n}S^{2})]^{2} - 2\lambda[(T^{*n}T)(S^{*n}S)] + \lambda^{2}$

$\geq [(T^{*2n}T^{2})(S^{*2n}S^{2})]^{2} - 2\lambda[(T^{*n}T)(S^{*n}S)] + \lambda^{2}$

$= (T^{*2n}T^{2})^{2} - 2\lambda[(T^{*n}T)(S^{*n}S)] + \lambda^{2}$

$[\because S^{*n}S = I]$

$\geq 0.$

Thus T is parahyponormal operator of order-n.

Section 5.0:

In this section we introduce quasi parahyponormal operator of order-n.

Definition: 4.1 Quasi-Parahyponormal operator of order-n

An operator $T\epsilon$B(H) is said to be quasi parahyponormal operator of order-n ,if

$$\|TT^{*n}x\|^{2} \leq \|T^{2}T^{*2n}x\|$$

Theorem 5.2:

Let $T\epsilon$B(H) be a quasi parahyponormal operator of order-n.If T commutes with isometric operator S then TS is quasi parahyponormal operator of order-n.

Proof:

Let A=TS for all real number λ.

$$(A^2A^{*2n})^2 + 2\lambda(AA^{*n})^2 + \lambda^2 \geq 0.$$
$$[(TS)^2(TS)^{*2n}]^2 + 2\lambda[(TS)(TS)^{*n}]^2 + \lambda^2 \geq 0.$$

$$[(T^2S^2)(S^{*2n}T^{*2n})]^2 + 2\lambda[(TS)(S^{*n}T^{*n})]^2 + \lambda^2 \geq 0.$$
$$\begin{bmatrix} \because TS = ST \\ S^*S = I \end{bmatrix}$$

$$[(S^2T^2)(S^{*2n}T^{*2n})]^2 + 2\lambda[(ST)(S^{*n}T^{*n})]^2 + \lambda^2 \geq 0.$$
$$(T^2T^{*2n})^2 + 2\lambda(TT^{*n})^2 + \lambda^2 \geq 0.$$

Therefore,A is quasi parahyponormal operator of order-n.

Theorem 5.3:

If a quasi parahyponormal operator of order-n T commutes with an isometric operator S then $\frac{T}{S}$ is quasi parahyponormal operator of order-n.

Proof:

Let A=$\frac{T}{S}$,

We have for any real number λ.

$$(A^2A^{*2n})^2 + 2\lambda(AA^{*n})^2 + \lambda^2 \geq 0.$$
$$[\left(\frac{T}{S}\right)^2\left(\frac{T}{S}\right)^{*2n}]^2 + 2\lambda[\left(\frac{T}{S}\right)\left(\frac{T}{S}\right)^{*n}]^2 + \lambda^2 I \geq 0.$$
$$[(T^2S^{-2})(T^{*2n}S^{-*2n})]^2 + 2\lambda[TS^{-1})(T^{*n}S^{-*n})]^2 + \lambda^2 I \geq 0.$$

$[\because T^2S^2 = S^2T^2$

and $S^2S^{-*2n} = I, SS^{-*n} = I]$

$$[(S^{-2}T^2)(S^{-*2n}T^{*2n})]^2 + 2\lambda[(S^{-1}T)(S^{-*n}T^{*n})]^2 + \lambda^2 I \geq 0.$$
$$(T^2T^{*2n})^2 + 2\lambda(TT^{*n})^2 + \lambda^2 I \geq 0.$$

A is quasi parahyponormal of order-n.

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