© 2018 IJSRSET | Volume 4 | Issue 4 | Print ISSN: 2395-1990 | Online ISSN : 2394-4099
Themed Section : Engineering and Technology

Adomian Decomposition Study of Unidimensional Flow in Unsaturated Porous Media

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ABSTRACT

In this paper, analytically discusses a one dimensional unsteady flow through unsaturated porous media by employing a two parameter singular perturbation technique. The mathematical formulation yields a nonlinear partial differential equations considering Adomian decomposition method.

Keywords: Adomian decomposition method, drainage, infiltration, evaporation, absorption.

I. INTRODUCTION

Nonlinear partial differential equations can be found in wide variety scientific and engineering applications. Many important mathematical models can be expressed in terms of nonlinear partial differential equations. The most general form of nonlinear partial differential equation is given by:

$$F(u, u_t, u_x, u_y, x, y, t) = 0 \quad \textbf{(1a)}$$

With initial and boundary conditions

$$u(x, y, 0) = \emptyset(x, y), \forall x, y \in \Omega, \Omega \in R^2 \quad \textbf{(1b)}$$

$$u(x, y, t) = f(x, y, t), \forall x, y \in \partial\Omega \quad \textbf{(1c)}$$

Where Ω is the solution region and $\partial\Omega$ is the boundary of Ω.

In recent years, much research has been focused on the numerical solution of nonlinear partial equations by using numerical methods and developing these methods (Al-Saif, 2007; Leveque, 2006; Rossler & Husner, 1997; Wescot & Rizwan-Uddin, 2001). In the numerical methods, which are commonly used for solving these kind of equations large size or difficult of computations is appeared and usually the round-off error causes the loss of accuracy.

The Adomian decomposition method which needs less computation was employed to solve many problems (Celik et al., 2006; Javidi & Golbabai, 2007). Therefore, we applied the Adomian decomposition method to solve some models of nonlinear partial equation, this study reveals that the Adomian decomposition method is very efficient for nonlinear models, and it results give evidence that high accuracy can be achieved.

Mathematical Methodology:

The principle of the Adomian decomposition method (ADM) when applied to a general nonlinear equation is in the following form(Celik et al., 2006; Seng et al., 1996):

$$Lu + Ru + Nu = g \quad \textbf{(2)}$$

The linear terms decomposed into $Lu+Ru$, while the nonlinear terms are represented by Nu, where L is an easily invertible linear operator, R is the remaining linear part. By inverse operator L, with $L^{-1}(\cdot) = \int_0^t (\cdot) dt$. Equation (2) can be hence as;

$$u = L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu) \tag{3}$$

The decomposition method represents the solution of equation (3) as the following infinite series:

$$\mathrm{u} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{u_n} \tag{4}$$

The nonlinear operator *Nu*= $\Psi(u)$ is decomposed as:

$$\mathrm{Nu} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{A_n} \tag{5}$$

Where A_n are Adomian's polynomials, which are defined as (Seng et al., 1996):

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[\psi(\sum_{i=1}^{n} \lambda^i u_i)]_{\lambda=0} \qquad n = 0,1,2,\ldots \tag{6}$$

Substituting equations (4) and (5) into equation (3), we have

$$\mathrm{u} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{u_n} = \mathrm{u_0} - \mathrm{L}^{-1}\left(\mathrm{R}(\sum_{\mathrm{n}=0}^{\infty} \mathrm{u_n})\right) - \mathrm{L}^{-1}(\sum_{\mathrm{n}=0}^{\infty} \mathrm{A_n}) \tag{7}$$

Consequently, it can be written as:

$$\left.\begin{aligned}
\mathrm{u_0} &= \varphi + \mathrm{L}^{-1}(\mathrm{g}) \\
\mathrm{u_1} &= -L^{-1}(R(u_0)) - L^{-1}(A_0) \\
\mathrm{u_2} &= -L^{-1}(R(u_1)) - L^{-1}(A_1) \\
&. \\
&. \\
&. \\
\mathrm{u_n} &= -L^{-1}(R(u_{n-1})) - L^{-1}(A_{n-1})
\end{aligned}\right\} \tag{8}$$

Where φ is the initial condition,

Hence all the terms of u are calculated and the general solution obtained according to

ADM as $u = \sum_{n=0}^{\infty} u_n$. The convergent of this series has been proved in (Seng et al., 1996). $n = 0$

However, for some problems (Celik et al., 2006) this series can't be determined, so we use an approximation of the solution from truncated series

$$U_M = \sum_{n=0}^{M} u_n \, with \lim_{M\to\infty} U_M = u$$

Statement of the Problem:

For definiteness of physical problem, we consider here that the recharge takes place over a large basin of such geological configuration that the sides are limited by rigid boundaries while the bottom is confined by a thick layer of water table. In this circumstances, water will flow vertically downward through unsaturated porous medium. It is assumed that the diffusivity coefficient is equivalent to an average value, over the whole range of moisture content, is small enough to be regarded as a perturbation parameter. Further, the permeability of the medium is considered to vary directly with moisture content and inversely as the square root of times. The mathematical formulation of the basic hydro dynamical equation yields nonlinear partial differential equation which has been reduced to an ordinary differential equation by using similarity transformation which is derived by using one – parameter stretching transformation. The resulting ordinary differential equation, contains two small parameter, is solved by applying two parameter singular perturbation method.

Mathematical Modeling:

The equation of continuity for an unsaturated porous medium is given by

$$\frac{\partial}{\partial t}(\rho_r \theta) = \nabla \cdot \vec{M} \tag{9}$$

Where ρ_r = the bulk density of the medium

θ = The moisture content on a dry weight basis

$\vec{M}$ = The mass flux of moisture

$\nabla =$ The vector differential operator and

$t =$ Time in second.

It has long been assumed |17| and was confirmed experimentally by Childs and Collis – George |18| and other that Darcy's law may hold for the flow of liquid, water, in an unsaturated media in a modified form in which K is a function of the volumetric moisture content. The theoretical validity of this concept depends on the assumption that the drag at air - water interfaces in the soil is negligibly small |19|. Thus from Darcy's law for the flow of water in an unsaturated porous medium, including soil, in the modified form we get

$$\nabla = -k(\theta)\nabla\emptyset \tag{10}$$

Where $\emptyset$ represent the gradient of the whole moisture potential. V is the volume flux of moisture and K, the coefficient of aqueous conductivity combining equation (9) and (10) we get,

$$\frac{\partial}{\partial t}(\rho_r\theta) = \nabla(\rho K\nabla\emptyset) \tag{11}$$

Where ρ is the fluid density.

Now for an unsaturated porous media, the total potential $\emptyset$ may be regarded as comprising moisture potential, φ which has been known variously as capillary potential; capillary pressure; moisture tension; moisture suction; negative pressure head; etc., |20 – 24|, and the gravitational component. Z, the height above some datum level. Thus,

$$\emptyset = \varphi(\theta) + Z \tag{12}$$

Substituting (3.3.4) in the equation (11), we then have

$$\rho_r\frac{\partial\theta}{\partial t} = \nabla(\rho_K\nabla\varphi) + \frac{\partial}{\partial z}(\rho K g) \tag{13}$$

Recalling that the flow takes places only in the vertical direction, we may write equation (13) as

$$\rho_r\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(\rho K\frac{\partial\varphi}{\partial z}\right) - \frac{\partial}{\partial z}(\rho K g) \tag{14}$$

The positive direction of z – axis is the same as that of gravity. Considering θ and φ to be connected by a single valued function of θ, we may introduce the quantity D such that

$$D = \frac{\rho}{\rho_r}K\frac{\partial\varphi}{\partial\theta} \tag{15}$$

Which is called diffusivity coefficient and therefore we may write (14) as

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial\theta}{\partial z}\right) + \frac{\rho_g}{\rho_r}\frac{\partial K}{\partial z} \tag{16}$$

Now replacing D by its average value D_a and assuming, as in |7|,

$$K = \frac{K_o\theta}{\sqrt{t}} \tag{17}$$

Equation (16) may be written as

$$\frac{\partial\theta}{\partial t} = D_a\frac{\partial^2\theta}{\partial z^2} + \frac{\rho_g K_o}{\rho_r\sqrt{t}}\frac{\partial\theta}{\partial z} \tag{18}$$

Considering water table, to be situated at a depth 'L' and setting

$$X = {}^{z}/_{L}\,; \qquad T = {}^{t}/_{L^2}\,; \qquad M = \frac{\rho_g K_o}{\rho_r} \tag{19}$$

Solution of the Problem:

$$\frac{\partial\theta}{\partial T} = D_a\frac{\partial^2\theta}{\partial X^2} + \frac{M}{\sqrt{T}}\frac{\partial\theta}{\partial X}$$

With the initial Condition

$$\theta(0,\ T) = \theta_0$$
$$\theta(1,\ T) = 1$$

Solution: In this problem, we have

$$N\theta = \Psi(\theta) = \frac{M}{\sqrt{T}}\frac{\partial\theta}{\partial X}$$

$g(X,t) = D_a \frac{\partial^2 \theta}{\partial X^2}$

$R\theta = 0$

$L\theta = \frac{\partial \theta}{\partial T}$

And $\phi = \theta(0,T) = \theta_0$

By using equation (6) Adomain's polynomials can be derived as follows:

$$\left.\begin{aligned} A_0 &= \frac{M}{\sqrt{T}} \frac{\partial \theta_0}{\partial X} \\ A_1 &= \frac{M_1}{\sqrt{T}} \frac{\partial \theta_0}{\partial X} - \frac{M}{2T^{3/2}} \frac{\partial \theta_0}{\partial X} + \frac{M}{\sqrt{T}} \frac{\partial \theta_1}{\partial X} \\ A_2 &= \left(\frac{4T^2M_2 - 3TM_1 + 3M}{4T^{5/2}}\right) \frac{\partial \theta_0}{\partial X} + \left(\frac{2TM_1 + \sqrt{T}M_1 - 2M}{2T^{3/2}}\right) \frac{\partial \theta_1}{\partial X} + \frac{M}{\sqrt{T}} \frac{\partial \theta_2}{\partial X} \\ &\vdots \\ &\vdots \end{aligned}\right\} \tag{20}$$

And so on. The rest of the polynomials can be constructed in similar manner.

By using Equation (8), we have

$\theta_{(0)} = \theta_0$

$\theta_{(1)} = -2M\sqrt{T} \frac{\partial \theta_0}{\partial X}$

$\theta_{(2)} = -\left[\left(\frac{2M_1T - M}{\sqrt{T}}\right) \frac{\partial \theta_0}{\partial X} + 2M\sqrt{T} \frac{\partial \theta_1}{\partial X}\right]$

$\theta_{(3)} = -\left[\left(\sqrt{T}M_2 - \frac{3}{4}\frac{M_1}{\sqrt{T}} + \frac{3}{4}\frac{M}{T^{3/2}}\right) \frac{\partial \theta_0}{\partial X} + \left(M_1\sqrt{T} + \frac{M_1}{2} \log T - \frac{M}{\sqrt{T}}\right) \frac{\partial \theta_1}{\partial X} + M\sqrt{T} \frac{\partial \theta_2}{\partial X}\right]$

$\vdots$

$\vdots$

Substituting these individual terms in equation (4) obtain

$$\theta(\xi,t) = \theta_0 - (8MT^2 + 8M_1T^2 - 4TM + 4M_2T^2 - 3M_1T + 3M)\frac{1}{4T^{3/2}} \frac{\partial \theta_0}{\partial X} - \left(4MT + 2M_1T + M_1\sqrt{T}\log T - 2M\right) \frac{\partial \theta_1}{\partial X} - M\sqrt{T} \frac{\partial \theta_2}{\partial X} - \cdots$$

Table 1

		T	$\boldsymbol{\theta(\xi,t)}$			T	$\boldsymbol{\theta(\xi,t)}$
$M = 1.2$	$\frac{\partial \theta_0}{\partial X} = 20$	1	21.1620	$M = 1$	$\frac{\partial \theta_0}{\partial X} = 20$	1	21.2833
		2	16.0867			2	19.3231
		3	10.8395			3	17.6079
$M_1 = 0.9$	$\frac{\partial \theta_1}{\partial X} = 1$	4	4.5109	$M_1 = 0.9$	$\frac{\partial \theta_1}{\partial X} = 1$	4	15.4090
		5	-3.0365			5	12.5864
		6	-11.8004			6	9.1141
		7	-21.7429			7	4.9990
$M_2 = 1$	$\frac{\partial \theta_2}{\partial X} = 2$	8	-32.8181	$M_2 = 0.8$	$\frac{\partial \theta_2}{\partial X} = 2$	8	0.2585
		9	-44.9798			9	-5.0870
		10	-58.1847			10	-11.0168

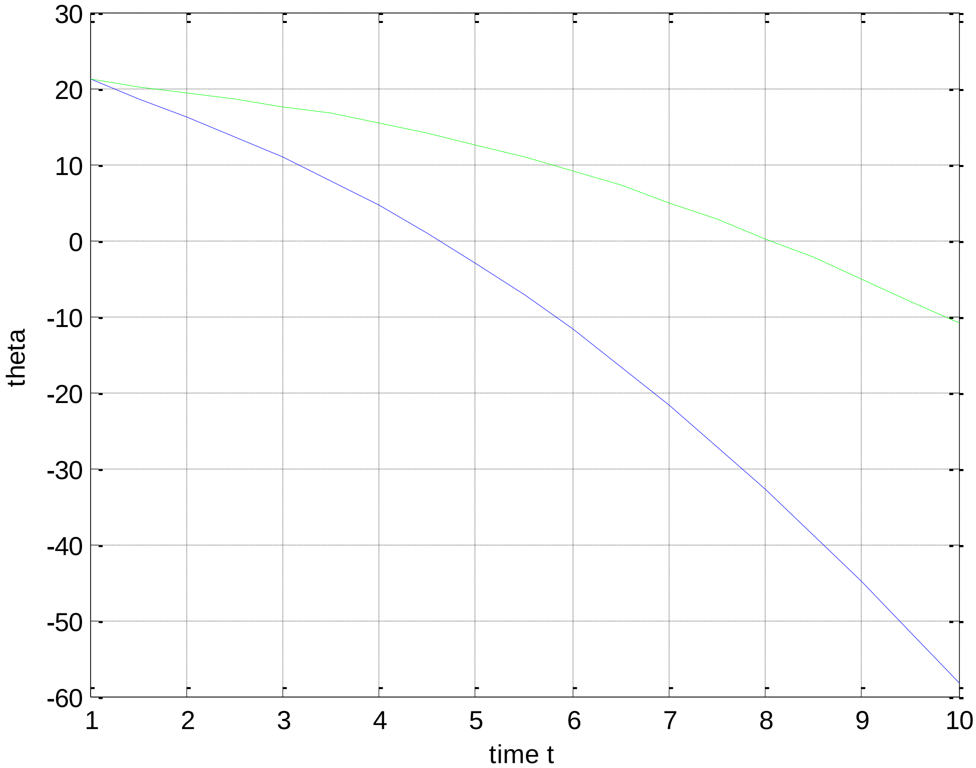

Figure 1. Time (T) → moisture(θ)

II. CONCLUSION

In this paper, we have discussed adomain decomposition method for the case of Instability phenomena and studied its saturation value in T and moisture(θ). The solution of $\theta(\xi,t)$ gives the distribution of moisture in unsaturated porous media and gives a uniformly valid asymptotic solution. From figure (1), it is clear that partial derivative fixed at each point and the mass flux of moisture are taking different fixed value, changing the time than get moisture content θ is decreasing at each point .

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