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Optimal Pricing, Quality Investment and Replenishment Policies for Perishable Items

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ABSTRACT

This paper discusses optimality of replenishment cycle time, selling price, quality investment cost and order quantity with simultaneous deterioration of quality and physical quantity. Most of models assume the quality of items does not decay before their expiration dates. In practice, the quality of many products often deteriorate over time, applying quality investment makes us able to maintain freshness of items. The quality of an item usually plays an important role in influencing factor of demand for product. Demand rate is dependent on the selling price and the quality of a product. The model seeks to maximize the total profit of the model by determining the optimality of decision variable. In order to analyse the behaviour of the model and numerical example accompanied by sensitivity analyses of key parameters of the model are provided.

Keywords : Inventory, pricing, replenishment policy, quality deterioration, physical deterioration, quality investment.

I. INTRODUCTION

E-commerce has become one of the vital parts of the modern life. Online payment is the supportive application for the payment of money for the products we buy. For the past years online security breach created a major problem and lots of money had been stolen. The proposed document deals by securing the payment through iris recognition [1]. This method also adds the method of using visual cryptography for securing the user credentials. This visual cryptography method was formerly invented by Moni Naor and Adi Shamir in 1994[6].

Most of the products deteriorate in nature. The deterioration of many items during storage period is a real fact but deterioration involves both quality and physical quantity. Quality of product starts to deterioration over time instantaneously. This reduction of quality level does not affect the physical quantity for a while. After a specific point of time, goods start to lose their utility due to quantity drops and subsequently non-instantaneous quantity deterioration stars. Freshness is a vital element of the produce has a significant impact on consumers purchase decision. In this direction, Qin et al (2014) formulated a model of the pricing and lot-sizing problem for products with quality and physical quantity deteriorating simultaneously, Jaggi et al (2015) presented an effect of deterioration on two-warehouse inventory model with imperfect quality, Panda and Modak (2016) provided an inventory model for imperfect quality items under demand fluctuation, Hsiao et al (2017) formulated a model to generate distribution plan for fulfilling customer requirements for various foods with pre-appointed quality levels at the lowest distribution cost and recently, Dey and Mukherjee (2018) presented an integrated single-vendor single-buyer production inventory model with stochastic demand, quality control and imperfect production process is investigated .

Deterioration rate of perishable items can be controlled and reduced through various efforts like procedural changes and specialized equipment acquisition. Therefore, according to the realistic situation, can make decisions on whether some quality investment could be adopted to reduce the deterioration rate. Xie et al (2011) investigated quality investment and price decision of a make-to-order supply chain with uncertain demand in international trade , Giovanni (2011) consider a marketing channel with a single manufacturer and a single retailer, where both advertising and quality improvement contribute to the built-up of goodwill, Jindal and Solanki (2016) presented quality improvement of defective items in the integrated vendor and buyer supply chain inventory model with backorder price discount involving controllable lead time, Vijayashree and Uthayakumar (2016) consider a vendor and buyer integrated production inventory model under quality improvement investment, Yu and Chen (2017) formulated a single vendor single-retailer production-inventory model for products with imperfect quality under the retailer gives a warranty policy of non-renewing free replacement for failure of perfect items within a warranty period and makes a quality improvement investment in defective percentage and Feng(2018) presented an optimal replenishment model with dynamic pricing and quality investment for perishable products, where the quality and physical quantity deteriorate simultaneously.

Researchers have taken the product quality as an important influential factor of demand. Most of researchers taking price dependent demand but with that some another factors are also effect the demand of the product like stock, quality, promotion effort, time etc. As Hsu at al (2010), Hsieh and Dye (2013), Herbon et al (2014), Liu et al (2015), Avinadav et al (2016), Bardhan et al (2017) and Shah et al(2018) adopted different demand patterns.

Inspired by the significance of the problem and related research gaps, this paper provides an optimal decision variable with simultaneous quality and quantity deterioration. Since quality of product deteriorates over time, applying quality investment cost makes us able to maintain freshness of items as long as possible. In addition, changes in price, demand rate is dependent on quality of product as well.

The remainder of the paper is structured as follows. In section 2, notations and assumptions of the problem are described. Section 3 represents the formulated mathematical model. Section 4 provides the numerical example and sensitivity analyses. Finally section 5 finishes the paper with conclusion and recommended future research directions.

II. Notations and Assumptions

Decision variables	
T	Cycle length time per unit
p	Selling price
ξ	Quality investment cost
Q	Order quantity

Parameters	
θ	Deterioration rate
h	holding cost
c	Purchasing cost
O	Ordering cost

Variables

α	deterioration rate
$f(\xi)$	Reduced decay rate of quality
$N(t)$	Quality level at time
$I(t)$	Inventory level during the period
$D(p,N(t))$	Demand rate
$\pi(T,p,\xi)$	Retailer's unit time profit

Assumptions

1. The inventory system involves single deteriorating item.
2. Demand rate $D(p,N(t))$ is a function of the selling price p and quality level $N(t)$ as

$$D(p,N(t)) = d(p) + N(t) = (x - yp) + \left(e^{-(\alpha - f(\xi))t}\right) \quad (1)$$

Where x, y and α are positive constant.

3. Deterioration involves both quality and physical quantity. Qualitative deterioration is instantaneous where physical deterioration follows non-instantaneous pattern.
4. The instantaneous deterioration rate of quality of inventory is

$$f(\xi) = \alpha\left(1 - e^{-a\xi}\right) \quad (2)$$

Where α and a are positive constant.

5. Replenishment rate is instantaneously infinite and lead time is zero.
6. Shortages are not allowed.

Model formulation

The inventory system evolves as follow: Accordingly, units of items arrive at the inventory system at the beginning of each cycle. The inventory level decline to zero owing to demand and two types of deterioration qualitative and quantitative. The qualitative deterioration rate is reduced by quality investment. The pattern of inventory level is depicted in figure.

During the time interval $[0,T]$ the quality status $N(t)$ starts to decrease over time instantaneously. The quality status is represented by the following differential equation.

$$\frac{dN(t)}{dt} = -(\alpha - f(\xi))N(t) \quad 0 \le t \le T \quad (3)$$

With boundary condition $N(0)=1$ solving equation (1) yields

$$N(t) = e^{-(\alpha - f(\xi))t} \quad (4)$$

The inventory level declines due to demand and physical deterioration during time interval $[0,T]$. Based on this description, the inventory status is represented by the following differential equation.

$$\frac{dI(t)}{dt} = -\theta I(t) - D(p,t) \quad (5)$$

With boundary condition $I(T)=0$ solving equation (4) yields:

$$I(t) = \frac{\left(d(p) + N(t,\xi)\right)\left(e^{-\theta t} - e^{-\theta T}\right)}{\theta e^{-\theta T}} \quad (6)$$

The seller's order quantity is

$$Q = I(0) = \frac{\left(d(p)+1\right)\left(1 - e^{-\theta T}\right)}{\theta e^{-\theta T}} \quad (7)$$

The components of total profit of the inventory system are defined as follows:

1. SR: The sales revenue

$$SR = p\int_0^T D(p,t)\,dt = p\int_0^T \left(d(p) + N(t,\xi)\right)dt \quad (8)$$

2. OC: The ordering cost

$$OC = O \quad (9)$$

3. HC : The inventory holding cost

$$HC = h\int_0^T I(t)\,dt$$

$$= h\int_0^T \frac{\left(d(p)+N(t,\xi)\right)\left(e^{-\theta t}-e^{-\theta T}\right)}{\theta e^{-\theta T}}dt \tag{10}$$

4. PC: The purchasing cost

$$PC = CI(0)$$

$$= \frac{C\left(d(p)+1\right)\left(1-e^{-\theta T}\right)}{\theta e^{-\theta T}} \tag{11}$$

5. QIC: The quality investment cost

$$QIC = T\xi \tag{12}$$

Therefore, the total profit per time unit $\pi(T,p,\xi)$ is given by:

$$\pi(T,p,\xi) = \frac{1}{T}\left(SR-OC-HC-PC-QIC\right)$$

$$= \frac{p\int_0^T \left(d(p)+N(t,\xi)\right)dt}{T} - \frac{O}{T} - \frac{h\int_0^T \left(d(p)+N(t,\xi)\right)\left(e^{-\theta t}-e^{-\theta T}\right)dt}{T\theta e^{-\theta T}} - \frac{C\left(d(p)+1\right)\left(1-e^{-\theta T}\right)}{\theta e^{-\theta T}} - T\xi \tag{13}$$

Numerical example and Sensitivity analysis

To provide a better understanding of the model and the solution procedure described above, we consider $N(t,\xi) = e^{-(\alpha - f(\xi))t}$ where $f(\xi) = \alpha\left(1-e^{-a\xi}\right)$. The algorithm was implemented in Maple 17.

Example 1. In this example we consider the linear function $d(p) = x - yp$, $x = 100$, $y = 0.9$, $C = \$35/$ unit, $O = \$80/$ order, $a = 0.1$, $\alpha = 2$, $h = 5$, $\theta = 0.01$. We have the optimal solution as follow: $T^* = 0.8862$, $p^* = 74.71$, $\xi = 16.44$ and $\pi^* = 1142.45$.

Example 2. We investigate the effects of changes in the value of scaling parameter (a) and constant parameter (α) on optimal solution. The identical set of input data are in Example 1. The optimal solution for different values of (a) and (α) is summarized in Table 1.

Table 1 Computational results for different values of a and α.

a	T	p	ξ	Π	Q
0.06	0.852217	74.59236	14.05886	1134.222923	28.98527
0.08	0.873296	74.67173	16.60134	1138.742872	29.64266
0.	0.886249	74.71486	16.44186	1142.450619	30.04974
0.12	0.895009	74.74249	15.68407	1145.385169	30.32572
0.14	0.901326	74.7618	14.81069	1147.737511	30.525
α	T	p	ξ	Π	Q
1.96	0.88624813	74.71486	16.2398	1142.65	30.04970
1.98	0.88624887	74.71485	16.3413	1142.55	30.04973
2	0.88624887	74.71485	16.4418	1142.45	30.04973
2.02	0.88624873	74.71485	16.5413	1142.35	30.04972
2.04	0.88624883	74.71485	16.6398	1142.25	30.04973

From Table 1, when the value of parameter (a) increases while optimal total profit (Π), optimal cycle time (T), optimal selling price (p) and optimal order quantity (O) increases but first of all optimal preservation technology cost (ξ) increases and then decreases.

On the other hand, as the value of parameter (α) increases, optimal preservation technology cost (ξ) increases but optimal cycle time (T) and optimal order quantity (O) first increases then decreases while optimal selling price (p) and optimal total profit (Π) decreases.

Example 3 In this example, we study the effect of constant parameter (x) and scaling parameter (y) on the optimal solution. The same set of input data are considered as in Example 1. Computational results are summarized in Table 2.

Table 2 Computational results for different values of x and y .

x	T	p	ξ	Π	Q
80	1.0488	63.80434	15.9138	508.9866	24.8581
90	0.9569	69.24618	16.1695	797.3534	27.5745
100	0.8862	74.71486	16.4418	1142.45	30.0497
110	0.8296	80.20111	16.7167	1544.02	32.3396
120	0.7829	85.69963	16.9877	2001.91	34.4813
y	T	p	ξ	Π	Q
0.54	0.8128	112.018	20.5916	2901.96	33.0609
0.72	0.8469	88.6876	18.1849	1786.22	31.5946
0.9	0.8862	74.7148	16.4418	1142.45	30.0497
1.08	0.9320	65.4265	15.1335	734.802	28.4112
1.26	0.9867	58.8218	14.1452	462.275	26.6588

In Table 2, we can observe that the optimal total profit (Π), optimal preservation technology cost (ξ), optimal selling price (p) and optimal order quantity (O) increase with an increase while optimal cycle time (T) decrease with an increase in the value of constant parameter (x). These results imply that as constant parameter (x) of the demand function increases with the total profit increases drastically.

On the other hand, we can observe that the optimal cycle time (T) increase with an increase in the value of parameter (y). Apparently, the reduction in the value of optimal total profit (Π), optimal preservation technology cost (ξ), optimal selling price (p) and optimal order quantity (O) found with an increase in the value of scaling parameter (y). Obviously, the demand rate decreases as the value of (y) increases. This result reveals that scaling parameter (y) increases in demand function which decreases the demand. So, there will be decrease in total profit.

Example 4. This example presents the impact of deterioration rate (θ) on optimal solution. We considered the same set of input data as in

Example 1. The optimal solution for different values of (θ) is summarized in Table 3.

Table 3 Computational results for different values of θ.

θ	T	p	ξ	Π	Q
0.006	0.8981	74.6979	16.5692	1144.65	30.4156
0.008	0.8921	74.7064	16.5051	1143.55	30.2309
0.01	0.8862	74.7148	16.4418	1142.45	30.0497
0.012	0.8804	74.7232	16.3793	1141.35	29.8718
0.014	0.8747	74.7315	16.3175	1140.26	29.6971

In Table 3, when a higher value of deterioration rate (θ) causes higher value of optimal selling price (p) while a lower value of optimal total profit (Π), optimal cycle time (T), optimal preservation technology cost (ξ) and optimal order quantity (O).

III. CONCLUSION

In this paper, we optimize the replenishment cycle time, the selling price, the preservation technology and the order quantity. The demand rate was dependent on the quality level of the product as well as selling price. Most of researchers focused on physical deterioration but we gave attention on quality deterioration of the product in this paper. To reduce this deterioration we use quality investment in it. Next, numerical example and sensitivity analyses of key parameters of the model were provided. Computational results indicates the total profit is more sensitive with respect to demand function parameter. For future research, this model can be extended to accommodate promotional effort cost, a shortage and so forth.

IV. REFERENCES

[1]. Avinadav, T., Chernonog, T., Lahav, Y., Spiegel, U. (2016) Dynamic pricing and promotion

[2]. expenditures in an EOQ model of perishable products. Annals of Operations Research. 248(1-2):1–17.

[3]. Bardhan, S., Pal, H., Giri, B.C. (2017) Optimal replenishment policy and preservation technology

[4]. investment for a non-instantaneous deteriorating item with stock-dependent demand. Operational Research, 17:1–22.

[5]. Dey O, Mukherjee A. (2018) An integrated imperfect production inventory model with lot-size-

[6]. Dependent lead-time and quality control. Springer Proceedings in Mathematics & Statistics. 225:303-314.

[7]. Feng L. (2018) Dynamic pricing, quality investment and replenishment model for perishable items.

[8]. International Transactions in Operational Research, https://doi.org/10.1111/itor.12505.

[9]. Giovanni P. D. (2011) Quality improvement vs. advertising support: Which strategy works better for

[10]. a manufacturer? European Journal of Operational Research. 208(2):119-130.

[11]. Herbon A., Levner E., Cheng T. C. (2014) Perishable inventory management with dynamic pricing

[12]. Using time-temperature indicators linked to automatic detecting devices. International Journal of Production Economics. 147:605-613.

[13]. Hsieh T. P., Dye C. Y. (2013) A production-inventory model incorporating the effect of preservation

[14]. technology investment when demand is fluctuating with time. Journal of Computational and Applied Mathematics. 239:25-36.

[15]. Hsiao Y. H., Chen M. C., Chin C. L. (2017) Distribution planning for perishable foods in cold chains

[16]. With quality concerns: Formulation and solution procedure. Trends in Food Science & Technology. 61:80-93.

[17]. Hsu P H., Wee H. M., Teng H. M. (2010) Preservation technology investment for deteriorating

[18]. Inventory. International Journal of Production Economics. 124(2):388-394.

[19]. Jaggi C. K., Tiwari S., Shafi A. (2015) Effect of deterioration on two-warehouse inventory model

[20]. with imperfect quality. Computers and Industrial Engineering. 88:378-385.

[21]. Jindal P., Solanki A. (2016) Integrated supply chain inventory model with quality improvement

[22]. involving controllable lead time and backorder price discount. International Journal of

[23]. Industrial Engineering Computations. 7:463-480.

[24]. Liu G, Zhang J., Tang W. (2015) Joint dynamic pricing and investment strategy for perishable foods

[25]. With price-quality dependent demand. Annals of Operations Research. 226(1):397-416.

[26]. Panda S., Modak N. M. (2016) An inventory model for random replenishment interval and imperfect

[27]. quality items under demand fluctuation. International Journal of Supply Chain and Inventory Management. 1(4):269.

[28]. Qin Y, Wang J., Wei C. (2014) Joint pricing and inventory control for fresh produce and foods with

[29]. Quality and physical quantity deteriorating simultaneously. International Journal of Production Economics. 152:42-48.

[30]. Shah N. H., Chaudhari U., Jani M. Y. (2018) Optimal control analysis for service, inventory and

[31]. preservation technology investment. International Journal of Systems Science: Operations & Logistics. https://doi.org/10.1080/23302674.2018.1447167

[32]. Vijayashree M., Uthayakumar R. (2016) An optimizing integrated inventory model with investment

[33]. For quality improvement and setup cost reduction. International Journal of Information Technology, Control and Automation. 6(3/4):1-13.

[34]. Xie G, Yue W., Wang S., Lai K. K. (2011) Quality investment and price decision in a risk-averse

[35]. supply chain. European Journal of Operational Research. 214(2):403-410.

[36]. Yu H F., Chen Y. (2017) An integrated inventory model for items with acceptable defective items

[37]. Under warranty and quality improvement investment. Quality Technology & Quantitative Management. 1-14.