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SPECIAL PAIRS OF SURD EQUATIONS

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ABSTRACT

Three special systems of double equations involving surds are considered to obtain their corresponding solutions in real numbers.

Keywords : System of indeterminate quadratic equations, pair of quadratic equations, system of double quadratic equation, irrational solutions.

I. INTRODUCTION

The pairs of quadratic equations of the form $ax+c=u^2$, $bx+d=v^2$ where a, b, c, d are non-zero distinct constants, have been investigated for solutions by several authors [1, 2] and with a few possible exceptions, most of the them were primarily concerned with rational solutions. Even those existing works wherein integral solutions have been attempted, deal essentially with specific cases only and do not exhibit methods of finding integral solutions in a general form. In [3], a general form of the integral solutions to the system of equations $ax+c=u^2$, $bx+d=v^2$ where a, b, c, d are non-zero distinct constants is presented when the product ab is a square free integer whereas the product cd may or may not a square integer. For other forms of system of double diophantine equations, one may refer [4-11].

In the above references, the equations are polynomial equations with integer coefficients which motivated us to search for solutions to system of equations with surds. This communication concerns with the problem of obtaining solutions a, b in real numbers satisfying each of the system of double equations with surds represented by

i) $a\sqrt{a}+b\sqrt{b}=\alpha^2$, $a\sqrt{b}+b\sqrt{a}=\beta^2$

ii) $a\sqrt{a}+b\sqrt{b}=\alpha^2+\beta^2$, $a\sqrt{b}+b\sqrt{a}=\beta^2$

iii) $a\sqrt{a}+b\sqrt{b}=N^2$, $a\sqrt{b}+b\sqrt{a}=3k^2+2kN$

where N is an integer.

II. METHOD OF ANALYSIS

System: 1

Let a, b be two positive distinct real numbers and N be a positive integer such that

$$a\sqrt{a}+b\sqrt{b}=\alpha^2$$

(1.1)

$$a\sqrt{b}+b\sqrt{a}=\beta^2$$

(1.2)

Consider the identity

$$\left(\sqrt{a}+\sqrt{b}\right)^3=\left(a\sqrt{a}+b\sqrt{b}\right)+3\sqrt{ab}\,\left(\sqrt{a}+\sqrt{b}\right)$$

(1.3)

The substitution of (1.1), (1.2) in (1.3) gives

$$\left(\sqrt{a}+\sqrt{b}\right)^3=\alpha^2+3\beta^2 \qquad (1.4)$$

Assume

$$\sqrt{a}+\sqrt{b}=p^2+3q^2 \quad (1.5)$$

Substituting (1.5) in (1.4) and employing the method of factorization, define

$$\left(p+i\sqrt{3}q\right)^3\left(p-i\sqrt{3}q\right)^3=\left(\alpha+i\sqrt{3}\beta\right)\left(\alpha-i\sqrt{3}\beta\right)$$

Equating the real and imaginary parts, we get

$$\alpha=p^3-9pq^2 \ , \ \beta=3p^2q-3q^3$$

From (1.2), note that

$$\sqrt{ab}=\frac{\left(3p^2q-3q^3\right)^2}{p^2+3q^2}$$

Employing the identity

$$\left(\sqrt{a}-\sqrt{b}\right)^2=\left(\sqrt{a}+\sqrt{b}\right)^2-4\sqrt{ab} \quad (1.6)$$

$$\Rightarrow \sqrt{a}-\sqrt{b}=\frac{1}{\sqrt{p^2+3q^2}}\sqrt{\left(p^2+3q^2\right)^3-4\left(3p^2q-3q^3\right)^2} \quad (1.7)$$

From (1.5) and (1.7), it is obtained that

$$a=\frac{1}{4\left(p^2+3q^2\right)}\left(\begin{array}{l}\left(p^2+3q^2\right)\sqrt{p^2+3q^2}\\ +\sqrt{\left(p^2+3q^2\right)^3-4\left(3p^2q-3q^3\right)^2}\end{array}\right)^2$$

$$b=\frac{1}{4\left(p^2+3q^2\right)}\left(\begin{array}{l}\left(p^2+3q^2\right)\sqrt{p^2+3q^2}\\ -\sqrt{\left(p^2+3q^2\right)^3-4\left(3p^2q-3q^3\right)^2}\end{array}\right)^2$$

System: 2

Let a, b be two positive distinct real numbers and N be a positive integer such that

$$a\sqrt{a}+b\sqrt{b}=\alpha^2+\beta^2 \quad (2.1)$$

$$a\sqrt{b}+b\sqrt{a}=\beta^2 \quad (2.2)$$

The substitution of (2.1), (2.2) in (1.3) gives

$$\left(\sqrt{a}+\sqrt{b}\right)^3=\alpha^2+4\beta^2 \quad (2.3)$$

Assume

$$\sqrt{a}+\sqrt{b}=p^2+4q^2 \quad (2.4)$$

Substituting (2.4) in (2.3) and employing the method of factorization, define

$$(p+i2q)^3(p-i2q)^3=(\alpha+i2\beta)(\alpha-i2\beta)$$

Equating the real and imaginary parts, we get

$$\alpha=p^3-12pq^2 \ , \ \beta=3p^2q-4q^3$$

From (2.2), note that

$$\sqrt{ab}=\frac{\left(3p^2q-4q^3\right)^2}{p^2+4q^2}$$

The identity (1.6) leads to

$$\sqrt{a}-\sqrt{b}=\frac{1}{\sqrt{p^2+4q^2}}\sqrt{\left(p^2+4q^2\right)^3-4\left(3p^2q-4q^3\right)^2} \quad (2.5)$$

From (2.4) and (2.5), it is obtained that

$$a=\frac{1}{4\left(p^2+4q^2\right)}\left(\begin{array}{l}\left(p^2+4q^2\right)\sqrt{p^2+4q^2}\\ +\sqrt{\left(p^2+4q^2\right)^3-4\left(3p^2q-4q^3\right)^2}\end{array}\right)^2$$

$$b=\frac{1}{4\left(p^2+4q^2\right)}\left(\begin{array}{l}\left(p^2+4q^2\right)\sqrt{p^2+4q^2}\\ -\sqrt{\left(p^2+4q^2\right)^3-4\left(3p^2q-4q^3\right)^2}\end{array}\right)^2$$

System: 3

Let a, b be two positive distinct real numbers and N be a positive integer such that

$$a\sqrt{a}+b\sqrt{b}=N^2 \quad (3.1)$$

$$a\sqrt{b}+b\sqrt{a}=3k^2+2kN \quad (3.2)$$

The substitution of (3.1), (3.2) in (1.3) gives

$$\left(\sqrt{a}+\sqrt{b}\right)^3=(N+3k)^2 \quad (3.3)$$

Assume $N=(s+1)^3-3k$ (3.4)

Substituting (3.4) in (3.3), we get

$$\sqrt{a}+\sqrt{b}=(s+1)^2 \quad (3.5)$$

From (3.2), note that

$$\sqrt{ab}=\frac{2k(s+1)^3-3k^2}{(s+1)^2}$$

The identity (1.6) gives

$$\sqrt{a}-\sqrt{b}=\frac{1}{s+1}\sqrt{(s+1)^6-8k(s+1)^3+12k^2} \quad (3.6)$$

From (3.5) and (3.6), it is obtained that

$$a=\frac{1}{4(s+1)^2}\left(\begin{array}{l}2(s+1)^6-8k(s+1)^3+12k^2\\ +2(s+1)^3\sqrt{(s+1)^6-8k(s+1)^3+12k^2}\end{array}\right)$$

$$b=\frac{1}{4(s+1)^2}\left(\begin{array}{l}2(s+1)^6-8k(s+1)^3+12k^2\\ -2(s+1)^3\sqrt{(s+1)^6-8k(s+1)^3+12k^2}\end{array}\right)$$

REFERENCES

[1] L.E. Dickson, History of the theory of Numbers, Vol.II, Chelsea publishing company, New York, 1952.

[2] B. Batta and A.N. Singh, History of Hindu Mathematics, Asia Publishing House, 1938.

[3] M. Mignotte, A. Petho, On the system of Diophantine equations $x^2-6y^2=-5$, $x=az^2-b$, Mathematica Scandinavica, 76(1), 50-60, (1995).

[4] M.A. Gopalan, S. Vidhyalakshmi and K. Lakshmi, On the system of double equations $4x^2-y^2=z^2$, $x^2+2y^2=w^2$, Scholars Journal of Engineering and Technology (SJET), 2(2A), 2014, 103-104.

[5] M.A. Gopalan, S. Vidhyalakshmi and R. Janani, On the system of double Diophantine equations $a_0+a_1=q^2$, $a_0a_1\pm 2(a_0+a_1)=p^2-4$, Transactions on Mathematics™, 2(1), 2016, 22-26.

[6] M.A. Gopalan, S. Vidhyalakshmi and A. Nivetha, On the system of double Diophantine equations $a_0+a_1=q^2$, $a_0a_1\pm 6(a_0+a_1)=p^2-36$, Transactions on Mathematics™, 2(1), 2016, 41-45.

[7] M.A. Gopalan, S. Vidhyalakshmi and E. Bhuvaneswari, On the system of double Diophantine equations $a_0+a_1=q^2$, $a_0a_1\pm 4(a_0+a_1)=p^2-16$, Jamal Academic Research Journal, Special Issue, 2016, 279-282.

[8] K. Meena, S. Vidhyalakshmi and C. Priyadharsini, On the system of double Diophantine equations $a_0+a_1=q^2$, $a_0a_1\pm 5(a_0+a_1)=p^2-25$, Open Journal of Applied & Theoretical Mathematics (OJATM), 2(1), 2016, 08-12.

[9] M.A. Gopalan, S. Vidhyalakshmi and A. Rukmani, On the system of double Diophantine equations $a_0-a_1=q^2$, $a_0a_1\pm(a_0-a_1)=p^2+1$, Transactions on Mathematics™, 2(3), 2016, 28-32.

[10] S. Devibala, S. Vidhyalakshmi, G. Dhanalakshmi, On the system of double equations $N_1-N_2=4k+2\,(k\succ 0)$, $N_1N_2=(2k+1)\alpha^2$, International Journal of Engineering and Applied Sciences (IJEAS), 4(6), 2017, 44-45.

[11] Dr. M.A. Gopalan, Dr. S. Vidhyalakshmi and S. Aarthy Thangam, Systems of double diophantine equations, KY Publications, Guntur, 2018.