

# Exact and Numerical Solution of ordinary differential equations using Highly Improved Euler's method and MATLAB

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## ABSTRACT

In this paper investigated about the general first order differential equation  $dy/dx = f(x,y)$  with the initial condition  $y(x_0)=y_0$  and introduce Highly Improved Euler's Method using MATLAB programming to solve the first order differential equation. And the Euler's solutions are compared with exact solution. And the errors are rectified using Calculating Error of the Approximation Method. Finally the results are reported as graph using MATLAB coding.

**Keywords :** Ordinary differential equation, Exact solution, Range, MATLAB coding.

## I. INTRODUCTION

In Mathematics, an ordinary differential equation is a differential equation containing one or more independent variable and its derivative. Many differential equations cannot be solved using symbolic computation for practical purpose, however such as in engineering a numeric approximation to the solution is often sufficient. In this paper we used the concepts of numerical methods for ordinary differential equation. It is the method used to find numerical approximation to the solution of ordinary differential Equation.

The Term "Numerical Analysis" implies that the goal is to obtain an accurate, but in most Cases approximate numerical solution to the problem under Consideration. The terms "Numerical Analysis", Numerical methods", Numerical Techniques", and Scientific Computing" each suggest a slightly different emphasis on the scale of the problem, the type of computing resources available, and the extent to

which one is interested in "how it works" as compared to "why it works" and "when it works".

The Computational tools available for numerical computation today range from paper and pencil, handheld Calculators, desktop personal Computers, and so on up to incredibly powerful super computers, some with parallel processing capabilities. There are also professionally developed software packages for solving many types of numerical problems. In this text MATLAB is used as the computing environment for several reasons numerical methods presented as computer programs written in MATLAB are essentially as easy to read and understand as those presented as algorithms or pseudo code. MATLAB has very convenient graphing capabilities. MATLAB has high qualities built in function for all of the most important numerical methods.

## EULER METHOD

To find the solution of the given ordinary differential equation using Euler method  $y' = xy$

**ANALYTICAL SOLUTION**

$$y' = xy$$

$$\frac{dy}{dx} = xy$$

$$\frac{dy}{y} = xdx$$

$$\int \frac{dy}{y} = \int xdx$$

$$y = e^{x^2/2}$$

**CALCULATING ERROR OF THE APPROXIMATION METHOD**

| Exact Solution – Euler Solution |

**EULER METHOD SOLUTION USING MATLAB CODING**

```
function []=euler(f,x0,y0,xn,n)
h=(xn-x0)/n;
x=x0;
y1=y0;
disp('x y')
for i=1:n
y1=y1+h*f(x,y1);
x=x+h;
disp([x y1])
end
disp('euler method:')
disp([y1])
plot[x y1]
end
```

Figure 1

**MODIFIED EULER METHOD USING MATLAB CODING**

```
function []=Meuler(f,x0,y0,xn,n)
h=(xn-x0)/n;
x=x0;
y3=y0;
disp('x y')
for i=1:n
y3=y3+h*f(x+0.5*h,y3+0.5*h*f(x,y3));
x=x+h;
disp([x y3])
end
disp('euler modified method:')
disp('MEM')
disp(y3)
plot[x y3]
end
```

Figure 2

**IMPROVED EULER METHOD USING MATLAB CODING**

```
function []=Ieuler(f,x0,y0,xn,n)
h=(xn-x0)/n;
x=x0;
y2=y0;
disp(' x y')
for i=1:n
y2=y2+(0.5*h*(f(x,y2)+f(x+h,y2+h*f(x,y2))));
x=x+h;
disp([x y2])
end
disp('euler improved method:')
disp('IEM')
disp(y2)
plot[x y2]
end
```

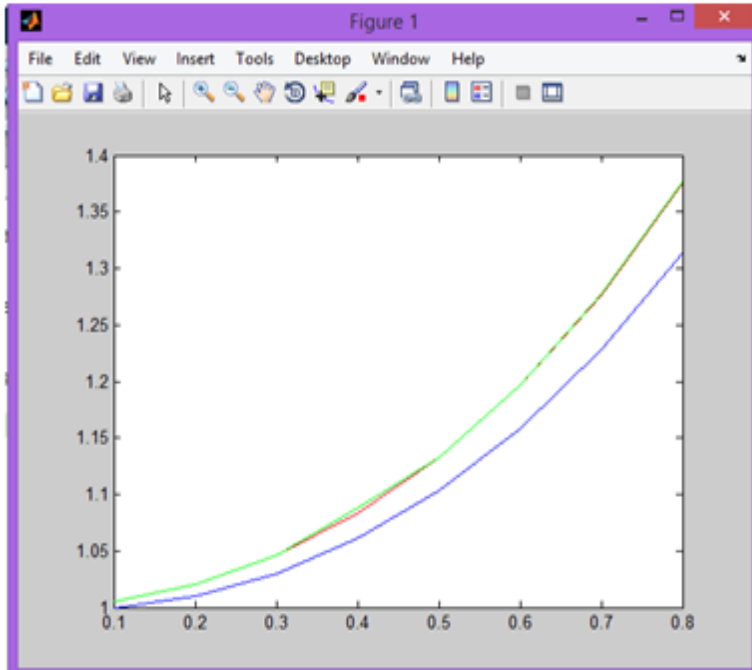
Figure 3

**FIRST ORDER ORDINARY DIFFERENTIAL EQUATION SOLUTION FOR EULER, MODIFIED, IMPROVED METHOD AND EXACT SOLUTION**

Table 1

| X   | Euler Method | Modified Euler Method | Improved Euler Method | Exact Solution | Calculating error of approximation method |
|-----|--------------|-----------------------|-----------------------|----------------|---|
| 0.1 | 1.0000       | 1.0050                | 1.0050                | 1.0050         | 0.005                                     |
| 0.2 | 1.0100       | 1.0202                | 1.0202                | 1.0202         | 0.0102                                    |
| 0.3 | 1.0302       | 1.0459                | 1.0460                | 1.0460         | 0.0158                                    |
| 0.4 | 1.0611       | 1.0831                | 1.0832                | 1.0833         | 0.0222                                    |
| 0.5 | 1.1036       | 1.1328                | 1.1331                | 1.1331         | 0.0295                                    |
| 0.6 | 1.1587       | 1.1966                | 1.1971                | 1.1972         | 0.0385                                    |
| 0.7 | 1.2283       | 1.2768                | 1.2774                | 1.2776         | 0.0493                                    |
| 0.8 | 1.3142       | 1.3759                | 1.3768                | 1.3771         | 0.0629                                    |

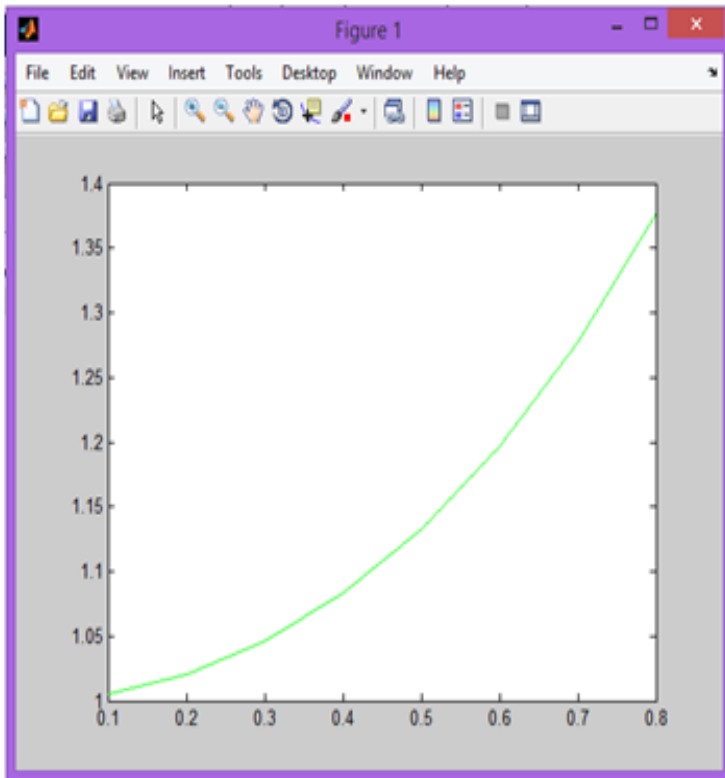
### MATLAB GRAPH FOR EULER, MODIFIED, IMPROVED METHOD



\_ Euler Method  
\_ Improved Euler Method  
\_ Modified Euler Method

Figure 4

### MATLAB GRAPH FOR EXACT SOLUTION



\_ Exact Solution

Figure 5

### COMPARING SOLUTION BETWEEN EULER'S AND EXACT

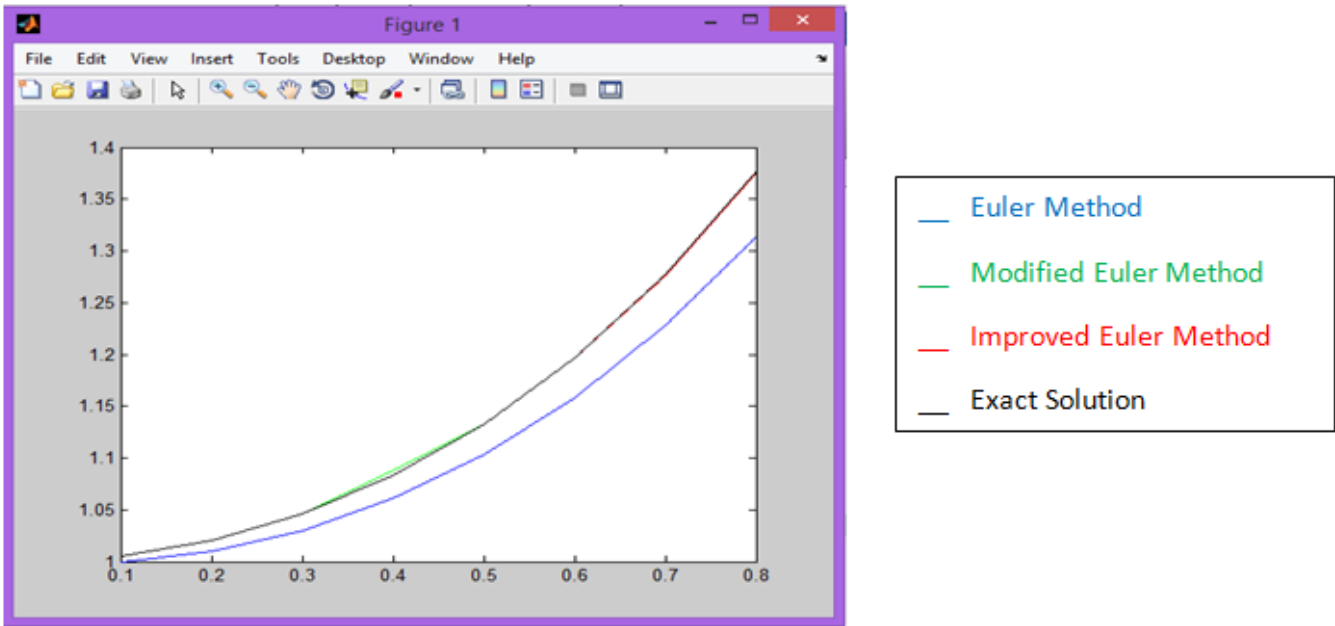


Figure 6

### HIGLY IMPROVED EULER METHOD MATLAB CODING

```
function []=Ijeuler(f,x0,y0,xn,n)
h=(xn-x0)/n;
x=x0;
y1=y0;
y2=y1;
disp(' x y1 y')
for i=1:n
y1=y1+h*f(x,y1);
y2=y2+(0.5*h*(f(x,y2)+f(x+h,y2+h*f(x,y2)))));
x=x+h;
disp([x y1 y2])
end
disp('NEW FOR OF EULER method:')
disp('IEM')
disp(y1)
disp(y2)
end
```

Figure 7

MATLAB GRAPH FOR HIGHLY IMPROVED EULER METHOD AND EXACT SOLUTION

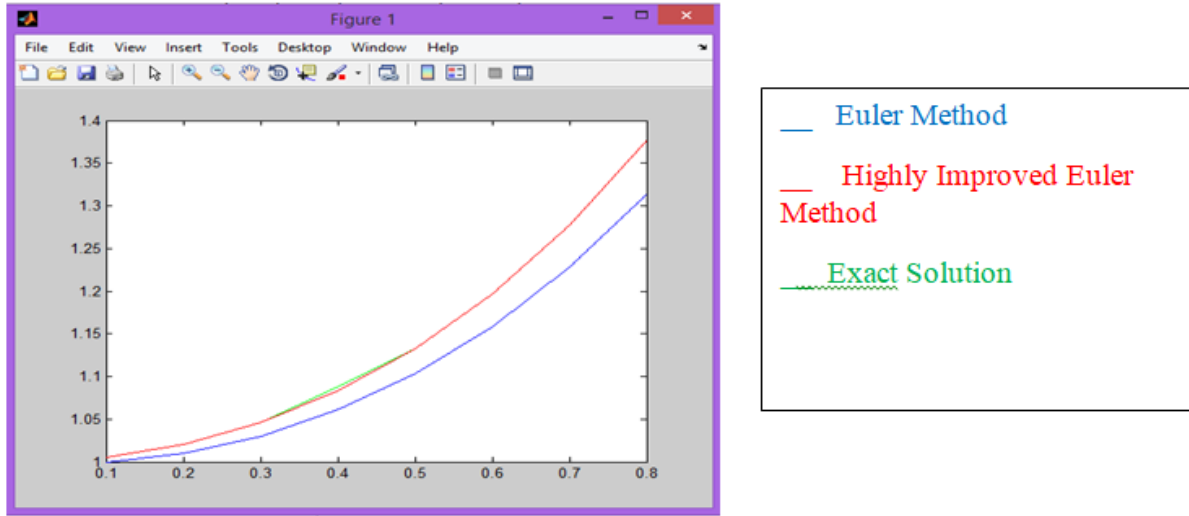


Figure 8

FIRST ORDER ORDINARY DIFFERENTIAL EQUATION SOLUTION FOR HIGHLY IMPROVED METHOD

Table 2

| X   | Highly Improved Euler Method(y) |
|-----|---------------------------------|
| 0.1 | 1.0050                          |
| 0.2 | 1.0202                          |
| 0.3 | 1.0460                          |
| 0.4 | 1.0832                          |
| 0.5 | 1.1331                          |
| 0.6 | 1.1971                          |
| 0.7 | 1.2774                          |
| 0.8 | 1.3768                          |

II. CONCLUSION

In this Paper Investigated about Solution of Ordinary Differential Equation with Initial Value condition using Highly Improved Euler’s Method. And the results are compared with Exact Solutions. Finally, the results are reported as Graph using MATLAB Coding.

III. REFERENCES

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