

# Two Person Zero Sum Game Problem Using Two Phase Simplex Method

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## ABSTRACT

Game theory is the study of the strategic interaction of two or more decision makers, or “players”, who are attentive that their actions affect each other. This paper deals with two person zero sum game problems with mixed strategies can be solved using two phase simplex method.

**Keywords :** Linear programming problem (LPP), two phase simplex method, Game problem, Value of the game.

## I. INTRODUCTION

Game theory is a branch of applied mathematics and it can be used to analyze the options, motivations and rewards involved in decision also used to represent a competitive (or) conflicting situation between two or more players, each player has no of choice called moves (or) pure strategies . A players select his moves without any knowledge of the moves chosen by the others players , the simultaneous choices of all players lead to the respective payoff of the game, if the sum of the payoff to all players is zero, then it is called zero sum game.

Two phase simplex method is a procedure of solving a linear programming problem involving artificial variables is divided into two phases. By using two phase simple method and game problem we have compared the solutions.

## II. PRELIMINARIES

**1.1 Player:** There are finite no of competitors called players.

**1.2 Strategy:** A strategy for a players is defined as a set of rules or alternatives courses is said to be strategy. A strategy may be two types,

- (i) Pure strategy
- (ii) Mixed strategy

**(i) Pure strategy:**

In a two person game if saddle point exists it is solved using pure strategy.

**(ii) Mixed strategy:**

In a two person game, if saddle point does not exist it is solved using mixed strategy.

**(1.3) Value of the game:**

It is the expected pay-off of the play when all the players of the game follow their optimal strategies. The game is called fair if the value of game is zero and unfair if it is non zero.

## III. PROBLEM

Consider a two person zero sum game without saddle point having the payoff matrix.

**Table 1**

PLAYER X	PLAYER Y		
	1	7	2
	6	2	7
	5	1	6

Hence, the matrix does not possess saddle point. Therefore, value of the game lies between 6 and 2. Thus the two phase simplex problem of the standard form can be written as,

$$\text{Maximize } Z = 0X_1 + 0X_2 + 0X_3 + 0S_1 + 0S_2 + 0S_3 - A_1$$

$$\text{Subject to } X_1 + 7X_2 + 2X_3 + S_1 + A_1 = 1$$

$$6X_1 + 2X_2 + 7X_3 + S_2 = 1$$

$$5X_1 + X_2 + 6X_3 + S_3 = 1$$

$$\text{And } X_1, X_2, X_3, S_1, S_2, S_3, A_1 \geq 0$$

**Solution:**

Maximin value=6 and Minimax value=2

**Initial Iteration:**

**Table 2**

$C_B$	$Y_B$	$X_B$	$X_1$	$X_2$	$X_3$	$A_1$	$S_1$	$S_2$	$S_3$
-1	$A_1$	1	1	7	2	1	1	0	0
0	$S_2$	1	6	2	7	0	0	1	0
0	$S_3$	1	5	1	6	0	0	0	1
$Z_j - C_j$			-1	-7	-2	0	1	0	0

$A_1 \Rightarrow$  Leaves the basis and  $X_2 \Rightarrow$  enter the basis

**First Iteration:**

**Table 3**

$C_B$	$Y_B$	$X_B$	$X_1$	$X_2$	$X_3$	$A_1$	$S_1$	$S_2$	$S_3$
0	$X_2$	1/7	1/7	1	2/7	1/7	1	0	0
0	$S_2$	5/7	40/7	0	45/7	2/7	0	1	0
0	$S_3$	6/7	34/7	0	40/7	-1/7	0	0	1
$Z_j - C_j$			0	0	0	1	0	0	0

**PHASE II:**

**Table 4**

$C_B$	$Y_B$	$X_B$	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	$S_3$
1	$X_2$	1/7	1/7	1	2/7	1	0	0
0	$S_2$	5/7	40/7	0	45/7	0	1	0
0	$S_3$	6/7	34/7	0	40/7	0	0	1
$Z_j - C_j$			-6/7	0	-5/7	1	0	0

$X_1 \Rightarrow$  Enters the basis and  $S_2 \Rightarrow$  leaves the basis

**Second Iteration:**

**Table 5**

$C_B$	$Y_B$	$X_B$	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	$S_3$
1	$X_2$	1/8	0	1	0	1	-1/280	0
1	$X_1$	1/8	1	0	9/8	0	7/40	0
0	$S_3$	-2/7	0	0	2/7	0	-17/140	0
$Z_j - C_j$			0	0	1/8	1	3/70	0

Two phase simplex method solution of LPP is  $X_1 = \frac{1}{8}, X_2 = \frac{1}{8}$ . We get sum of the value of solution (i.e)  
 $X_1 + X_2 = 1/4$

Finding the value of the game,

PLAYER X	PLAYER Y		
	1	7	2
	6	2	7
	5	1	6

By dominance property rule, reducing the number of matrix and finding the value of the game,  
Hence row 3 is dominated by row 2 and it is deleted.

PLAYER X	PLAYER Y	
	1	2
	6	7

Column 3 is dominated by column 1 and its is deleted

PLAYER X	1	7
	6	2

$$v = \frac{2 \cdot 1 - 7 \cdot 6}{2 + 1 - (7 + 6)} = \frac{40}{10} = 4$$

Value of the game (v) for players is=4

**Compared solutions:**

- (i) Two phase simplex solution of LPP =1/4. Reciprocal of Two phase simplex solution of LPP is the value of the game.

- (ii) Value of the game=4

#### **IV. CONCLUSION**

In this paper, a game theory problem was successfully solved using two phase simplex method. Thus it can be concluded that the value of game for players are same when comparing to the solution of two phase simplex method.

#### **APPLICATIONS:**

As a method of applied mathematics, game theory has many applications in subjects such as economics, political science, psychology, etc...

#### **ECONOMICS AND BUSINESS:**

Game theory is a major method used in mathematical economics and business for modeling competing behaviors of interacting agents. Applications include a wide array of economic phenomena and approaches, such as auctions, bargaining, mergers etc...

#### **POLITICAL SCIENCE:**

The applications of game theory to political science is focused in the overlapping areas of fair division, political economy, public choice, war bargaining etc...

#### **V. REFERENCES**

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