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δ^*- closed sets in Topological Spaces

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ABSTRACT

Abstract: To introduce a new class of closed sets called δ^* - closed sets and investigate some properties of these sets in topological spaces.
Keywords: δ^* -closed sets, δ - closed sets.

I. INTRODUCTION

Topology is the Mathematical study of the properties that are preserved through deformation, twisting and stretching of objects. A topology on a set X is a collection τ of subsets of X. This can be studied by considering a collection of subsets collect open sets topology developed as a field of study out of geometry & set theory, through analysis of concepts such as space, dimension & transformation. I have introduce and investigate a new class of closed set namely δ^* - closed set.

II. BASIC CONCEPTS

Definition 2.1:

Any of the subsets of a topological space x that comprise a topology on x are called open

Definition 2.2:

A subset a of a topological space x is called closed if and only if its complement A^C in X is open (i.e) X-A is open.

Definition 2.3:

Let (X,τ) be topological space. Then a subset δ of a space (X, τ) is said to be

(i)an δ^*- Closed set if $\delta \supseteq$ (limit(cl(limit$^\theta$(δ)))

(ii)an δ^*- Open set if $\delta \subseteq$ (limit(cl(limit$^\theta$(δ)))

Lemma 2.4:

Let δ be a subset of a space(X, τ). Then the following statements are:

(1)Every θ- Closed set is an δ^*- Closed set.

(2)Every δ^*- Closed set is an θ- Semi closed set.

(3)Every δ^*- Closed set is an δ- Closed set.

Proof:

(i) Let δ be an θ- Closed set

Then δ =limit$_\theta$ (δ)

limit$^\theta$ (δ) $\supseteq$ limit (δ) $\supseteq$ δ

hence δ= limit (δ)

Since δ =limit$_\theta$ (δ) $\supseteq$ cl(limit$_\theta$ (δ)), then δ = limit(δ) $\supseteq$ limit(cl(limit$_\theta$(δ))) Thus δ is δ^* - Closed .

(ii) obvious from the definition .

(iii) let δ be δ^* - Closed. Then δ $\supseteq$ limit (cl (limit_θ (δ)))$\supseteq$ cl (limit_θ (δ)) $\supseteq$ cl (limit_θ (δ)) $\cap$ limit(cl_δ(δ)).

Hence δ is an δ^* - Closed set.

Example:

Let X={U,V,W} with topology τ ={X,ϕ,{U}{V}{UV}} Then δ= {u,w} is an δ- closed set and θ- semi closed set, but it is not δ^* - closed.

Lemma 2.5:

Let (X,τ) be a topological space. Then the following statements are

(1)The finite intersection of δ^* - closed sets is δ^*-closed.

(2)The arbitary union of δ^*- open set is δ^*-open.

Proof:

(1)Let $\{\delta_i : i \in I\}$ be a family of δ^* - closed set.

Then $\delta_i \supseteq$ limit(cl(limit$_\theta$ (δ_i))) for all $i \in I$.

Then $\cup_{i\in I}\ \delta_i \supseteq \cup_{i\in I}$ limit (cl(limit_θ (δ_i))) $\supseteq$ limit (cl(limit$_\theta$($\cup_{i\in I}\ \delta_i$))).

Hence $\cup_{i\in I}\ \delta_i$ is δ^*closed.

Lemma 2.6:

For a topological space (X,τ) the family of all δ^* - closed set of ∪ forms a topology denoted by T_{δ^*} for X.

Proof : It is obvious that X,ϕ are in $\delta^*\ O$(X) and we've arbitrary intersection of δ^*- closed set is δ^*-closed.

Let U & V be δ^*- closed set.

Then U $\supseteq$ limit (cl (limi t_θ (U))) & V $\supseteq$ limit (cl(limit_θ(V)))

And hence

U ∪ V $\supseteq$ limit (cl(limit$_\theta$(U))) ∪ limit (cl (limit$_\theta$ (V)))

$\supseteq$ limit (cl(limit$_\theta$(U))) ∪ cl (limit$_\theta$ (V)))

$\supseteq$ limit (cl(limit$_\theta$ (U) ∪ limit$_\theta$ (V)))

$\supseteq$ limit (cl(limit$_\theta$ (U ∩ V)))

Hence the finite union of δ^*- closed set is δ^*-closed and hence T_{δ^*} is a topology for X.

Definition 2.7:

For a subset A of a topological space (x,τ)

1) U is an δ^* - closed set iff U = δ^* limit(U)
2) U is an δ^* - open set iff U = δ^* int(U)

Definition 2.8:

For a subset of topological spaces (X,τ)

1) U is an δ^*-closed set iff U = limit (u)
2) U is an δ^* -open set iff U = δ^*int (u)

Lemma 2.9:

1) δ^* -cl(X| δ) = X|δ^* -limit(δ)
2) δ^* -limit(X| δ) =X|δ^* -cl(δ)
3) If U $\supseteq$ V then δ^* -cl(A) $\supseteq$ δ^* -cl (V) and δ^* -limit (δ) $\supseteq$ δ^* -limit (V)
4) δ^*-limit (δ^* -cl(δ)) = δ^*-cl(U) and -limit(δ^* -limit(δ)) = δ^* -limit (U)
5) δ^*-cl(U) ∩ δ^*- cl(V) $\supseteq$ δ^*-cl(A ∩ B) and δ^*-limit(U) ∩ δ^*-limit(V) $\supseteq$ δ^*-limit(A ∩ B).
6) δ^*-cl(U) ∩ δ^*- cl(V) $\supseteq$ δ^*-cl(A ∪ B) and δ^*-limit(U) ∩ δ^*-limit(V) $\supseteq$ δ^*-limit(A ∩ B).

Proof:

(1)By the definition (2) (1)

δ^*- cl(X|δ) = (X| δ) ∩ (cl(limit(cl_θ(X| δ))))

= (X| δ) ∩ ((X|limit(cl(limit$_\theta$(δ))))

= (X| δ)(δ ∪ limit (cl(limit$_\theta$ (δ))))

= X|δ^*-limit(δ)

(2)& (3) Follows from the definitions

(4) By the definition (1) (2)

δ^*-cl(δ^*-cl(U)) =cl (limit(cl_θ (δ^*-cl(U)

=cl (limit(cl_θ(U ∩ cl (limit(cl_θ(δ))))))

$\supseteq$cl (limit(cl_θ (U ∩ cl_θ(limit(cl_θ(δ)))))

$\supseteq$cl (limit (cl_θ(U)))

$\supseteq \delta^*$-cl (U)

But δ^* -cl (U) δ^*-clδ^*-cl (U))

Hence δ^*-cl (U)= δ^*-cl(δ^*-cl(U))

(5) By the definition (2)(3)

δ^*-cl (U) ∩ δ^*-cl(V) = (U ∩ cl(limit(cl_θ(U))))∩(V ∩ cl(limit(cl_θ

= (U ∩ V) ∩ (cl(limit(cl_θ(U)))) ∩ cl(limit(cl_θ(V)))

= (U ∩V) ∩ cl(limt(cl_θ(A ∩ B)))

= δ^*-cl(A ∩ B)

(6) By the definition (3) (4)

δ^* -limit(U ∪ V) = (U ∪ V) ∪ limit (cl(limit_θ((U ∪ V)))

= (U ∪ V) ∪ limit (cl(limit_θ(U) (limit_θ(V)))

⊇ (U ∪ limit (cl(limit_θ(U)))) ∪ (V ∪ limit (cl(limit_θ(v))))

= δ^* -limit(U) ∪ δ^* - (limit_θ(V)

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