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A Study on Energy of A Graph

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ABSTRACT

In this paper, different types of energy of the graph is compared. Energy f he graph is defined as the sum of the absolute values of its Eigen values of G. If A(G) is the adjacency matrix of G and $\lambda_1, \lambda_2, \dots \lambda_n$ are the Eigen values of A(G), then E(G)= $\sum_{i=1}^{n}|\lambda_i|$. Comparison between Energy of a graph, Laplacian Energy of a graph and Maximum Eccentricity of the graph are derived. It is found that Maximum Eccentricity of a graph has the maximum energy when compared to the other two energies.

Keywords : Energy of a graph, Laplacian Energy of a graph, Maximum Eccentricity of a graph.

I. INTRODUCTION

The energy E(G) of a graph, defined as the sum of the absolute values of its Eigen values that belongs to the most popular graph invariants in chemical graph theory. It originates from the π-electrons energy in Huckel molecular orbital model, but has also gained purely mathematical interest. In this paper we study about different kinds of energies of the graphs. If $\{v_1, v_2, \dots v_n\}$ is the set of vertices of G, then the adjacency matrix A(G) = $[a_{ij}]$ is an n×n matrix where $a_{ij} = 1$ if v_i and v_j are adjacent and $a_{ij} = 0$, otherwise. Here we study about the energy of a graph, the Laplacian energy of a graph and maximum eccentricity energy of a graph. In this paper, all the graphs are assumed to be simple, connected, finite and undirected.

The energy, E(G), of a graph G is defined as the sum of the absolute values of the Eigen values of G. If A(G) is the adjacency matrix of G and $\lambda_1, \lambda_2, \dots \lambda_n$ are the Eigen values of A(G), then E(G)= $\sum_{i=1}^{n}|\lambda_i|$. The set $\{\lambda_1, \lambda_2, \dots \lambda_n\}$ is spectrum of G and is denoted by Spec G.

Let G = (V,E) be a simple graph of order n with m edges. The Laplacian energy of a graph G is defined as

$$\text{LE} = \text{LE}(G) = \sum_{i=1}^{n}\left|\lambda_i - \frac{2m}{n}\right|,$$

Where $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{n-1}, \lambda_n = 0$ are the Laplacian Eigen values of a graph G

The maximum eccentricity energy of a graph is defined as the maximum eccentricity Eigen values of G and the Eigen values of $M_e(G)$. It is defined as $EM_e(G) = \sum_{i=1}^{n}|\lambda_i|$.

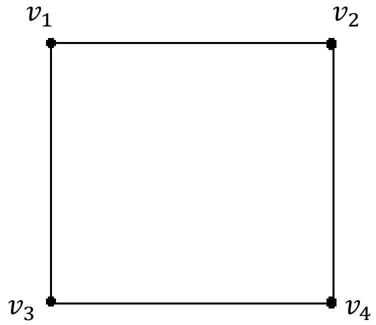
II. ENERGY OF A GRAPH

The energy, E(G), of a graph G is defined as the sum of the absolute values of the Eigen values of G. If A(G) is the adjacency matrix of G and $\lambda_1, \lambda_2, \dots \lambda_n$ are the eigen values of A(G), then E(G) = $\sum_{i=1}^{n}|\lambda_i|$. Now the set $\{\lambda_1, \lambda_2, \dots \lambda_n\}$ is the spectrum of G. All the graphs have at most 2(n-1) energy. But this was then disproved. Graphs for which the energy s greater than 2(n-1) are called hyper energetic graphs. if E(G) ≤ 2n-1,G is called non-hyper energetic graphs. In theoretical chemistry the π electron energy of a conjugated carbon molecule, computed using the Huckel theory, coincides with the energy as defined.

The energy of the graph K_n -H, where H is a Hamilton cycle of G. There exist an infinite number of values of n for which k-regular graphs exist whose energies are arbitrarily small compare to the known sharp bound k+$\sqrt{k(m-1)(n-k)}$ for the energy of k regular graphs on n vertices.

To illustrate this concept, we study the following examples

Example:



$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 & 1 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 1 \\ 1 & 0 & 1 & -\lambda \end{vmatrix}$$

$= \lambda^4 - 4\lambda^2$

λ_1=-0, λ_2= 0, λ_3=2, λ_4= -2

E(G) = $\sum_{i=1}^{n} |\lambda_i|$

= | -2| +| 2|

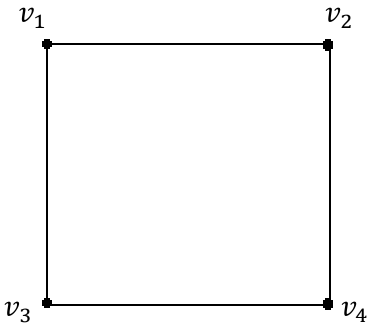
E(G) = 4

III. LAPLACIAN ENERGY OF A GRAPH

Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{n-1} \lambda_n$ be the eigenvalues of L(G). Then the Laplacian Energy, LE(G) is defined as

$$LE(G) = \sum_{i=1}^{n} \left|\lambda_i - \frac{2m}{n}\right|$$

Example



$$D(G) = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

L(G) = D(G) – A(G)

$$L(G) = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 0 & -1 \\ -1 & 2-\lambda & -1 & 0 \\ 0 & -1 & 2-\lambda & -1 \\ -1 & 0 & -1 & 2-\lambda \end{vmatrix}$$

= (2-λ)[-$\lambda^3 + 6\lambda^2 - 10\lambda + 4] - 2\lambda^2 + 8\lambda - 8$

$= \lambda^4 - 8\lambda^3 + 20\lambda^2 - 16\lambda$

$= \lambda(\lambda^3 - 8\lambda^2 + 20\lambda - 16)$

$= \lambda(\lambda - 2)(\lambda^2 - 6\lambda + 8)$

$\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 2, \lambda_4 = 4$

LE(G) = $\sum_{i=1}^{n} \left|\lambda_i - \frac{2m}{n}\right|$

Here, m=4, n=4

So, LE(G) = $\sum_{i=1}^{n} \left|\lambda_i - \frac{2(4)}{4}\right|$

Therefore,

LE(G) = 4

IV. THE MAXIMUM ECCENTRICITY ENERGY OF A GRAPH

Let G be a simple graph with n vertices $v_1, v_2, \ldots v_n$ and let e (v_i) be the eccentricity at a vertex v_i, i = 1,2,…..n. The maximum eccentricity matrix of G defining as, where

$$e_{ij}=\begin{cases} max\{e(v_i), e(v_j)\}, If\ v_i, v_j \in E(G); \\ 0, otherwise. \end{cases}$$

The characteristics polynomial of the maximum eccentricity matrix M_e(G) is defined by

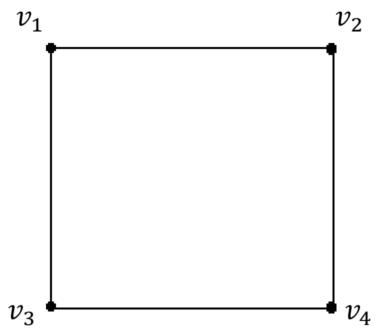
$$P(G,\lambda) = \det(\lambda I - M_e(G))$$

Where I is the unit matrix of order n . the maximum eccentricity Eigen values of G are the Eigen values ofM_e(G). Since M_e(G) is real and symmetric with the zero trace, then its Eigen values are real number with sum equals to zero. We label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$.The maximum eccentricity energy of a graph G is defined as

$$EM_e\ (G) = \sum_{i=1}^{n} |\lambda_i|$$

To illustrate this concept, we study the following examples.

Example



$$A(G) = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 2 & 0 & 2 \\ 2 & -\lambda & 2 & 0 \\ 0 & 2 & -\lambda & 2 \\ 2 & 0 & 2 & -\lambda \end{vmatrix}$$

$= \lambda^4 - 4\lambda^2$-$4\lambda^2$-$4\lambda^2 - 4\lambda^2$

$= \lambda^2(\lambda^2 - 16)$

$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 4, \lambda_4 = -4$

$E(G) = 8$

V. COMPARISION BETWEEN LAPLACIAN ENERGY OF A GRAPH AND ENERGY OF A GRAPH

We calculate the energy, Laplacian energy for few graphs.

GRAPH	LAPLACIAN ENERGY	ADJACENCY ENERGY
G1	0.00	0
G2	0.00	0
G3	2.00	2
G4	0.00	0
G5	2.67	2
G6	3.33	2.82842
G7	4.00	4

From the above table, it s clear that the Laplacian energy of a graph is always greater than or equal to the Energy of a graph.

That is, L(G) ≥ E(G)

THEOREM

The Maximum Eccentricity energy of a graph is always greater than the Laplacian energy of a graph and Energy of a graph.

Proof

Part 1

Here we prove that the maximum eccentricity energy of a graph is always greater than the Laplacian energy of a graph.

We know that,

$$\text{LE(G)} = \sum_{i=1}^{n} \left| \lambda_i - \frac{2(m)}{n} \right|$$

And,

$$\text{E}M_e(G) = \sum_{i=1}^{n} |\lambda_i|$$

Comparing the above two equations and from the example we discussed above, we get

$$\text{E}M_e(G) > \text{LE(G)}$$

(i.e) $\sum_{i=1}^{n} |\lambda_i| > \sum_{i=1}^{n} \left| \lambda_i - \frac{2(m)}{n} \right|$

The result is trivial since m and n are positive integers.

Part 2

Here we prove that the maximum eccentricity energy of a graph is always greater than the energy of a graph.

Let G be a graph of order n and $\lambda_1, \lambda_2, \dots \lambda_n$ be the maximum eccentricity Eigen values of G.

We know that $\sum_{i=1}^{n} \lambda_i = 0$.

Then let $\lambda_1, \lambda_2, \dots \lambda_r$ be the positive eigen values of G and $\lambda_{r+1}, \lambda_{r+2}, \dots \lambda_n$ be the negative Eigen values of G.

Then,

$\text{E}M_e(G) = \lambda_1, \lambda_2, \dots \lambda_r - (\lambda_{r+1}, \lambda_{+2}, \dots \lambda_n)$

$= 2\ (\lambda_1, \lambda_2, \dots \lambda_r)$

Since, $\lambda_1, \lambda_2, \dots \lambda_r$ are algebraic numbers, which is the Eigen values of energy of a graph. Therefore, we conclude that the maximum eccentricity of a graph is 2 times that of the energy of a graph.

Hence, the maximum eccentricity energy of graph is always greater than the energy of a graph and Laplacian energy of a graph.

VI. CONCLUSION

In this paper we conclude that the Maximum Eccentricity energy is always greater than the Laplacian energy of a graph and Energy of a graph and also that the Laplacian energy of a graph is always greater than or equal to the Energy of a graphs.

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