

Adaptive Control Scheme for Coupled Tank Process

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ABSTRACT

The objective of this paper is to develop a Direct Model Reference Adaptive Control (DMRAC) algorithm for a MIMO process by extending the MIT rule adopted for a SISO system. The controller thus developed is implemented on Laboratory interacting coupled tank process through simulation. This can be regarded as the relevant process control in petrol and chemical industries. These industries involve controlling the liquid level and the flow rate in the presence of nonlinearity and disturbance which justifies the use of adaptive techniques such as DMRAC control scheme. For this purpose, mathematical models are obtained for each of the input-output combinations using white box approach and the respective controllers are developed. A detailed analysis on the performance of the chosen process with these controllers is carried out. Simulation studies reveal the effectiveness of proposed controller for multivariable process that exhibits nonlinear behaviour.

Keywords : Coupled tank system, MIMO, DMRAC, MIT rule

I. INTRODUCTION

Most of the industrial processes are Two Input Two Output (TITO) systems which have cross coupling between the process inputs and outputs. Comparing with Single Input Single Output (SISO) counterparts, TITO processes are more difficult to control due to the existence of interactions between input and output variables. Many techniques have been suggested in the past for controlling the TITO systems. Decoupling controllers are used to transform TITO plant transfer function model into SISO form. Thus two degrees of freedom PI controller for each of the SISO models can be designed by Root Locus Technique [1].

Chatchaval et al.[2] has implemented a decentralized fuzzy logic controller for TITO coupled tank process. Suparoek et al. [3] has presented a design methodology of auto-adjustable PI controller using Model Reference Adaptive Control (MRAC)

technique for solving the problem of coupled tank process as fixed controllers are not able to provide efficient control in the presence of disturbance and variation in parameters.

Recently Asan Mohideen et al. [4] has designed a MRAC with a very good steady state and transient performance for a nonlinear process such as the hybrid tank process. Rathikarani et al. [5] proposed an auto tuning of the PI controller using MRAC concept for a nonlinear air flow process. Bharathi et al. [6] have considered continuous time adaptive control to a nonlinear two tank non-interacting SISO system.

The present work provides the design procedure of DMRAC based on negative gradient approach. This method enables adjustment of controller parameters in response to changes in plant and disturbance by referring to the model that specifies properties of the desired control system. Hence, this technique is

convenient for controller design satisfying the system requirement.

The paper is organized as follows: The laboratory interacting coupled tank process setup chosen for the study is detailed first. Then the methodology used for modeling and the validation of multiloop process is outlined. After a brief introduction about multiloop PI controller, the procedure involved in developing DMRAC is presented. The performance analysis of the proposed controller is reported before providing the conclusion

II. PROCESS DESCRIPTION

The schematic diagram of the chosen Interacting coupled tank system is illustrated in Fig. 1.

The mass balance equations of tank1 and tank2 are given in Equations 1 and 2. The rate of change of liquid volume in each tank is equal to the net flow of liquid into the tank. The volumetric inflow rate into the tank1 and tank2 are q_{in1} and q_{in2} . The volumetric flow rate from the tank1 and tank2 are q_{o1} and q_{o2} . Flow rate between tank1 and tank2 is q_{12} . The height of the liquid level is h_1 in tank1 and h_2 in tank2.

$$A_1 \frac{dh_1}{dt} = q_{in1} - q_{o1} - q_{12} \quad (1)$$

$$A_2 \frac{dh_2}{dt} = q_{in2} - q_{o2} + q_{12} \quad (2)$$

The system model can be formulated by ordinary differential equation using Bernoulli's law as shown in Equations 3 and 4.

$$\frac{dh_1}{dt} = \frac{q_{in1}}{A_1} - \frac{a_1}{A_1} \sqrt{2gh_1} - \frac{a_{12}}{A_1} \sqrt{2g(h_1 - h_2)} \quad (3)$$

$$\frac{dh_2}{dt} = \frac{q_{in2}}{A_2} - \frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_{12}}{A_1} \sqrt{2g(h_1 - h_2)} \quad (4)$$

The cross sectional area of tank1 and tank2 are $A_1=A_2=1130.4\text{cm}^2$, restriction areas in the outlet pipes of tank1 and tank2 are $a_1=a_2= 3.9\text{cm}^2$. Restriction area of interconnecting pipe is $a_{12}=1.27\text{cm}^2$ and g is the specific gravity. The maximum capacity of two tanks is 25cm. Equations 3 and 4 describe the coupled tank system dynamics. To design the control systems for

this process the equations are linearized by considering small variations in q_{in1} , q_{in2} , h_1 and h_2 [3].

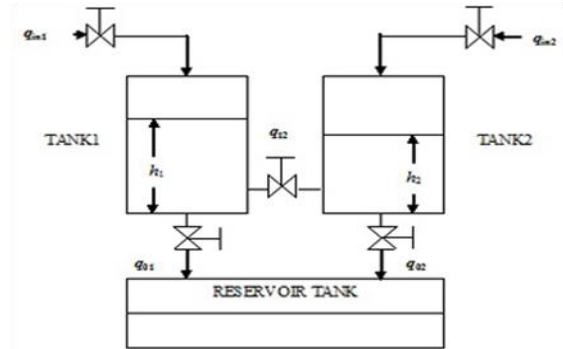


Fig. 1. Schematic diagram of coupled tank process

The variations are measured with respect to nominal operating conditions. The hand valves are adjusted so that the levels in both the tanks are brought to nominal condition initially. Nominal values of q_{in1} , q_{in2} are 26 and 20.75 l/hr and for h_1 and h_2 are 12.5 and 12.1cm respectively.

The linearized models of the process are given below in Equations 5. Based on this structure, the reference model for the MRAC is to be decided.

Rearranging the equations (3) & (4) and then taking laplace transform on both sides, we get

$$h_1(s) = \frac{R_1}{\tau_1 s + 1} [q_{in1}(s) - q_{12}(s)] \quad (5)$$

$$h_2(s) = \frac{R_2}{\tau_2 s + 1} [q_{in2}(s) + q_{12}(s)]$$

III. MULTILoop PROCESS

The four models relating the two controlled outputs h_1 and h_2 with two manipulated inputs q_{in1} and q_{in2} are essential to design the multiloop controllers [5]. The model transfer functions with the flow rates as manipulated inputs and the levels as controlled outputs can be written as follows:

$$\begin{aligned} G_{11}(s) &= \left(\frac{h_1}{q_{in1}} \right)_{q_{in2}} ; G_{21}(s) = \left(\frac{h_2}{q_{in1}} \right)_{q_{in2}} ; \\ G_{12}(s) &= \left(\frac{h_1}{q_{in2}} \right)_{q_{in1}} ; G_{22}(s) = \left(\frac{h_2}{q_{in2}} \right)_{q_{in1}} \end{aligned} \quad (6)$$

The open loop response of the process for a 10% change in q_{in1} is plotted in Fig. 2. This causes the level in tank1 to change from 12.5cm to 20cm. Due to interaction, level in tank2 reaches the steady state value 16.5cm from its nominal value. Figure 3

Fig. 2. Openloop response for the process (Δq_{in1})

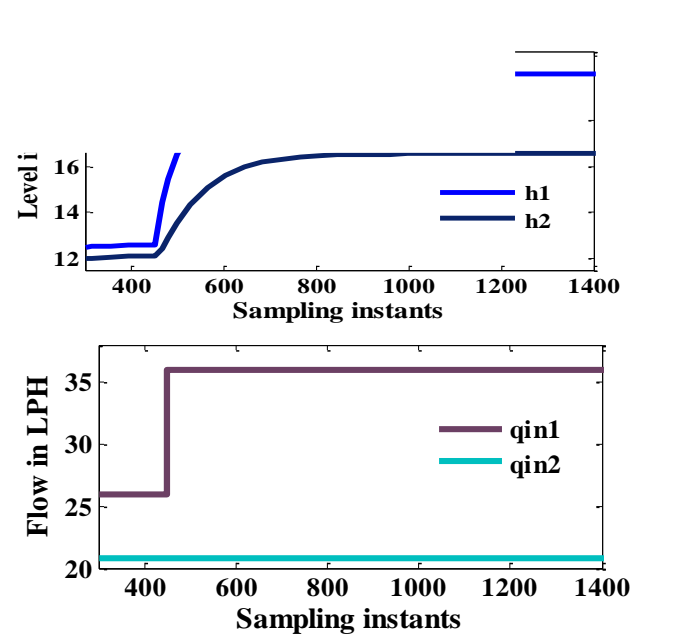


Fig. 3. Variation of q_{in1}

In the same manner, the open loop responses (as in Fig. 4) are obtained by making 10% change in q_{in2} maintaining q_{in1} in nominal condition. Here also the level in tank1 reaches steady state at 18cm. Figure 5 displays the graphs for variation in q_{in2} and q_{in1} in nominal condition.

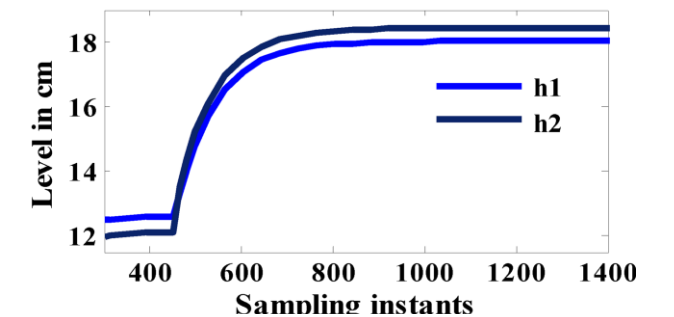


Fig. 4. Openloop response for the process (Δq_{in2})

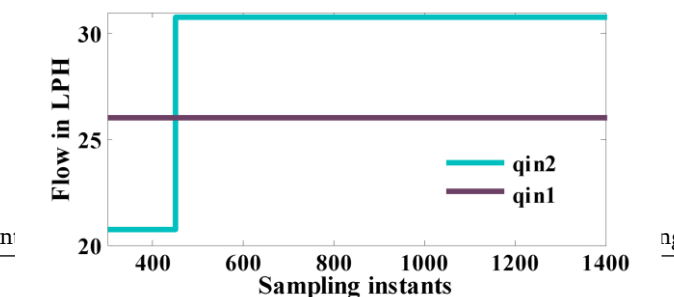


Fig. 5. Variation of q_{in2}

The Transfer functions are computed and represented in matrix form as follows:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} \frac{0.737}{61s+1} & \frac{0.583}{111s+1} \\ \frac{0.443}{106s+1} & \frac{0.633}{81s+1} \end{bmatrix} \quad (7)$$

A .Validation

Model validation is the most important step of model building and is accomplished by matching simulated output from white box approach (Actual output) with transfer function model (model output). Implementation of the model without validation may lead to erroneous and misleading results. So it is essential to verify the model. Figures 6(a) and (b) shows the time domain validation for the models G_{21} and G_{22} respectively.

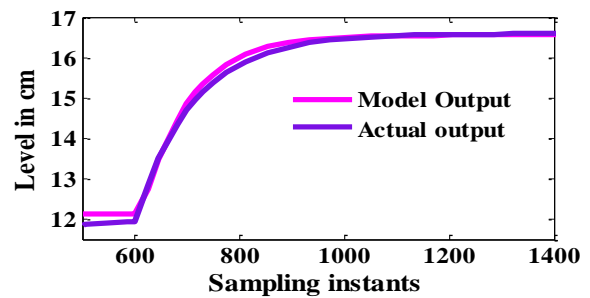


Fig. 6(a). Time domain validation for the model G_{21}

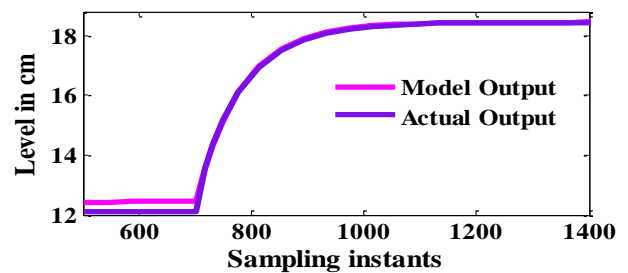


Fig. 6(b). Time domain validation for the model G_{22}

The block diagram of Multiloop control system of a coupled tank process employed with G_{c1} and G_{c2} as PI controllers for tank1 and tank2 is shown in Fig. 7 [7].

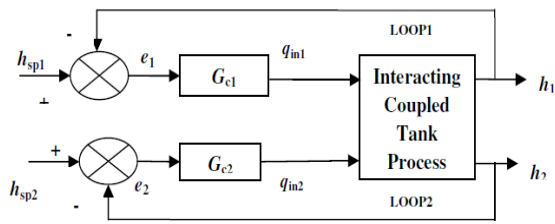


Fig. 7. Multiloop PI control system

Synthesis Method of Tuning

The PI controller parameters are tuned using synthesis method for the two first order processes G11 and G22 obtained from modelling. Kc is proportional gain. Ti is integral time. Ki is integral gain. τ_c is closed loop time constant of the process. Controller parameters are determined and tabulated in Table I.

Table I. Controller Parameters for PI Controller

Parameters	Controller	
	G _{c1}	G _{c2}
$K_c = \frac{\tau_p}{k_p \tau_c}$	2.357	2.579
$K_i = \frac{K_c}{T_i}$	0.038	0.032

IV. Direct Model Reference Adaptive Controller

The block diagram of Multiloop DMRAC system is shown in Fig. 8. This system consists of a reference model, parameter adjustment mechanism and controller in each loop in order to control the two controlled outputs. The reference model describes the desired input/output character of the closed loop system. The controller drives the control signal so that the closed loop characteristics from the setpoint (h_{spi}) to the process output h_i (where $i=1,2$ represents the control loops) is equal to the dynamics of the reference model (h_m). In this work, the reference model is selected with the gain 1 and time constant 0.5. Matching the process and reference model dynamics guarantees the convergence of the modeling error (e_i) to zero. The controller drives the difference (error) between the process response and

desired model output to zero asymptotically at a rate constrained by the adaptation gain (γ).

The designed controller has a conventional inner loop followed by a adaptive outer loop to adjust the controller parameters for the respective loops based on the modeling error such a way that the coefficients of the model and the closed loop plant are equal [8].

Here the modeling error,

$$e_i = h_i - h_m \tag{8}$$

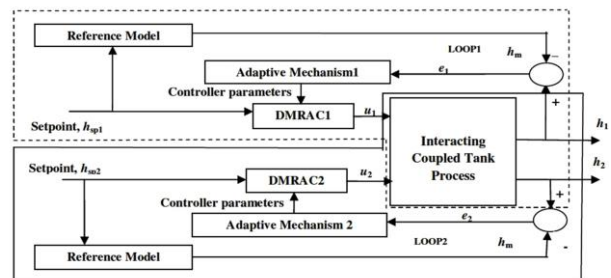


Fig.8. Block diagram of Multiloop DMRAC system

A.MIT Algorithm

The controller parameters (θ_{ni}) is adjusted with the constraint given by following loss function,

$$J(\theta_{ni}) = \frac{1}{2} e_i^2 \tag{9}$$

where $n=1,2$ represents controller parameter number. In order to minimize the loss function J, the parameters can be changed in the direction of negative gradient of J. The following parameter adjustment mechanism is called MIT algorithm [9].

$$\frac{d\theta_{ni}}{dt} = -\gamma e_i \frac{\partial J}{\partial \theta_{ni}} = -\gamma e_i \frac{\partial e_i}{\partial \theta_{ni}} \tag{10}$$

The quantity $\frac{\partial e_i}{\partial \theta_{ni}}$ is the sensitivity derivative of the error with respect to controller parameters. The parameter, γ determines the adaptation rate. Based on the apriori knowledge, the linearized model of the chosen nonlinear process around a specific operating condition is represented as follows:

$$G_{pi} = \frac{h_i}{u_i} = \frac{b_i}{s + a_i} \tag{11}$$

$$u_i = \theta_{1i} h_{spi} - \theta_{2i} h_i \tag{12}$$

By substituting equation 12 in 11, the closed loop transfer function with MRAS is obtained as,

$$\frac{h_i}{h_{spi}} = \frac{b_i \theta_{1i}}{s + a_i + b_i \theta_{2i}} \quad (13)$$

Based on the equation 13, the transfer function of the model is given by

$$G_m = \frac{h_m}{h_{spi}} = \frac{b_m}{s + a_m} \quad (14)$$

where b_m, a_m are the model parameters. Under the exact model following condition, the controller parameters are given by equation 14.

$$\theta_{1i} = \frac{b_m}{b_i} \quad \text{and} \quad \theta_{2i} = \frac{a_m - a_i}{b_i} \quad (15)$$

By substituting equations 13 and 14 in equation 8, the modeling error is obtained as follows:

$$e_i = \frac{b_i \theta_{1i}}{s + a_i + b_i \theta_{2i}} h_{spi} - \frac{b_m}{s + a_m} h_{spi} \quad (16)$$

The sensitivity derivatives are obtained by taking partial derivatives with respect to the controller parameters.

$$\frac{\partial e_i}{\partial \theta_{1i}} = \frac{b_i}{s + a_i + b_i \theta_{2i}} h_{spi} \quad \text{and} \quad \frac{\partial e_i}{\partial \theta_{2i}} = \frac{-b_i}{s + a_i + b_i \theta_{2i}} h_i \quad (17)$$

By substituting equations 17 in equation 10, the rate of change of controller parameters $\frac{d\theta_{1i}}{dt}, \frac{d\theta_{2i}}{dt}$ are given by equations 18.

$$\frac{d\theta_{1i}}{dt} = -\gamma e_i \frac{a_m}{s + a_m} h_{spi} \quad \text{and} \quad \frac{d\theta_{2i}}{dt} = \gamma e_i \frac{a_m}{s + a_m} h_i \quad (18)$$

By varying adaptation gain (γ), tracking speed and the convergence rate of controller parameters are varied.

V. Simulation Results

The Fig.9 shows the enlarged servo responses of interacting tanks (Δh_1). The adaptation gain used in the controllers is 0.2. Figure 10(a) and (b) shows the servo and regulatory responses of interacting tanks. Positive change in h_{sp1} (Servo 1) from 12.5 to 17cm is given at 1000th sampling instant as shown in Fig. 10(a). Negative change of 1cm in h_{sp1} (Servo 2) is applied at 2000th sampling instant. Due to the action of DMRAC1 in loop1, the level in tank1 tries to track the setpoint. Due to interaction h_2 slightly oscillates and settles in its nominal value 12.1cm). Positive

change in h_{sp2} (Servo 3) from 12.1 to 14.6cm is given at 3000th sampling instant. Negative change of 1cm in h_{sp2} (Servo 4) is applied at 5000th sampling instant. Due to the action of DMRAC2 in loop2, the level in tank2 tries to track the setpoint. Due to interaction h_1 oscillates and settles in 16cm. Disturbances are applied to the tanks by varying the position of the hand valves. In Fig. 10(b), negative disturbance (d_1) and positive disturbance (d_2) are applied to tank1 at 8000th and 9000th sampling instant respectively. Negative disturbance (d_3) and positive disturbance (d_4) are applied to tank2 at 10000th and 11000th sampling instant respectively. Due to the action of the controllers in both the loops the levels are brought back to the setpoint.

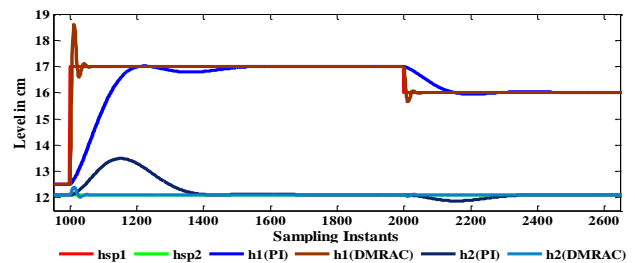


Fig. 9. Servo responses of interacting tanks (Δh_1)

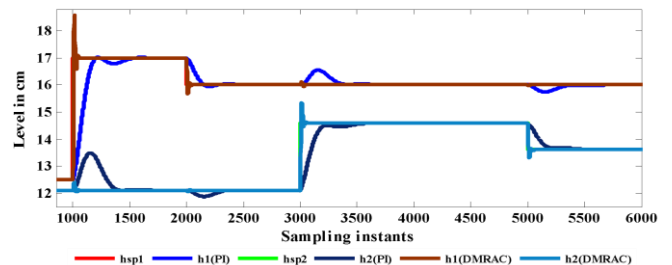


Fig. 10(a). Servo responses of interacting tanks

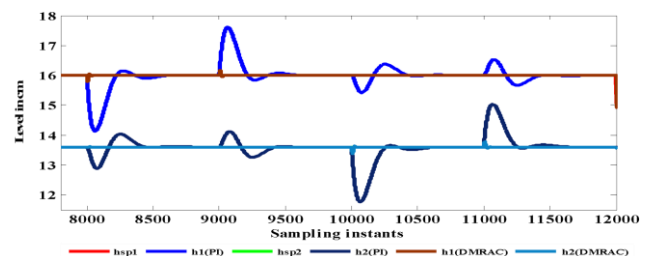


Fig. 10(b). Regulatory responses of interacting tanks

The corresponding responses of the controllers are shown in Fig. 11(a) and 11(b).

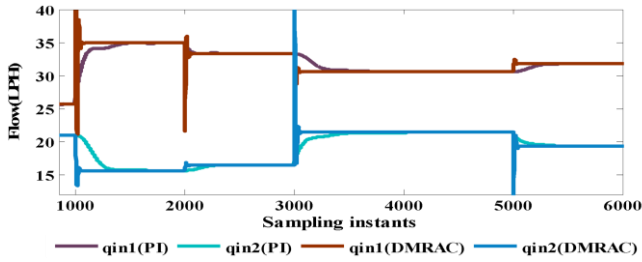


Fig. 11(a). Responses of the controllers (Servo)

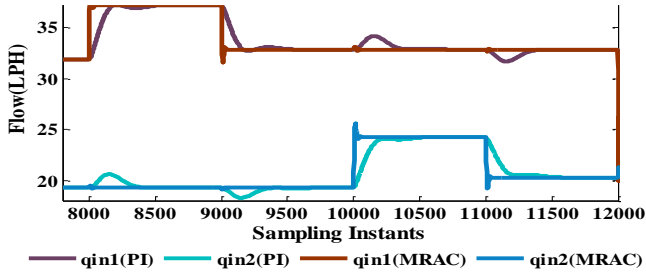


Fig. 11(b). Responses of the controllers (regulatory)

When the adaptation gain (γ) is reduced, then the spike in the controller output disappears as shown in Fig. 12(a) and (b).

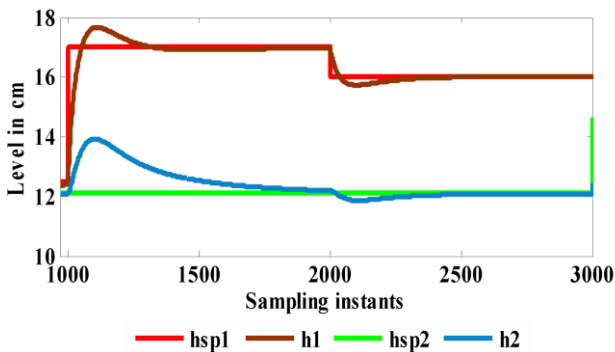


Fig. 12(a). Servo responses of interacting tanks (Δh_1)

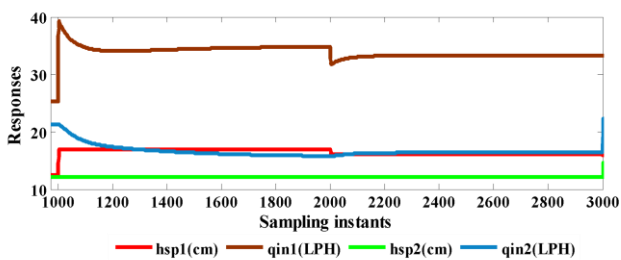


Fig. 12(b). Responses of the controllers

The vanishing nature of adaptation of controller parameters (Th_1, Th_2) of both DMRAC1 and DMRAC2 for servo and regulation can be visualized in Fig. 13 and 14 respectively.

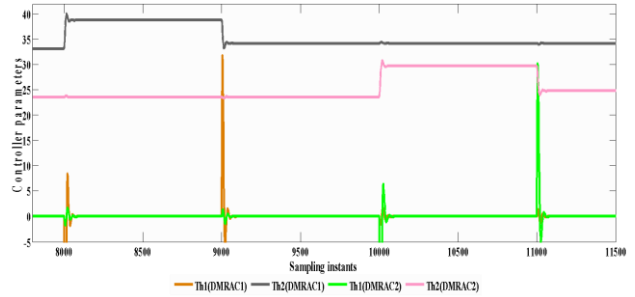
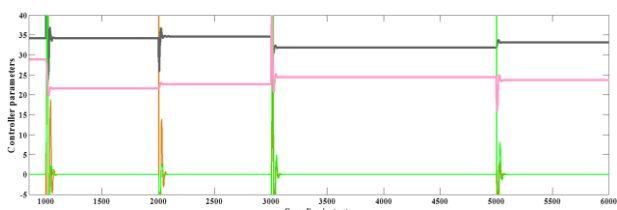


Fig. 14. Adaptation of the controller parameters for load regulation

Simple performance criteria and Time integral criteria for servo is compared for the process with PI and DMRAC controllers in Tables II and III [10].

Table II Comparison of Simple Performance Criteria (Servo)

Controller	Peak Overshoot		Undershoot		Settling Time			
	Tank1	Tank2	Tank1	Tank2	Tank1		Tank 2	
	Servo1	Servo3	Servo2	Servo4	Servo1	Servo2	Servo 3	Servo4
PI	-	-	-	-	600	300	600	500
DMRAC	0.09	0.05	0.02	0.01	50	40	60	45

Table III Comparison of Time Integral Criteria (Servo)

Controller	ISE				IAE			
	Tank1		Tank2		Tank1		Tank 2	
	Servo1	Servo2	Servo3	Servo4	Servo1	Servo2	Servo 3	Servo4
PI	1137	49.05	359.9	60.83	411.6	81.49	261	111.8
DMRAC	69.1	3.355	21.53	3.428	43.14	9.551	23.53	9.475

Simple performance criteria and Time integral criteria for regulation is compared for the process with PI and DMRAC controllers in Table IV and Table V.

Table IV Comparison of Simple Performance Criteria (Regulatory)

Settling Time			
Tank1		Tank 2	
d_1	d_2	d_3	d_4
615	527	560	440
49	47	50	60

Table V Comparison of Time Integral Criteria
(Regulatory)

Controller	ISE				IAE			
	Tank1		Tank2		Tank1		Tank 2	
	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4
PI	338.9	244.2	347.1	214	265.3	224	268.7	213.3
DMRAC	1.017	0.672	1.267	0.806	5.292	4.304	6.127	4.885

VI. Conclusion

The performance of coupled tank process has been investigated using DMRAC concept and MIT rule. Interaction effects are projected for both servo tracking and load disturbance rejection. From the plots, it is clear that the overall system performance with DMRAC is observed to have better tracking and disturbance rejection than that of the system with PI controller. From Table III, it is observed that, for both tank1 (servo1) and tank2 (servo3) DMRAC has reduced ISE to 6% and IAE to 10% when compared to PI controller. From Table V, it is observed that, for both tank1 (d_1) and tank2 (d_3) DMRAC has reduced ISE to 0.3% and IAE to 2% when compared to PI controller. Even though there are slight peak overshoots and undershoots, all the Integral errors are significantly low and less settling time indicates that the response of DMRAC is appreciable. The resulting performance could be improved by a better choice of the adaptation gain.

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