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A Study to Estimate Head Count Index in Small Areas with M-quantile Regression Model Case Study : Poverty in Bogor District Year 2015

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ABSTRACT

Poverty should be overcome with data. Problem arises when poverty is identified in sub-distric level, yet the data are in district level. Alternatively, M-quantile regression modeling in small area estimation as an indirect estimation approach can be done to measure poverty level in sub-district region with the use of district-scaled or even province-scaled data. In this paper, a Monte Carlo simulation will be conducted to empirically evaluate M-quantile modeling which contaminated area random effect and individual random effect to estimate head count index. M-quantile modeling is chosen because it is quantile-based semiparametric method which guarantees statistical estimation to be robust. Both direct and indirect estimations are performed and the the results of both estimations will be compared in each scenarios. The goodness of fit will be measured with bias and root mean squared error (RMSE). The result shows that M-quantile modeling is effective when there are outliers in individual random effect. Finally, results of application of M-quantile regression modeling to National Socio-economic Survey in Indonesia are presented.

Keywords : Monte Carlo simulation, M-quantile Modeling, Head Count Index, Outliers, Random Effect.

I. INTRODUCTION

In the new global economy, poverty has become one of central issues of the world's Sustainable Development Goals. Poverty eradication can be supported by an accurate use of data so that a country's government can design policies effectively and efficiently. Problem arises when the government wants to identify poverty at sub-district level, while the data available are at district or even at province level.

The main challenge faced by many researchers is doing direct estimation at sub-district level in small areas. The term 'small areas' is used when sample sizes of certain areas are not adequate to do direct estimation with some specific accuracy (Rao, 2003). Directly estimating poverty statistic at sub-district level will be at much risk because sample sizes are often found to be too small or even no sample is available, which will lead to a large predictive variance.

Extensive research has shown that small area estimation can be carried out with indirect estimation approach through modeling (Girinoto, 2017). With this method, poverty statistic can be estimated in sub-distric level by utilizing data from district-scaled or even province-scaled national survey. Small area estimation method can be used not only to estimate mean and total, but also to estimate model-based poverty indicators.

It has previously observed by world-class reseachers that model-based poverty indicators can be estimated in small areas. Elbers et al. (2003) investigated the estimation of poverty indicators in small area by building mixed linear model which utilized significantly correlated neighboring variables from previous census data. Molina et al. (2010) published a research which discussed about the estimation of poverty indicators and certain quantiles in small areas distribution function. Chambers and Tzavidis (2006) estimated poverty indicators in small areas with M-quantile modeling approach.

Goodness of fit in small area estimation are measured by bias and mean squared error (MSE). Chambers and Tzavidis (2006) estimated MSE of small area means with M-quantile modeling. Marchetti et al. (2012) developed Chambers' and Tzavidis' research by estimating MSE of poverty indicators estimation in small area with non-parametric bootstrap approach. In Indonesia, Girinoto (2017) estimated poverty indicators of 40 sub-districts in Bogor District with M-quantile modeling approach.

In closing the gaps, this research is designed to evaluate the performance M-quantile regression modeling in different population conditions and prove the robustness of this method against the presence of outliers. The specific objectives of this research are the application of Monte Carlo simulation in small area estimation through M-quantile modeling technique in different population conditions, as well as conducting case study using national survey data and administrative data.
Throughout this paper, poverty indicator will only refer to head count index (P0), which are the proportion of poor households below poverty line, which describes poverty condition based on basic needs. This research provided an important opportunity to advance the understanding of M-quantile regression modeling as an indirect small area estimation to design the best policies to eradicate poverty as one of the world's Sustainable Development Goals.

II. METHODS AND MATERIALS

This research includes both model-based Monte Carlo simulation study and case study to to gain deeper insights to understand M-quantile regression modeling in small area estimation. The main advantage of using computer simulation was that allowed an empirical evaluation to test in which sampling condition M-quantile regression modeling was best performed. A case study was used to conduct exploratory study in real dataset to estimate poverty indicator in Bogor District with its estimated RMSE.

A. Simulation Study

In what follows, subscript j indentifies small areas (G) j = 1, 2, .., 10 and i identifies individual units in given area i = 1, 2, .., n_j. Population (N_j) and sample (nj) sizes were generated in 10 areas (G=10) ranging between $1000 \leq N_j \leq 1500$ and between $10 \leq n_j \leq 15$. Two different scenarios which contaminated area random effect and individual random effect are shown in Table I.

TABLE I
SIMULATION SCENARIOS

No	Scenarios	Area Random Effect Distribution	Individual Random Effect Distribution
1	[0,0]: no outliers	v ~ N(0, 0.15)	e ~ N(0, 0.6)
2	[0,e]: outliers in area random effect	v ~ N(0, 0.15)	e_j ~ δN(0, 0.6) + (1-δ)N(25, 50) where j=1,…,10. δ is population proportion with Pr(δ=1)=0.95

Variables of X_1 and X_2 are independently generated where x_1~uniform[0, 1] and x_2~ lognormal(1, 1).

Population data are generated differently for each scenarios by an equation as follows

$$y_{ij} = e^{15-5x_{1ij}+0.015x_{2ij}+v_i+e_{ij}} \quad (1)$$

where v_{ij} area random effect and e_{ij} is individual random effect as shown in Table 1.

Poverty line is set by z = 0.6*median(Y), with Y from the first scenario, which is then applied for both scenarios. Parameter value of P0 is calculated with the following formula

$$P_0 = \frac{1}{N}\sum_{i=1}^{q}\left[\frac{z-y_i}{z}\right]^0 \quad (2)$$

where y_i is the mean value below poverty line (i=1, 2,.., q; y_i<z), q is number of units below poverty line, and N is number of units.

Simple random sampling with replacement is used to take samples from population with 50 iterations (R=50) performed in each scenarios. In each iteration, P0 is estimated with direct estimation with formula as follows

$$\hat{P}_{0,j}^{Dir} = \frac{1}{n_j}\sum_{i=1}^{n_j} I\left(y_{ij} \le t\right) \quad (3)$$

where $\hat{P}_{0,j}^{Dir}$ is the direct estimate of P0 in j-th area, y_{ij} is per-capita expenditure in i-th unit of j-th area, t is poverty line, n_j is sampling size of j-th area, and $I\left(y_{ij} \le t\right) = 1$ if $y_{ij} \le t$ and $I\left(y_{ij} \le t\right) = 0$ if $y_{ij} > t$.

The result will be evaluated with bias and RMSE with formulas as follows

$$Bias = \sum_{j=1}^{R}\frac{\hat{P}_{0,j}^{Dir} - P_{0j}}{P_{0j}} \quad (4)$$

$$RMSE = \sqrt{\frac{\sum_{j=1}^{R}\left(\hat{P}_{0,j}^{MQ} - P_{0j}\right)^2}{R}} \quad (5)$$

where $\hat{P}_{0,j}^{Dir}$ is direct estimate of P0 in j-th area, P_{0j} is parameter of P0 in j-th area, and R is the number of iteration (R=50).

The next step is to estimate P0 with M-quantile regression modeling which algorithm function was designed by Marchetti et. al (2012) as follows.

1. Creating M-quantile model from $y_{ii} \mid x_{ij}$ (Tzavidis and Brown, 2010) $MQ_{yi}\left(q \mid x_{ij;\Psi}\right)$ = $X_{ij}{}^T\beta_\Psi\left(q\right)$ where y_{ij} is treated as per-capita expenditure in i-th unit of j- th area, $\mathbf{X}_{ij}{}^T$ was the auxiliary variables matrix (including vector 1) in i-th unit of j-th area, and $\hat{\beta}_\Psi\left(q\right)$ was the estimate of $\beta_\Psi\left(q\right)$ in certain number of q.
2. Calculating M-quantile coefficients of q_{ij} for pairs $\left(y_{ij}, X_{ij}\right)$ by interpolating sample data which fulfilled equation $y_{ij} = \boldsymbol{X}_{ij}^T\hat{\beta}_\Psi\left(q_{ij}\right)$ where $y_{ij}, \mathbf{X}_{ij} \grave{o}\ s_j$.
3. Calculating M-quantile coefficients of area with following formula.
$$\hat{\theta}_j = n_j^{-1}\sum_{i \grave{o} s_j} q_{ij} \quad (6)$$
4. Calculating the estimate of $\hat{\beta}_\Psi$ in coefficient $\hat{\theta}_j$ with following formula.
$$y_{ij} = \boldsymbol{X}_{ij}^T\beta_\Psi\left(\hat{\theta}_j\right) \quad (7)$$
5. Calculating estimation errors $e_{ij} = y_{ij} - \hat{y}_{ij}, i\grave{o}s_j$ where $\hat{y}_{ij} = \boldsymbol{X}_{ij}^T\beta_\Psi\left(\hat{\theta}_j\right)$.
6. Forming a model of $y_{kj}^* = \boldsymbol{X}_{kj}^T\hat{\beta}_\Psi\left(\hat{\theta}_j\right) + e_{ij}^*$ where e_{ij}^* was a random sample with replacement of n_j from obtained error of e_{ij} and $\mathbf{X}_{kj}$ was population matrix of auxiliary variables.
7. Running iteration process to get the estimate of $P0_j$ as follows
$$\hat{P}0_j^{*R} = \frac{1}{N_j}\sum_{i=1}^{N_j} I(y_{kj}^* \le t) \quad (8)$$

$\hat{P}0_j^{*R}$ was the estimate of P0 in j-th area of R-th iteration.

8. Calculating mean value of the estimate of P0 in each iteration with formula as follows.

$$\hat{P}0_j = \frac{1}{R}\sum_{R=1}^{50}\hat{P}0_j^{*R} \quad (9)$$

9. Calculating bias and RMSE with formulas as follows

$$Bias = \sum_{j=1}^{R}\frac{\hat{P}_{0,j}^{MQ} - P_{0j}}{P_{0j}} \quad (10)$$

$$RMSE = \sqrt{\frac{\sum_{j=1}^{R}\left(\hat{P}_{0,j}^{MQ} - P_{0j}\right)^2}{R}} \quad (11)$$

where $\hat{P}_{0,j}^{MQ}$ was the estimate of P0 with M-quantile regression modelling, P_{0j} was the parameter of P0, and R was the number of iteration (R=50).

10. Comparing the results of estimation between direct estimation and M-quantile regression modeling.

B. Application Case Study

Response variable used in this case study is per-capita expenditure from Year 2015 National Socio-economic Survey (Susenas 2015) of Republic of Indonesia which produced a distric-scaled data. The auxiliary variables are obtained from Year 2014 Village Potential (Podes 2015) administrative data which its smallest unit is a village. The selection of auxiliary variables were refered to Girinoto (2017) which contributed significantly to per-capita expenditure as written in Table II.

TABLE II.
LIST OF AUXILIARY VARIABLES

No	Codes	Variables	Unit Value
1	Y	Household per-capita expenditure	Rupiah
2	X_1	Main source of income	1 = Agriculture 0 = else
3	X_2	Numbers of electricity users from State Electricity Company	Household
4	X_3	Numbers of community health insurance for poor people recipients	Individual
5	X_4	Numbers of grocery stalls	Units

The estimation steps start with estimating P0 with direct estimation and M-quantile regression modeling which steps are the same as in simulation. RMSE of M-quantile regression modelling can be estimated with non-parametric boostrap explained as follows.

1. Sample clusters $\left(y_{ij}, X_{ij} \in s_j\right)$ are taken without replacement from finite population U.
2. M-quantile models are formed to estimate small areas using sample clusters to obtain $\hat{\theta}_j$ and $\hat{\beta}_{\Psi}\left(\hat{\theta}_j\right)$.
3. Bootstrap populations (U*b) are generated and samples from each bootstrap population are also taken by R iterations with simple random sampling technique without replacement. In each small areas $n_j^* = n_j$ are calculated the estimate of $P0_j$.
4. The estimates of bias and variance are calculated by following formulas.

$$\hat{B}\left(\hat{P}_{\alpha j}\right) = B^{-1}R^{-1}\sum_{b=1}^{B}\sum_{R=1}^{R}\left(\hat{P}_{\alpha j}^{*br} - \hat{P}_{\alpha j}^{b}\right) \quad (12)$$

$$\hat{V}\left(\hat{P}_{\alpha}\right) = B^{-1}R^{-1}\sum_{b=1}^{B}\sum_{R=1}^{R}\left(\hat{P}_{\alpha j}^{*br} - \overline{\hat{P}}_{\alpha j}^{br}\right)^2 \quad (13)$$

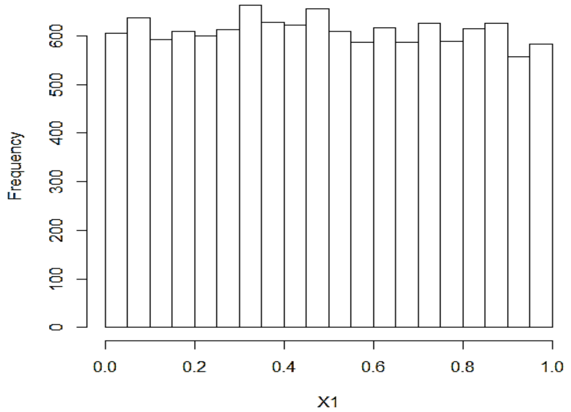
5. The estimate of RMSE is calculated with following formula.

$$RMSE\left(\hat{P}_{\alpha j}\right) = \sqrt{\hat{B}\left(\hat{P}_{\alpha j}\right)^2 + \hat{V}\left(\hat{P}_{\alpha}\right)} \quad (14)$$

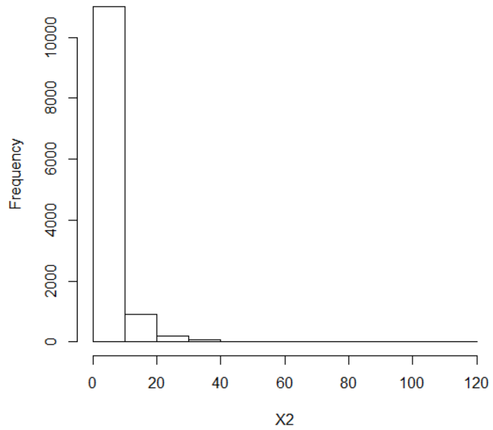
III. Results and Discussion

A. Simulation Study

Population sizes in each areas are randomly generated ranging between $1000 \leq N_j \leq 1500$; j=1,..,10 and total population obtained are 12214 units. Sample sizes in each areas are randomly generated ranging between $10 \leq n_j \leq 15$; j=1,.,10 and total samples obtained are 118 units. Auxiliary variables x_{1ij} and x_{2ij} are independently generated with U[0,1] and LN(1,1) distributed as following Figure 1.



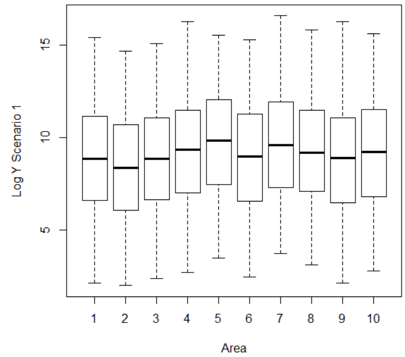
(a)



(b)

Figure 1 : Distribution of auxiliary variables with (a) uniform distribution U[0,1] and (b) lognormal distribution LN(1,1)

In each scenarios, variable Y has lognormal distribution according to the characteristics of per-capita expenditure of Bogor District. The distributions of variable Y in each scenarios is different which can be seen in the presence of outliers. Figure 1 shows that scenario 1 is the condition without outliers, whereas scenario 2 is the condition with the presence of outliers in individual level.



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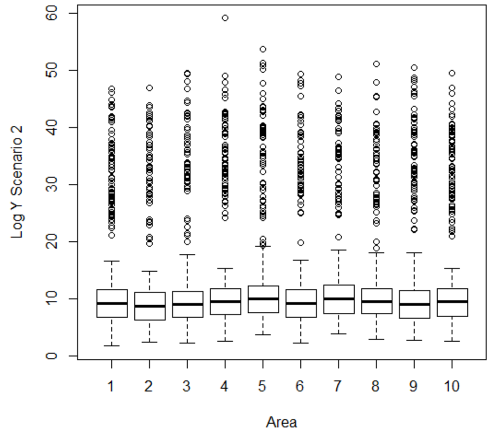

(b)

Figure 2: Distribution of response variables Y in (a) scenario 1and (b) scenario 2

Monte Carlo simulation is used to estimate P0, bias, and RMSE with 50 iterations (R=50) which estimation procedure referes to R syntax of M-quantile regression modeling by Marchetti, et.al (2012). In what follows is presented the result of simulation study using scenario 1 and scenario 2. The application case study will be discussed afterwards.

1) Results for Scenario 1: Scenario 1 is normal condition when no outliers is detected. Area random effect (v) and individual random effect (e) are randomly and independently generated with N(0, 0.15) and N(0, 0.6). Table III presents the estimation results of P0 along with its bias and RMSE in both direct estimation and M-quantile regression modeling.

TABLE III.
THE ESTIMATION OF P0, BIAS, AND RMSE IN SCENARIO 1

Are	Paramet	Direct	M-quantile

a	er	Estimation					
	P0	P0	Bias	RMSE	P0	Bias	RMSE
1	47.6	45.9	1.8	14.8	26.6	18.8	19
2	52.6	56	3.4	14.1	33.2	21.9	22
3	47.4	46.8	0.6	15.9	34.4	17.5	17.9
4	42.3	45.4	3.1	16.3	26.1	16.5	16.7
5	37.7	41.3	3.7	15.5	23.4	12.5	12.9
6	45.6	48.8	3.2	13.9	27.7	17.9	18.2
7	37.2	36	1.2	15.9	25.1	11.1	11.4
8	42.1	44.7	2.6	13.7	27.3	15.6	15.9
9	47.5	49	1.5	13.5	33.1	18.6	18.9
10	43.1	42	1.1	15.2	24.8	15.2	15.5
Mean	44.3	45.6	0.1	14.9	28.2	16.7	16.8

Based on Table III, it can be seen that the estimates of P0 with M-quantile regression modeling are very distorted from its parameter values. The estimates of bias and RMSE with M-quantile are also bigger than direct estimation. This is hypothetically an indication that M-quantile regression modeling is less suitable to be used to estimate poverty indicator (P0) in a condition where no outliers is detected.

2) Results for Scenario 2: Scenario 2 is a condition when outliers are found in each areas as individual random effects. As many 95% of population in each area are distributed normally with N(0, 0.6), while the rest of 5% of population are outliers with N(25, 50). Table IV presents the estimation results of P0 along with its bias and RMSE in both direct estimation and M-quantile regression modeling in scenario 2.

TABLE IV.
THE ESTIMATION OF P0, BIAS, AND RMSE IN SCENARIO 2

Area	Parameter	Direct Estimation			M-quantile		
	P0	P0	Bias	RMSE	P0	Bias	RMSE
1	44.5	42.3	2.2	13.3	31.7	12.8	12.8
2	49.4	50.6	1.2	15	40.5	8.9	8.9
3	44.8	43.4	1.4	18.1	39.7	5.2	5.2
4	40.4	43.2	2.8	15.9	35.5	4.9	4.9
5	35.6	39.3	3.8	16.1	43.8	8.2	8.2
6	43.9	46.6	2.7	16.4	48.9	5.1	5.1
7	37	36.7	0.3	16.1	48.9	11.9	11.9
8	40.6	42.7	2.1	13.2	52.9	12.3	12.3
9	46	49.2	3.2	14.5	56.6	10.6	10.6
10	41.4	41.1	0.4	15.7	58.5	17.1	17.1
Mean	42.4	43.5	2	15.4	45.7	9.7	9.7

Based on Table IV, it can be seen that the estimates of P0 with M-quantile regression modeling are much closer to its parameter values than scenario 1 from Table III, so that the estimates of bias are generally lower than scenario 1. The estimates of RMSE with M-quantile modeling are lower than the estimates of RMSE with direct estimation. These results suggest that M-quantile regression modelling is suitable to estimate poverty indicator (P0) when there are outliers in individual random effects.

B. Application Case Study

Bogor District is located in West Java, Indonesia, which consists of 40 sub-districts and 434 villages. Eventhough located very close to the capital city of

Indonesia, Bogor District is one of the poorest districts in West Java, so poverty still becomes the main focus of district government to be eradicated (jawapos.com, 2017). The 2015 National Socio-economic Survey (Susenas 2015) of Indonesia included 108 villages and 1119 households from all sub-districts which were assumed to be sampled with simple random sampling technique.

Codes to the 40 sub-districts were given without any consideration to ensure the neutrality of estimation. Refering to Girinoto (2017), the estimate of household population in Bogor District was 1.374.056 units. Per-capita expenditures in sub-districts level based on the survey data was graphed in Figure 3.

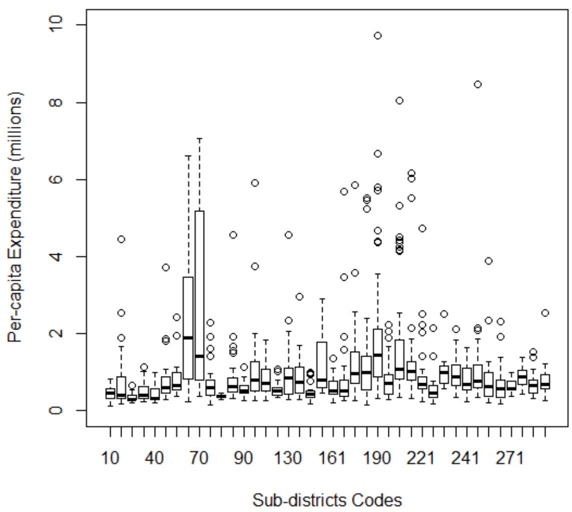

Figure 3: Distribution of per-capita expenditures of the selected survey samples on 40 sub-districts (Susenas 2015 data)

Figure 3 shows that the distribution of per-capita expenditures of selected survey samples on 40 sub-districts is not normal and there are outliers in each sub-districts. This reflects that M-quantile regression modeling can be applied to the survey data because of the robustness of this method. Poverty line used in this research was 290.874 IDR which was estimated and published by Central Bureau of Statistics as an estimated poverty line for 2015. This number becomes a baseline to estimate P0 with M-quantile regression modelling. The estimated head count index in Bogor District in 2015 was 8.96% (bogorkab.bps.go.id/).

1) The Estimation of P0 with Direct Estimation : Since direct estimation is only using data from response variable to estimate a statistical value, the RMSE value was estimated through non-parametric bootstrap method. The estimated results of P0 and RMSE with direct estimation was written in Table V. Table V reveals the fact that as many 15 sub-districts have the estimated poverty proportion of 0%, meant that there is no poor households in those sub-districts. The other 15 sub-districts are identified as having poverty proportions below the district estimate. These results indicate that most sub-districts in Bogor District are prosperous ones. However, this conclusion has to be interpreted very carefully, especially in sub-districts which have poverty proportion of 0%, because it does not necessarily reflect the actual condition. This is one of the reasons why indirect estimation is very crucial in small area estimation.

TABLE V.
THE ESTIMATION OF P0 AND RMSE IN 40 SUB-DISTRICTS WITH DIRECT ESTIMATION (SUSENAS 2015 AND PODES 2014 DATA)

Kode Kecamatan	P0		Kode Kecamatan	P0	
	P0	RMSE		P0	RMSE
10	20.69	8.32	161	16.67	8.52
20	20.51	6.36	170	13.79	6.25
21	30	14.21	180	2	2.14
30	6.67	4.23	181	3.7	3.29
40	26.32	10.91	190	0	0
50	5.41	3.73	200	0	0
51	0	0	210	0	0
60	6.67	4.4	220	0	0
70	0	0	221	3.57	3.39
71	6.67	4.55	230	20	8.54
80	12.5	10.16	231	0	0

81	0	0	240	0	0
90	5.26	5.49	241	5.88	6.31
100	3.7	4.46	250	0	0
110	3.57	3.17	260	5.56	5.56
120	0	0	270	15.38	6.35
130	2.38	2.32	271	0	0
140	10	9.49	280	0	0
150	2.4	6.88	290	0	0
160	0	0	300	5.26	5.18

2) The Estimation of P0 with M-quantile : In an indirect estimation, auxiliary variables from the same or different sources of data are needed to support the estimation of statistical value. From Podes 2014 data as auxiliary variables, it is assumed that all households in a village have the same number, so population matrix of auxiliary variables can be formed. The estimated results of P0 and RMSE with indirect estimation as M-quantile regression modeling is written in Table VI as follows.

TABLE VI.
THE ESTIMATION OF P0 AND RMSE IN 40 SUB-DISTRICTS WITH M-QUANTILE MODELLING (SUSENAS 2015 AND PODES 2014 DATA)

Kode Kecamatan	P0		Kode Kecamatan	P0	
	P0	RMSE		P0	RMSE
10	19.9	5.2	161	16.8	7.1
20	20.5	5.5	170	13.5	6
21	41.8	14.4	180	2.2	1.6
30	25.4	5.6	181	5.2	5.7
40	15	6.5	190	0	1.3
50	11.6	6	200	6.4	8.4
51	12.8	9.6	210	1	2.8
60	4.5	4.4	220	1.6	2.3
70	0.1	1.8	221	5.4	7.4
71	12.2	5.1	230	20.9	12.9
80	39.2	17.8	231	2.1	10.5
81	9.2	5.9	240	3.9	4.2
90	16.7	5.4	241	8.6	8.7
100	5.5	5.6	250	4.8	3.3
110	10.5	5.9	260	15.5	8.2
120	20.6	12.9	270	17	6
130	6.3	3.4	271	21.3	10.3
140	3.6	7.7	280	6.7	10.8
150	34.8	5.8	290	12.8	6.5
160	3.6	9.7	300	10	7

Table VI shows that most sub-districts are identified as having poverty proportions above the district estimate. This is contradictive to the direct estimation which informed otherwise. Moreover, with M-quantile regression modeling, poverty proportion in some sub-districts are able to be estimated which claimed to be 0% by direct estimation.

3) Evaluation of Both Estimations :

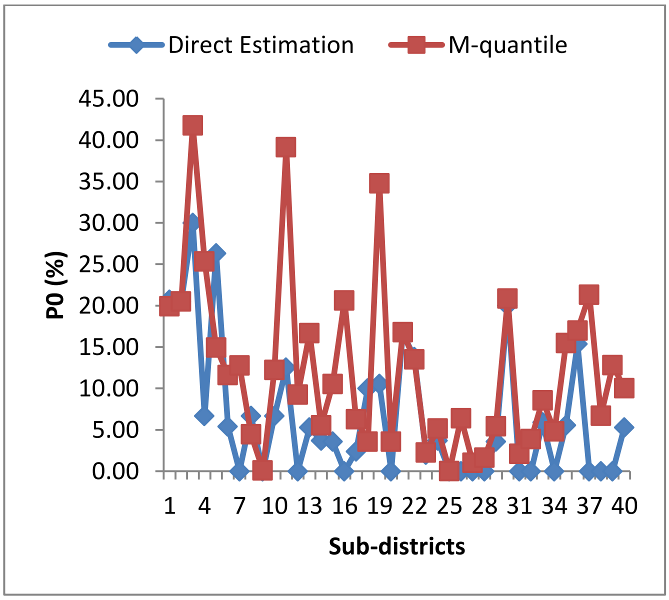

Figure 3 : Distribution of per-capita expenditures of the selected survey samples on 40 sub-districts (Susenas 2015 data)

Figure 4 is the comparison of estimated P0 between direct estimation and M-quantile regression modeling. It shows that M-quantile regression modeling as indirect estimation in small areas were able to estimate P0 which claimed to be 0% by direct estimation. This result indicates the usefulness of auxiliary variables in making indirect estimation in small areas, especially using M-quantile regression modeling.

IV. CONCLUSION

This paper sets out to apply Monte Carlo simulation in small area estimation through M-quantile regression modeling technique in different population conditions, as well as conducting case study using survey data and administrative data. This study has found that M-quantile regression modeling generally can be used to estimate head count index (P0) in small areas. M-quantile regression modeling is best used in the condition of data which tend to have outliers in individual random effects.

As the most obvious finding in this research, M-quantile regression modeling is able to produce P0 estimation which is closer to its parameter value when there are outliers in each areas. The estimates of RMSE with M-quantile are also generally smaller than direct estimation when outliers are present. This indicates that M-quantile regression modeling is more stable in estimating P0.

In application case study, the estimation result of direct estimation has to be interpreted very carefully because it does not necessarily depict the actual condition of some sub-districts. In fact, by using auxiliary variables from other data source, M-quantile regression modeling is able to estimate poverty proportion in some sub-districts which claimed to be 0% by direct estimation.

However, the estimation has not been applied to a more complex poverty indicator as poverty severity index or P2 because of computational limitation of an R syntax function by Marchetti et.al (2012). This research is also limited by the consideration of geographically weighted variable of Indonesia's Central Bureau of Statistics, as well as the sampling technique which supposed to be multistage sampling. Greater efforts are needed for the next researchers who wanted to study M-quantile regression modeling in small areas to improve their work by considering these limitations.

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